Application of fuzzy set theory in the selection of underground mining method

by A. Karadogan*, A. Kahriman* and U. Ozer*

Synopsis

Decision making can be defined as the selection process of the best routes from the alternatives in order to achieve a goal, and mostly involves uncertainty. Additionally, mostly qualitative variables (weak rock, massive ore deposit, etc.) are in question. Mine planning engineers often use their intuition and experience in decision making. As fuzzy set theory has been used since 1970, these uncertainties are easily evaluated in the decision-making process. Real world study is decision making under vague constraints of different importance, involving uncertain data (qualitative variables), where compromises between antagonistic criteria are allowed.

This paper presents a new approach in the selection of an underground mining method based on fuzzy set theory for the Ciftalan Lignite Site located close to Istanbul in Turkey. The physical parameters such as geology and the geotechnical properties of ore, hanging and foot wall, economic effects, environmental effects, are established with field and laboratory tests together with the determination of other qualitative variables. Meanwhile, some qualitative variables dealing with the matter was described according to the view of a number of experts. Then fuzzy set theory is applied to these parameters, considering the available underground methods in order to choose the proper method. At the end of the evaluations, the room and pillar method with filling was determined as the most suitable method for the test site.

Keywords: Decision making, fuzzy set theory, underground, mining, method selection

Introduction

Available deposits should be evaluated carefully in an optimum manner. In the process, the selection of the most appropriate underground mining method is of great importance from the economic, technique, and safety points of view. In the method selection process, many controllable and uncontrollable parameters should be taken into account. Therefore, these parameters must be obtained with scientific and technical studies for each ore deposit.[1,2]

Up to now, research dealing with underground mining method selection was carried out by many scientists such as Boskhov-Wright (1973), Hartman (1987), Laubscher (1981), Nicholas-Narek (1981).[3,4] Classical method evaluations generally produce a complex situation and take a long period of time. The decision-making process is made harder by the question of many parameters and the uncertain elements of underground mining method selection. New methods for decision making processes have enabled decision-makers to decide more quickly, easily and sensitively.

Fuzzy set theory

Fuzzy logic is an appropriate methodology for investigating a number of problems characterized by unreliable data, imprecise measures, ambiguous language and unclear decision rules. Over nearly the past three decades, fuzzy logic has been advanced as a formal means of handling the implicit imprecision in a wide range of problems, e.g. in industrial control, military operations, economics, engineering, medicine, reliability, pattern recognition and classification.[5,6]

Mine planning engineers often use their intuition and experience in decision making. Decision makers, however, may not know how to deal with quantative variables such as weak rock or ore with massive dimensions. The uncertainties in question can be easily evaluated by applying fuzzy set theory in the decision making process.[7]

A fuzzy set can be described as a value appointment that shows membership degree in the fuzzy set to whatever an existence in pronunciation universe as mathematical.[8] The membership degree shows the adaptation degree to the properties that are described by

* *Mining Engineering Department, Istanbul University, Turkey.*

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the fuzzy set of this existence. There is not a exact definition between elements of the fuzzy set and outside remaining of these elements. Thus, a fuzzy set can be described as concepts/objects having uncertain limits between them. Namely, Fuzzy set theory is interested in fuzziness that appears within these uncertain limits. Old people, high temperature and small numbers can be given as examples of this fuzziness.[9]

In classical set theory, an element is either a member of a set or not. There is no partial membership. This situation can be explained with an example given in Figure 1. In this example, the temperatures below 30°C are not considered as hot. According to this, even 29.5°C is not considered as hot. Therefore, there is no flexibility in this logic.

In the real world, these limits are not as sharp as that.[10] It is required that events should have a certain flexibility. The sharp limits of bilateral variables such as cold-hot, fast-slow, high-low in classical sets is softened by fuzzy logic with the application of flexible characterizations such as little cold-little hot, little high-little low. Figure 2 shows fuzzy set theory that gives the value of variables such as temperature close to the value observed in the real world. According to this, the membership degree of the hot fuzzy set is revealed for the temperature values between 20°C and 40°C. The membership degree decreases gradually from 1 at 30°C to 0 at 20°C. According to Figure 2, little-hot state will appear as the temperature decreases. For example, 25°C temperature is qualified as little hot while 30°C temperature is qualified as very hot and 20°C temperature is not qualified as hot. Therefore, 20°C temperature is not an element of the hot fuzzy set.[10]

The next step of fuzzy set theory is given in Figure 3. In this Figure, hot fuzzy set membership degree gains cold fuzzy set membership identity at 0.5. The degree of the cold fuzzy set membership increases when temperature decreases. According to this, temperature between 0 and 15°C is qualified as very cold and this part has cold fuzzy set entire membership. There is a gradual membership of cold fuzzy set for the temperature between 15°C and 25°C. It appears that there is an intersection of cold and hot fuzzy sets for the temperature between 20°C and 25°C. Elements in this part can be taken as members of either the hot or the cold fuzzy set[10].

Although these examples are valid for the non-fuzzy inputs, sometimes inputs can be fuzzy in fuzzy logic. In this situation, fuzzy set membership degree is determined from the area being covered between fuzzy set and fuzzy input value. This situation is shown in Figure 4 and membership degree is about 0.3.

Fuzzy multiple attribute decision-making

Decision-making is explained as a selection process of the best alternative from alternative sets in order to reach an aim or aims. There are many methods of decision-making. The focus of this paper is on Yager's method, which is one of the methods of fuzzy multiple attribute decision-making. This method is also based on the Analytical Hierarchy Process (AHP).[11]

AHP, developed by Thomas Saaty (1988), has been used successfully in practice for the solutions of the various problems related to decision-making and planning. This technique has also been used to solve the problems of

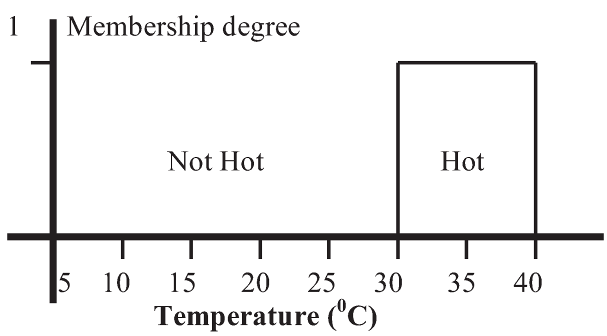

Figure 1—Classical set theory

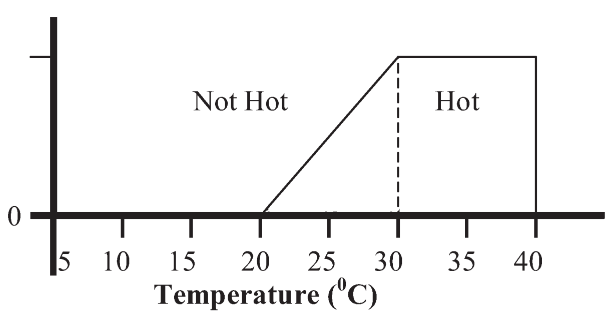

Figure 2—Fuzzy set theory

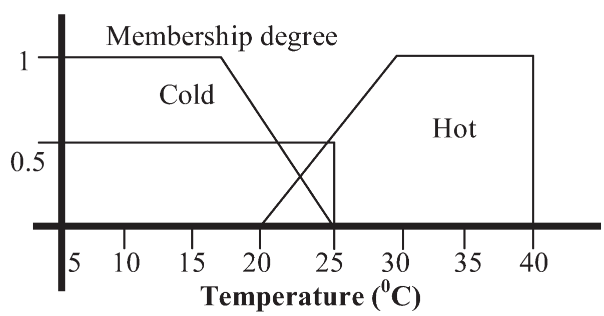

Figure 3—Being covered in fuzzy set

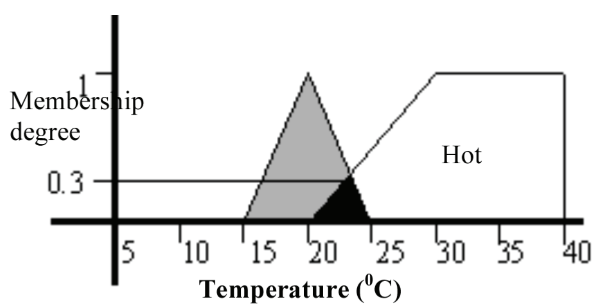

Figure 4—Membership degree of the fuzzy input values

decision-makers in different areas such as politics, defence, town planning, communication and psychology. AHP is based on the fact that problems can be solved more effectively by dividing the problems into plausible and smaller sub-parts. This process is formed by the following stages.[12]

- ➤ Clear description of the problem
- ➤ The determination of sub-aims
- ➤ The determination of the factors affecting sub aims
- ➤ Analysis of the model results according to alternatives.

The development by Saaty of a mathematical analysis of pairwise comparisons of user responses has continued to exert a profound influence on computer applications designed to enhance interactive human cognition. Saaty discovered that if users are asked to measure the contribution of one or more decision elements to two or more potential decisions, and if the decision element evaluations are entered pairwise in a two-dimensional matrix as they are obtained from the user, then the response matrix can be solved for its principal eigenvalue and for the eigenvector containing this principal eigenvalue. These mathematical solutions yield direct estimates of data consistency within the response matrix, and a normalized estimate of the contribution of each decision element to each potential decision. The use of the eigenvector method also preserves the ordinal rank of each of the decision elements when data within the response matrix are incomplete or inconsistent, a situation often encountered during the selection evaluation of a system[6].

The practical effect of such matrix cognition during the selection evaluation is to record the user's choices of potential decisions for consideration, the user's choices of decision elements for analysis, and the user's choices of consultants for participation in the analysis. Each consultant is then asked to estimate the quantitative contribution of each decision element to each potential decision, and this estimate is then weighted by the calculated eigenvector obtained by solving the response matrix of each consultant. Each consultant's quantitative scaling of each potential decision is then calculated, ranked in order, and combined with the results of the other consultants to yield a consensus best decision and ranking of alternative decisions[6].

Formally, let $A = \{A_1, A_2, \ldots, A_n\}$ be the set of alternatives, $C = \{C_1, C_2, \ldots, C_m\}$ be the set of criteria, which can be given as fuzzy sets in the space of alternatives, and G the goal, which can also be given by a fuzzy set. Firstly, the membership degrees of alternatives for each criterion are determined by expert views.

Consider a simple example:

$$G = [0.5/A_1,\ 0.8/A_2,\ 0.3/A_3] \quad [1]$$

$$C_1 = [0.7/A_1,\ 0.9/A_2,\ 0.5/A_3] \quad [2]$$

$$C_2 = [0.4/A_1,\ 0.2/A_2,\ 0.9/A_3] \quad [3]$$

For the criterion weights to be determined, we used the judgement scale (Table I) determined by decision-maker.

Yager suggests the use of Saaty's method for pairwise comparison of the criteria (attributes). A pairwise comparison of attributes (criteria) could improve and facilitate the assessment of criteria importance. Saaty developed a procedure for obtaining a ratio scale for the elements compared[14]. To assess the importance the decision-maker (w) is asked to judge the criteria in pairwise comparisons and the values assigned are $w_{ij} = 1/w_{ij}$. Where i is the column number and j is the line number in m x m matrix. Having obtained the judgements, the m x m matrix B is constructed so that: (a) $b_{ii} = 1$; (b) $b_{ij} = w_{ij}$; (c) $b_{ji} = b_{ij}$[14]. To sum up, Yager suggests that the resulting eigenvector should be used to express the decision-maker's empirical estimate of importance. The reciprocal matrix in which the values are given by the decision-maker for each criteria in the decision and criteria 1 and 2, respectively C_1 and C_2, are three times as important as G, and the pairwise comparison reciprocal matrix is:

Table I

Judgement scale determined by decision-maker

Numerical scale	Meanings
1	Equally important
3	Weakly more important
5	Strongly more important
7	Demonstrably more important
9	Absolutely more important
2, 4, 6 , 8	Compromise judgements

$$\begin{array}{c} \\ G \\ C_1 \\ C_2 \end{array} \begin{array}{c} \begin{array}{ccc} G & C_1 & C_2 \end{array} \\ \begin{bmatrix} 1 & 1/3 & 1/3 \\ 3 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix} \end{array}$$

Hence, the eigenvalues of the reciprocal matrix are $\lambda = [0, 3, 0]$ and therefore $\lambda_{max} = 3$. All values except one are zero (as stated in Saaty[14]). The weights of the criteria are finally achieved in the eigenvector of the matrix,

$$\text{Eigenvector} = \begin{bmatrix} 0.299 \\ 0.688 \\ \end{bmatrix} \text{ with } \lambda_{max}. \quad [4]$$

The eigenvector corresponds to the weights to be associated with the memberships of each attribute/feature/goal. Thus, the exponential weighting is $\alpha_1=1/3$, $\alpha_2=2/3$, $\alpha_3=2/3$ and the final decision (membership decision function) about the site location is given as follows:

$$\mu_D(A) = D(A) = \min \{\mu_G(X),\ \mu_{C1}(X),\ \mu_{C2}(X), \ldots\ldots, \mu_{CM}(X)\} \quad [5]$$

There are some cases where the importance of criteria is not equally alike and weighting coefficients are required. The decision function with the relative importance of criteria, omitting the membership signal μ for simplification, is[13]:

$$D = \min(\omega_0 G, \omega_1 C_1, \omega_2 C_2, \ldots\ldots\ldots, \omega_m C_m) \text{ with } \sum_{j=1}^{m} \omega = 1 \quad [6]$$

Thus, the fuzzy decision function in the example is

$$D(A) = \min \{ G^{1/3}, C_1^{2/3}, C_2^{2/3} \} \quad [7]$$

$G = [0.5/A_1, 0.8/A_2, 0.3/A_3]^{0.229} = [0.85/A_1, 0.95/A_2, 0.76/A_3]$

$C_1 = [0.7/A_1, 0.9/A_2, 0.5/A_3]^{0.688} = [0.78/A_1, 0.93/A_2, 0.62/A_3]$

$C_2 = [0.4/A_1, 0.2/A_2, 0.9/A_3]^{0.688} = [0.53/A_1, 0.33/A_2, 0.93/A_3$

$$D(A) = \{ 0.53/A_1, 0.33/A_2, 0.62/A_3 \} \quad [8]$$

and the optimal solution, corresponding to the maximum membership 0.62, is A_3 $(D(A^*)$

$$D(A) = 0.62/A_3). \quad [9]$$

Case study

This study is based on the optimum underground mining method selection for the Çiftalan lignite site located 35 km. north of Istanbul, Turkey. The location of the test site is shown in Figure 5.

Although all open pit mines operated at the sites surrounding the Çiftalan lignite site had stopped their lignite production, the Çiftalan lignite site had stayed untouched and no coal has ever been produced at this site. It was understood that the company had planned to extract the coal by open pit mining despite the fact that the profitable coal seam lies under the Çiftalan village at a depth of 55 m. A major proportion of the land above the coal seam was owned by the company (approximately 2 217 000 m^2). However, the rest of the area was owned by the village people and village common public property (approximately 172 000 m^2). The company could not extract the coal by open pit mining because of the fact that the village refused to sell these properties to the company. This situation still continues. This leaves the company with no option but underground mining. However, The Kutman Company did not have any plans to extract the coal by underground mining. The goal of this study is to select the most suitable underground mining method for this site with a new approach by applying fuzzy set theory.

In accordance with the main objective of this study, the needed physical parameters such as geologic and geotechnical properties of ore, hanging- and footwall, economic effects, environmental effects, which are established by field and laboratory tests, together with uncertain variables, were determined. Meanwhile some uncertain variables dealing with the matter were described according to the view of some experts. The generated parameters, which are needed for the method selection, are given briefly in Table II, together with related criterion.

The optimum method was selected from among the methods listed in Table III. These methods were defined as more applicable methods for these conditions at the end of the preliminary election considering the expert comments.

Figure 5—Location of the Çiftalan lignite site in Turkey

Table II

Technical parameters determined as selection criterion

Criterion	Description
Geometric shape of the lignite deposit	Plate state (layered)
Thickness of the lignite seam	2.3 m (average)
Seam inclination	5° (average)
Excavation dept	55 m (average)
Soundness degree of the lignite	Low strength (compressive strength: 41 MPa)
Contact state of the lignite seam-hanging and footwall	Not clear
Soundness degree of the hangingwall	Hangingwall is marl-clay-sand. Low strength (compressive strength: 28 MPa)
Soundness degree of the footwall	Footwall is clay. Low strength (shear strength: 2.2 kg/cm^2)
Subsidence effect	Seam is near to the surface (55 m) and hangingwall is low strength (compressive strength: 28 MPa). Therefore, there is a subsidence risk
Support necessity	However, hangingwall and footwall became low strength, support is necessary
Nearness of the settlement areas	There is a Çiftalan village over the lignite seam (55 m)
Burning property of the lignite	The lignite has burning property by itself and burning property is high.
Hydraulic conditions	There is the Black Sea at the north of lignite site (about 500 m) and there is a water problem. Because elevation of the seam is about -10

Some of the qualitative results were produced from varied solution methods such as linear programming, expert systems, expert views, etc. Each alternative (method) has shown its advantages. In this case, it did not appear that an easy solution to the problem will be obtained. From the solution point of view, application of the fuzzy set theory would be a proper choice and, therefore, used in this paper. It was given some of the analysis that was determined by experts below:

- ➤ According to dimension, A_3 is the best method.
- ➤ Seam thickness averages 2.3 m. Therefore, A_3 is the best method.
- ➤ According to seam inclination, A_5 is appropriate.
- ➤ According to the soundness degree of the hangingwall, A_5 is more appropriate than other methods.
- ➤ From the viewpoint of appropriateness, hydraulic condition, A_1 is the best method.

Table III

Alternative underground mining methods

A1	Longwall methods with filling (direction of inclination rising)
A2	Longwall methods with filling (direction of inclination decline)
A3	Longwall method with filling (progressed)
A4	Longwall method with filling (returned)
A5	room and pillar method with filling

The selection criteria are presented in Table IV and the optimum underground mining method selection procedure is given below.

Let $A = \{A_1, A_2, A_3, A_4, A_5\}$ be the set of alternative systems and $C = \{C_1, C_2, C_3, \ldots, C_{18}\}$ be the set of criteria. The decision-maker is then asked to define the membership degree of each criterion that is conferred with experts on this subject. Following that procedure the membership degree of each criterion is given below in detail:

$$C_1 = \{0.80/A_1, 0.75/A_2, 0.95/A_3, 0.90/A_4, 0.85/A_5\}$$
$$C_2 = \{0.75/A_1, 0.80/A_2, 0.88/A_3, 0.85/A_4, 0.82/A_5\}$$
$$C_3 = \{0.70/A_1, 0.65/A_2, 0.87/A_3, 0.85/A_4, 0.92/A_5\}$$
$$C_4 = \{0.70/A_1, 0.75/A_2, 0.90/A_3, 0.80/A_4, 0.65/A_5\}$$
$$C_5 = \{0.55/A_1, 0.60/A_2, 0.70/A_3, 0.75/A_4, 0.85/A_5\}$$
$$C_6 = \{0.50/A_1, 0.55/A_2, 0.65/A_3, 0.75/A_4, 0.85/A_5\}$$
$$C_7 = \{0.70/A_1, 0.65/A_2, 0.85/A_3, 0.75/A_4, 0.90/A_5\}$$
$$C_8 = \{0.40/A_1, 0.50/A_2, 0.70/A_3, 0.80/A_4, 1.00/A_5\}$$
$$C_9 = \{0.65/A_1, 0.75/A_2, 0.85/A_3, 0.60/A_4, 0.95/A_5\}$$

Table IV

Parameters taken into consideration for underground mining method selection

Criterion	Selection parameters	Criterion	Selection parameters
C1	Geometric shape of the lignite deposit	C10	Support necessity
C2	Thickness of the lignite seam	C11	Nearness of the settlement areas
C3	Seam inclination	C12	Burning property of the lignite
C4	Excavation dept	C13	Methane existence
C5	Soundness degree of the lignite	C14	Hydraulic conditions
C6	Contact state of the lignite seam-hanging and footwall	C15	Mining cost
C7	Soundness degree of the hangingwall	C16	Capital cost
C8	Soundness degree of the footwall	C17	Production ratio
C9	Subsidence effect	C18	Labour cost

$C_{10} = \{ 0.60/A_1, 0.55/A_2, 0.85/A_3, 0.65/A_4, 0.80/A_5 \}$
$C_{11} = \{ 0.80/A_1, 0.75/A_2, 0.90/A_3, 0.65/A_4, 0.95/A_5 \}$
$C_{12} = \{ 0.78/A_1, 0.70/A_2, 0.90/A_3, 0.75/A_4, 0.65/A_5 \}$
$C_{13} = \{ 0.50/A_1, 0.72/A_2, 0.80/A_3, 0.60/A_4, 0.85/A_5 \}$
$C_{14} = \{ 0.85/A_1, 0.45/A_2, 0.75/A_3, 0.60/A_4, 0.50/A_5 \}$
$C_{15} = \{ 0.60/A_1, 0.55/A_2, 0.80/A_3, 0.70/A_4, 0.95/A_5 \}$
$C_{16} = \{ 0.60/A_1, 0.55/A_2, 0.80/A_3, 0.65/A_4, 0.90/A_5 \}$
$C_{17} = \{ 0.75/A_1, 0.70/A_2, 0.82/A_3, 0.80/A_4, 0.90/A_5 \}$
$C_{18} = \{ 0.65/A_1, 0.70/A_2, 0.80/A_3, 0.75/A_4, 0.60/A_5 \}$

Additionally, the m x m matrix (Figure 6) was constructed to express the decision-makers' empirical estimate of importance for each criterion. Then, the maximum eigenvector was obtained using Matlab.[15] The judgement scale used here is: 1 equally important; 3 weakly more important; 5 strongly more important; 7 demonstrably more important, 9 absolutely more important.

It turns out that the maximum eigenvalue of the reciprocal matrix is $\lambda = 21.3488$. The weights of the criteria are finally obtained in the eigenvector of the matrix.

Eigenvector = {0.3543, 0.4855, 0.5369, 0.1637, 0.3069, 0.1298, 0.2990, 0.2320, 0.0782, 0.0450, 0.0430, 0.0777, 0.0313, 0.0288, 0.1564, 0.1600, 0.0353, 0.0197} λ_{max}

The eigenvector corresponds to the weights to be associated with the memberships of each attribute/feature/goal. Thus, the exponential weighting is $\alpha_1 = 0.3543$, $\alpha_2 = 0.4855$, $\alpha_3 = 0.5369$, $\alpha_4 = 0.1637$, $\alpha_5 = 0.3069$, $\alpha_6 = 0.1298$, $\alpha_7 = 0.2990$, $\alpha_8 = 0.2320$, $\alpha_9 = 0.0782$, $\alpha_{10} = 0.0450$, $\alpha_{11} = 0.0430$, $\alpha_{12} = 0.0777$, $\alpha_{13} = 0.0313$, $\alpha_{14} = 0.0288$, $\alpha_{15} = 0.1564$, $\alpha_{16} = 0.1600$, $\alpha_{17} = 0.0353$, $\alpha_{18} = 0.0197$ and the final decision is obtained as follows:

$C_1 = \{ 0.80/A_1, 0.75/A_2, 0.95/A_3, 0.90/A_4, 0.85/A5 \}^{0.3543}$
$= \{ 0.92/A_1, 0.90/A_2, 0.98/A_3, 0.96/A_4, 0.94/A5 \}$
$C_2 = \{ 0.75/A_1, 0.80/A_2, 0.88/A_3, 0.82/A_4, 0.85/A5 \}^{0.4855}$
$= \{ 0.87/A_1, 0.90/A_2, 0.94/A_3, 0.91/A_4, 0.92/A5 \}$
$C_3 = \{ 0.70/A_1, 0.65/A_2, 0.87/A_3, 0.85/A_4, 0.92/A5 \}^{0.5369}$
$= \{ 0.83/A_1, 0.79/A_2, 0.93/A_3, 0.92/A_4, 0.96/A5 \}$
$C_4 = \{ 0.70/A_1, 0.75/A_2, 0.90/A_3, 0.80/A_4, 0.65/A5 \}^{0.1637}$
$= \{ 0.94/A_1, 0.95/A_2, 0.98/A_3, 0.96/A_4, 0.93/A5 \}$
$C_5 = \{ 0.55/A_1, 0.60/A_2, 0.70/A_3, 0.65/A_4, 0.85/A5 \}^{0.3069}$
$= \{ 0.83/A_1, 0.85/A_2, 0.89/A_3, 0.87/A_4, 0.95/A5 \}$
$C_6 = \{ 0.50/A_1, 0.55/A_2, 0.65/A_3, 0.75/A_4, 0.85/A5 \}^{0.1298}$
$= \{ 0.91/A_1, 0.92/A_2, 0.94/A_3, 0.97/A_4, 0.98/A5 \}$
$C_7 = \{ 0.70/A_1, 0.65/A_2, 0.85/A_3, 0.75/A_4, 0.90/A5 \}^{0.2990}$
$= \{ 0.90/A_1, 0.88/A_2, 0.95/A_3, 0.92/A_4, 0.97/A5 \}$
$C_8 = \{ 0.40/A_1, 0.50/A_2, 0.70/A_3, 0.80/A_4, 1.00/A_5 \}^{0.2320}$
$= \{ 0.81/A_1, 0.85/A_2, 0.92/A_3, 0.97/A_4, 1.00/A_5 \}$
$C_9 = \{ 0.65/A_1, 0.75/A_2, 0.85/A_3, 0.60/A_4, 0.95/A_5 \}^{0.0782}$
$= \{ 0.97/A_1, 0.98/A_2, 0.99/A_3, 0.96/A_4, 1.00/A_5 \}$
$C_{10} = \{ 0.60/A_1, 0.55/A_2, 0.85/A_3, 0.65/A_4, 0.80/A_5 \}^{0.0450}$
$= \{ 0.97/A_1, 0.97/A_2, 0.99/A_3, 0.98/A_4, 0.99/A_5 \}$
$C_{11} = \{ 0.80/A_1, 0.75/A_2, 0.90/A_3, 0.65/A_4, 0.95/A_5 \}^{0.0430}$
$= \{ 0.99/A_1, 0.99/A_2, 0.99/A_3, 0.98/A_4, 1.00/A_5 \}$
$C_{12} = \{ 0.78/A_1, 0.70/A_2, 0.90/A_3, 0.75/A_4, 0.65/A_5 \}^{0.0777}$
$= \{ 0.98/A_1, 0.97/A_2, 0.99/A_3, 0.98/A_4, 0.97/A_5 \}$
$C_{13} = \{ 0.50/A_1, 0.72/A_2, 0.80/A_3, 0.60/A_4, 0.85/A_5 \}^{0.0313}$
$= \{ 0.98/A_1, 0.99/A_2, 0.99/A_3, 0.98/A_4, 1.00/A_5 \}$
$C_{14} = \{ 0.85/A_1, 0.45/A_2, 0.75/A_3, 0.60/A_4, 0.50/A_5 \}^{0.0288}$
$= \{ 1.00/A_1, 0.98/A_2, 0.99/A_3, 0.99/A_4, 0.98/A_5 \}$
$C_{15} = \{ 0.60/A_1, 0.55/A_2, 0.80/A_3, 0.70/A_4, 0.95/A_5 \}^{0.1564}$
$= \{ 0.92/A_1, 0.91/A_2, 0.97/A_3, 0.95/A_4, 0.99/A_5 \}$
$C_{16} = \{ 0.60/A_1, 0.55/A_2, 0.80/A_3, 0.65/A_4, 0.90/A_5 \}^{0.1600}$
$= \{ 0.92/A_1, 0.91/A_2, 0.96/A_3, 0.93/A_4, 0.98/A_5 \}$
$C_{17} = \{ 0.75/A_1, 0.70/A_2, 0.82/A_3, 0.80/A_4, 0.90/A_5 \}^{0.0353}$
$= \{ 0.99/A_1, 0.99/A_2, 0.99/A_3, 0.99/A_4, 1.00/A_5 \}$
$C_{18} = \{ 0.65/A_1, 0.70/A_2, 0.80/A_3, 0.75/A_4, 0.60/A_5 \}^{0.0197}$
$= \{ 0.99/A_1, 0.99/A_2, 1.00/A_3, 0.99/A_4, 0.99/A_5 \}$

$D(A) = \min \{ 0.81/A_1, 0.79/A_2, 0.89/A_3, 0.87/A_4, 0.92/A_5 \}$

and the optimal solution, corresponding to the maximum membership 0.92, is A_5 ($D(A^*)$

$D(A^*) = 0.92/A_5$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}
C_1	1	1/3	1/3	3	1	7	3	3	3	5	9	5	9	7	5	5	7	9
C_2	3	1	1	7	3	5	3	3	5	7	9	7	9	9	5	5	7	9
C_3	3	1	1	7	5	7	3	3	7	9	9	7	9	9	5	5	7	9
C_4	1/3	1/7	1/7	1	1/7	1	1/7	1/7	5	7	7	5	5	5	3	3	3	7
C_5	1	1/3	1/5	7	1	5	1	1	7	7	7	7	7	7	3	3	5	9
C_6	1/7	1/5	1/7	1	1/5	1	1/7	1/7	3	7	1	1/5	5	5	3	3	5	7
C_7	1/3	1/3	1/3	7	1	7	1	1	5	7	7	3	7	9	3	3	9	9
C_8	1/3	1/3	1/3	7	1	7	1	1	3	5	5	1	7	7	1/3	1/3	7	9
C_9	1/3	1/5	1/7	1/5	1/7	1/3	1/5	1/3	1	5	3	3	3	3	1/3	1/3	3	5
C_{10}	1/5	1/7	1/9	1/7	1/7	1/7	1/7	1/5	1/5	1	3	1/3	3	1	1/5	1/5	3	5
C_{11}	1/9	1/9	1/9	1/7	1/7	1	1/7	1/5	1/3	1/3	1	1/5	3	3	1/3	1/5	1	5
C_{12}	1/5	1/7	1/7	1/5	1/7	1/5	1/3	1	1/3	3	5	1	3	5	1/3	1/3	3	7
C_{13}	1/9	1/9	1/9	1/5	1/7	1/5	1/7	1/7	1/3	1/3	1/3	1/3	1	1	1/7	1/7	3	3
C_{14}	1/7	1/9	1/9	1/5	1/7	1/5	1/9	1/7	1/3	1	1/3	1/5	1	1	1/5	1/5	1/3	3
C_{15}	1/5	1/5	1/5	1/3	1/3	1/3	1/3	3	3	5	3	3	7	5	1	3	5	7
C_{16}	1/5	1/5	1/5	1/3	1/3	1/3	1/3	3	3	5	5	3	7	5	3	1	5	7
C_{17}	1/7	1/7	1/7	1/3	1/5	1/5	1/9	1/7	1/3	1/3	1	1/3	1/3	3	1/5	1/5	1	3
C_{18}	1/9	1/9	1/9	1/7	1/9	1/7	1/9	1/9	1/5	1/5	1/5	1/7	1/3	1/3	1/7	1/7	1/3	1

Figure 6—Criteria comparison matrix

At the end of the evaluations, the room and pillar method with filling (A_5) was determined to be the most suitable method with 0.92 membership degree as an optimum underground mining method for the Çiftalan lignite Site. Other methods with lower membership degrees than A5 have lowered the chosen probability.

To reach the result of this study, we benefited from expert views through a questionnaire to determine the alternative methods and criteria weights. When experts were asked for the optimum method for application to this site, most of experts suggested the room and pillar method with filling.

Conclusions

The development of computer technology and programming of colloquial language with expert systems has considerably reduced the decision-maker's burden. With regard to classic methods, decision-making with fuzzy set theory enabled one to reach the aim in a quicker, easier, and more sensitive way.

This paper has discussed decision-making in a fuzzy environment (uncertain qualitative data variables involved in the systems) for solving multiple attribute problems of optimum underground mining method selection. Since the focus is on fuzzy multiple attribute problems, a detailed discussion of the most important methods for solving these problems was presented.

At the end of the evaluations, the room and pillar method with filling (A_5) was determined to be the most suitable underground mining method for the Çiftalan lignite site. Results of this study used to determine underground mining method selection with fuzzy set theory will show the way to the most appropriate method selection studies at other mines.

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