An Image Encryption Based on Confusion-Diffusion Using Two Chaotic Maps and Frobenius Endomorphism

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***Abstract*— The chaotic map including 2D-Henon and Tent maps, have been widely used in modern cryptography, due to its high level of security and low cost of computation, compared to traditional algorithms. This study suggests a method for sharing images that are dependent on both confusion and diffusion for securing a gray-scale image. First, a new perturbation method of pixels based on Henon map to generate the permutation matrix for scrambling the plain image is proposed. The confused image should be encrypted, in which we combine the Frobenius endomorphism and Tent chaotic map to generate a key encryption. Using the eXclusive-OR operation, the substitution process is realized between the key encryption and confused image. The simulation results of the proposed scheme maintain a lossless encryption quality, and the security analysis, which includes differential attack, entropy, correlation coefficient, and histogram analysis illustrate that the proposed approach reveals a high performance.**

***Index Terms*—2D-Henon map, Chaotic cryptography, Frobenius endomorphism, Image encryption, Tent map**

I. Introduction

With the advent of both public and hybrid cloud, ensuring a secure and reliable sharing of secret image information has become a challenge for security managers. In cryptography, most research papers improve the system security to comply with the current image encryption standards, which call for a lot of redundant information, a lot of data, and a significant of correlation [1]. In general, the encryption process depends on two phases: the permutation(confusion) phase, followed by the diffusion phase. The confusion phase is intended to adjust a pixel in a basic image in order to decrease the association between pixels [2], and the pixel values are changed during the diffusion phase. Such as Advanced Encryption Standard (AES), Elliptic Curve Cryptography (ECC) and Rivest-Shamir-Adleman (RSA), various encryption algorithms have been proposed to sharing a secret image [3-5]. However, these traditional encryption methods remain limited in front of the current image encryption requirements [6]. In addition, some authors have proposed to realize confusion and diffusion approaches based on other techniques such as Zaslavskii and Hilbert Space Filling Curve algorithms [7] or the principle of the Josephus and the filtering technology [8]. With the introduction of chaos theory in cryptography, the chaotic encryption systems become the best solution to enhance security level of image information. Several research studies have been proposed to combine two existing chaotic systems for confusion and diffusion [9, 10], or multiple chaotic maps [11], others are based on mixed multi chaos to propose new chaotic systems [12, 13]. Numerous hypothesized chaotic systems exist, some of which are one dimensional such as 1D logistic map, and others are high-dimensional such as 3D chaotic systems [14], 4D [15] and 5D hyperchaotic system [16]. This paper provides a novel method for encrypting grayscale images that uses two signals produced by a 2D Henon chaotic system to pervert the original image. The 2D chaotic system has been chosen because the use of one-dimensional chaotic systems for encryption may be vulnerable to brute-force attack, high dimensional chaotic systems, however, it is expensive to implement and its performance assessment is complicated [17]. The image's pixel values are completely out of place in their position via a new transformation method to replace the values of plain image in their new positions. To realize the diffusion phase, we employ a Tent map to create a chaotic sequence. To improve the level of security, this sequence is applied with Frobenius Function to create the encryption key. Finally, each pixel's value in a scrambled image is replaced using the encryption key and an exclusive-OR operation. This research paper is structured as follows: The second section is about a literature review on image encryption approaches. The third section sheds light on some of the used methods: Henon map, Tent map and Frobenius method.

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Section four describes the proposed approach. The simulation results and performances analysis are detailed in section five. The sixth and final sections conclude the paper.

II. Related Works

Overall, in the literature, several algorithms based on both confusion and diffusion using chaotic map have been proposed. The authors in [9]and [10] have proposed a chaotic scheme by using two kinds of chaotic systems, the first one is for confusion and the second is for diffusion. The position of images pixels is reorganized applying 2D Arnold cat map. Then, they have been encrypted the new reorganized image using a 1D logistic map-generated chaotic sequence. But in [10] the scrambled image is encrypted pixel by pixel using a preprocessed signal produced by a Lorenz chaotic system. In [12], a coupled map lattice (CML) is used for image encryption by combining a CML map with a logistic map and a tent map. The authors in [13] have combined the Henon and Chebyshev maps to create a new two-dimensional Henon-Chebyshev modulation map (2D-HCMM). Reference [18] has shown a novel technique for breaking images into blocks that uses a zigzag pattern [19], rotation, and random permutation to jumble the image blocks. Then, a chaotic logistic map provides a key sequence to disperse the scrambled image. In [11], Sura has proposed to split the secret image into four equally blocks, each block is rotated 90 degrees in the direction of rotation and then, numerous chaotic maps are used for confusion and diffusion with each block. The authors in [20] have shuffled the image with Zaslavskii and Hilbert Space Filling Curve algorithms [7], which consists of the confusion process. An encryption approach using the principle of the Josephus for confusion and the filtering technology for diffusion is given in [8]. The overview of the related study reveals that the construction of schemes is based on the use of multiple chaotic systems known of confusion and diffusion, or else to combine various chaotic systems in order to create a new system. Although, these methods have made it possible to provide the desired level of security, it is mainly due to the high redundancy and the data capacity presented by an image. To overcome the above-stated in terms of security, this research paper proposes a more effective technique for image encryption using 2D-Henon map, Tent map and Frobenius endomorphism. The primary innovations of this study are: first to propose a new approach which can disturb the image pixels in a random way, to be able to solve the issue of strongly correlated between two adjacent pixels, and second to create an encryption key by combining between Tent choatic map and Frobenius endomorphism. The proposed approach shows a high sensitivity to any change in encrypted images, in addition to the other properties such as randomness, correlation of pixels and lossless of the final output image.

III. Review Of The Used Methods

The proposed approach in this paper is based on three methods: 2D Henon map, Tent map and Frobenius endomorphism. This section reviews the above-mentioned methods.

A. 2D-Henon Map

One of the most used chaotic maps in modern cryptography is the 2D-Henon map. The map was proposed by Michel Henon which is a streamlined illustration of the Poincare section of the Lorenz model which is described in reference [21] by

$$\begin{cases} x_{n+1} = 1 - ax_n^2 + y_n \\ y_{n+1} = bx_n \end{cases} \quad (1)$$

Where, the chaotic system's parameters are a and b, and the chaotic map's initial values are a_0 and b_0. a=1.4 and b=0.3 represent the standard parameters values to be chaotic for Henon map.

B. Tent Map

The one-dimensional Tent map is a simplest chaotic system that indicates chaotic behavior which is defined in reference [22] by

$$x_{n+1} = \begin{cases} rx_n, & if \quad x_n < 1/2 \\ r(1-x_n), & if \quad x_n \geq 1/2 \end{cases} \quad (2)$$

Where, $r \in [0,2]$

C. Frobenius Endomorphism

The Frobenius endomorphism is a special endomorphism defined in reference [23] by

$$\begin{aligned} &Frob_A : A \to A \\ &x \to x^p \end{aligned} \quad (3)$$

Where A is a finite field and p is prime number.

A frobenius element is a power of the frobenius endomorphism for the composition low of map.

IV. The Proposed Approach

In this section, the two principles processes, encryption and decryption, are presented. For the encryption process, the plain image is scrambled using our confused method. Then, both the Tent chaotic map and Frobenius endomorphism are used to create a key encryption to encrypt the confused image. Even so, the processes described in the encryption process are reversed in the decryption procedure. Table I shows the notations used in the rest of this paper.

TABLE I
NOTATION USED IN THIS PAPER

Notation	Description
P	Plain image
L	Confused image
M	Rows of image
N	Column of image
$X= (X_1, X_2, \ldots, X_M)$	Chaotic sequence vectors
$Y= (Y_1, Y_2, \ldots, Y_N)$	generated by 2D Henon map
$IX= (I_{X1}, I_{X2}, \ldots, I_{XM})$	Indexes of X and Y
$IY= (I_{Y1}, I_{Y2}, \ldots, I_{YN})$	sequences respectively
A'← sort (A)	A function that sorting in ascending order the vector A and produce the A' as output

A. Encryption Process

To minimize the correlation between the original image's neighboring pixels, which is in general higher, a new confusion method is presented. Fig. 1 shows the schematic for both the confusion and diffusion methods.

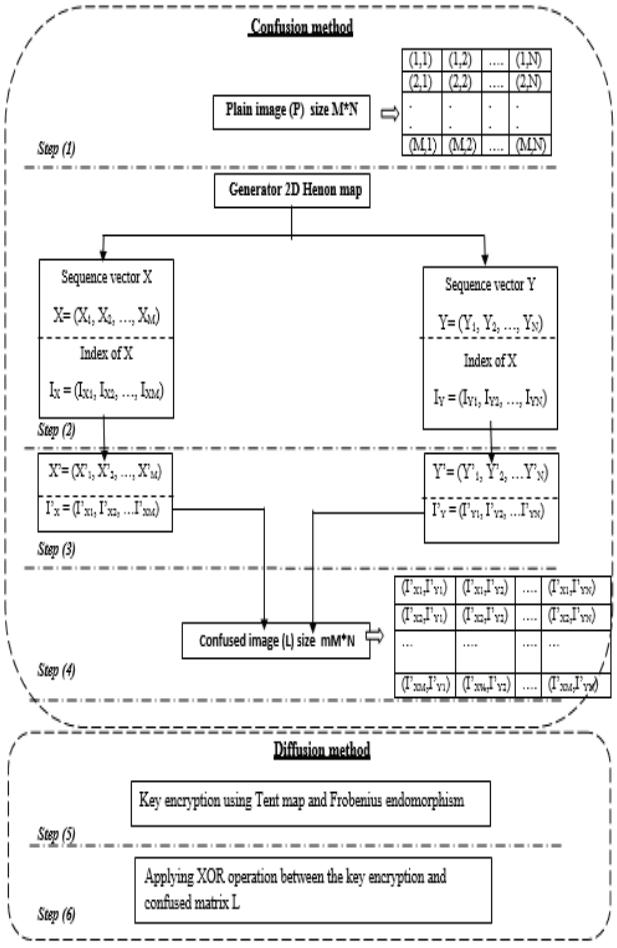

Fig. 1. Diagram of the confusion and diffusion methods

The specific permutation steps are organized as follows:

Step (1): Let P be a plain image of M*N size.

Step (2): We generate two chaotic sequence vectors X and Y based on the 2D-Henon system, there sizes are also M and N respectively and the I_x and I_y are the indexes vectors of X and Y respectively.

Step (3): $(X'; I'_X)$ and $(Y'; I'_Y)$ are sorted in ascending order according to 4.

$$\begin{cases} (X', I'_X) = sort(X, I_X) \\ (Y', I'_Y) = sort(Y, I_Y) \end{cases} \quad (4)$$

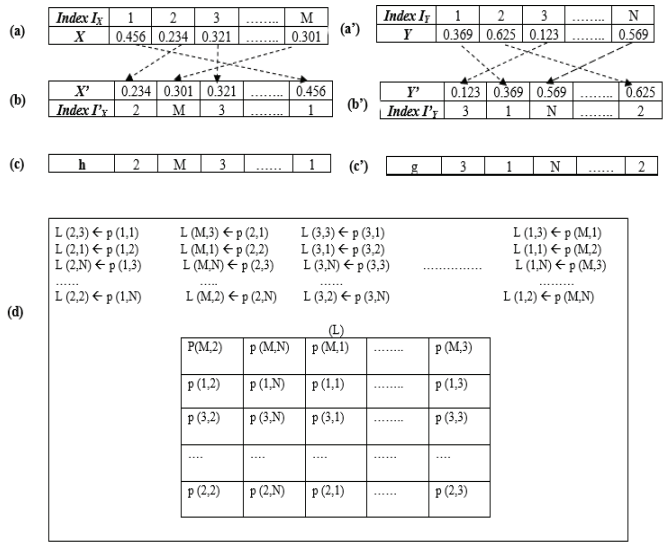

Fig. 2. Confusion process

Step (4): It produces a new matrix L of size M*N. Filling out the new matrix L according to Algorithm 1 where the plain image's row and column of pixel positions are k and v, respectively. We fill the vectors h and g according to new indexes generated by X' and Y', then we fill the confused matrix L with the position of two vectors h and g.

Algorithm 1 Encryption process

For k=1: M **do**
For v=1: N **do**
h ←I'x (k, 1)
g ←I'y (v, 1)
L (h, g) ← p (k, v)
End for
End for

Fig. 2 shows the detailed confusion process:

(a) and (a'): generation of two sequence vectors X and Y using 2D-Henon map, there size are M and N respectively with their indexes I_X and I_Y.

(b) and (b'): sorting X and Y in ascending order to produce X' and Y' with their new indexes I'_x and I'_Y

(c) and (c'): summarized two new indexes I'_X and I'_Y in two

vector h and g.

(d): filled the confused matrix L from the plain image P according to new position indicated in vectors h and g.

Fig. 3 shows a detailed example of the confusing approach.
Fig. 3 (a) shows a plain image of 4*4. The two sequences chaotic vectors are generated by 2D henon map as displayed

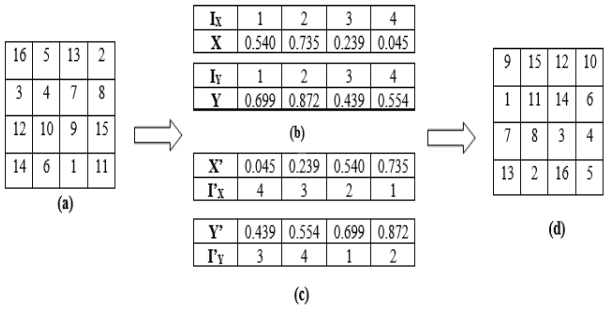

Fig. 3. Example of the confusion process

in Fig. 3 (b). However, Fig. 3 (c) shows the X' and Y' sorting from the respective X and Y vectors in ascending order. The confused matrix is shown in Fig. 3 (d) by applying the steps detailed in both Algorithm 1 and Fig. 2. The confusion method successfully disturbs the pixel values of the plain image when it is applied to some examples of images chosen for this study, as shown in Fig. 4.
The proposed method minimizes the correlation among adjacent pixels of the input image and enhances the chaos. From Fig. 1, steps 5 and describe the diffusion method used to

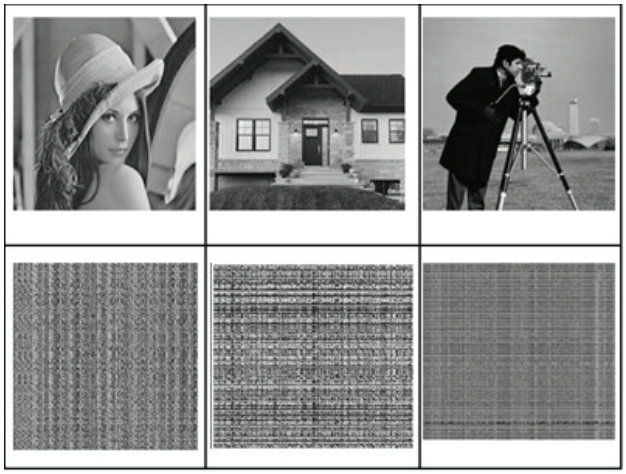

Fig. 4. Examples of the confused images

encrypt the confused image including the key generation and diffusion process.

Step (5): a chaotic vector T= {T1, T2,....T_{M*N}} is generated from the Tent Map, We apply the Frobenius function to the chaotic sequence T as follows: describe the diffusion method used to encrypt

$$ph = Frob(T) \tag{5}$$

Then, we create the key encryption using the following formula:

$$K(i) = \mathrm{mod}((ph*10^5), 256) i = 1 : M*N \tag{6}$$

Step (6): In the diffusion process, to change the pixel value, the key K and the confused image L are combined using the eXclusive-OR (XOR) operation to produce the encrypted image C as follows:

$$C = L \oplus K \tag{7}$$

Fig. 5 explains the key construction procedure.

B. Decryption Process

The output plain image can be recovered using the key and the encrypted image. The method of decryption can be done as follow:

1- XOR operation is used to create the dL image confusion using the K and encrypted image.

$$dL = K \oplus C \tag{8}$$

2- According to Algorithm 2, the procedure to extract the plain image from the confused image involves the steps described in the encryption process in reverse order.

T	0.045	0.365	0.023	0.876	0.654	0.098	0.342	0.067	0.123
T ×10^5	4500	36500	2300	87600	65400	9800	34200	6700	12300
K(i)	148	148	252	48	120	72	152	44	12

Fig. 5. Example of the construction of the key

Algorithm 2 Decryption process

For k=1: M **do**
For v=1: N **do**
h ←I'x (k, 1)
g ←I'y (v, 1)
dP (k, v) ← dL (h,g)
End for
End for

V. Simulation Results And Performance Analysis

Table II summarizes the parameters used in the implementation of the proposed method. For the 2D Henon map and Tent map, we choose the standard values which are chaotic.
Fig. 6 shows the three plain images (Image1, Image2 and Image3), encrypted images and decrypted images respectively. According to Fig. 6, no details regarding the original image

TABLE II

LIST OF PARAMETERS USED IN THE SIMULATION

Notation	Parameters	Description
Image1	220*220	Test with different size of images
Image2	225*225	
Image3	512*512	
a	1.4	The standard value of Henon which is chaotic
b	0.3	
A	0.5	The standard value of Tent which is chaotic
B	1.99	
n	5	A prime number chosen randomly for the Frobenius Function

can be obtained from the encrypted images, moreover; the

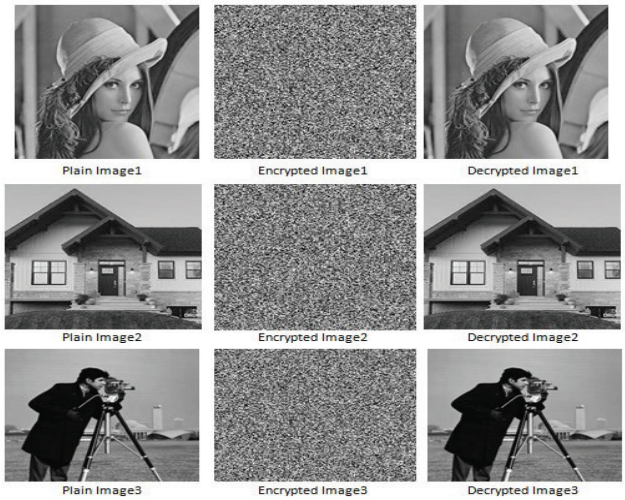

Fig. 6. Three gray-scale images selected as example for encryption and decryption process

decrypted images restore the plain images without distortion. This section evaluates the performance of the suggested technique in terms of PSNR (Peak Signal to Noise Radio), histogram, correlation, entropy, and differential attack.

A. PSNR

The PSNR is described in [24] as the ratio of the maximum possible signal power to the maximum possible signal plus corrupted noise power. The PSNR is given by

$$PSNR(dB) = 20\log_{10}\left(\frac{\max_i}{\sqrt{MSE}}\right) \tag{9}$$

Where, $\max_i$ represents the greatest possible pixel's value of the image, it is equal to 255. The MSE (Mean Square Error) is the average square of the error between the decrypted and plain image is known as the mean squared error. High PSNR value indicates a lower variation between the input and output image, theoretically, PSNR can be infinite if MSE is equal to 0, in this case, there is no difference among the two images, i.e., corresponding pixels of both images have similar values. When the PSNR value for the encrypted image is compared to the plain image, the PSNR is low, which results in greater encryption quality. According to Table III, the obtained encrypted images' PSNR values are really low (<9.5 dB). However, the decrypted images show infinite PSNR values which mean that they have similar values compared to the ones of the plain images. Consequently, with the proposed

TABLE III

OBTAINED PSNR VALUES OF BOTH ENCRYPTED AND DECRYPTED IMAGES

	PSNR (dB)	
	Encrypted images	Decrypted images
Image1	9.4717	Inf
Image2	8.3488	Inf
Image3	8.4855	Inf

approach, the encrypted image cannot provide any details on the secret image and can only be used to recreate the original secret image without distortion.

B. Histogram Analysis

The histogram presents a visual interpretation of both plain and encrypted image distribution. Fig. 7 shows the uniformly distributed histograms of the encrypted images, contrary to plain image histograms which are randomly distributed. This designates that the encryption method can be more resistant to attacks.

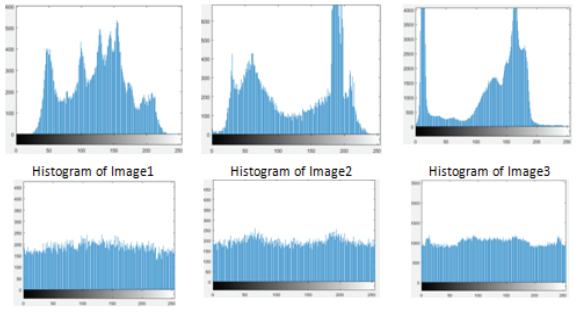

Fig. 7. Histograms of plain and encrypted images

C. Correlation Analysis

This research aims to demonstrate how closely the plain and encrypted image are alike. Equations (10), (11), (12) and (13) can be used to express the nearby pixel's correlation coefficient factor [11]

$$r_{xy} = \frac{\mathrm{cov}(x, y)}{\sqrt{D(x)}\sqrt{D(y)}} \tag{10}$$

$$\operatorname{cov}(x,y)=\frac{1}{N}\sum_{i=1}^{N}(x_i-E(x))(y_i-E(y)) \tag{11}$$

$$D(x)=\frac{1}{N}\sum_{i=1}^{N}\left(x_i-E(x)\right)^2 \tag{12}$$

$$E\left(x\right)=\frac{1}{N}\sum_{i=1}^{N}x_i \tag{13}$$

Where, in the plain or encrypted images, x and y are two adjacent pixel values, E(x) is x's average value, D(x) is the difference from the mean, cov (x, y) is the evaluation of the covariance between neighboring pixels x and y, and $r_{\{xy\}}$ is the correlation coefficient among x and y. To achieve a stronger encryption image, the correlation coefficient should be very close to zero or extremely low. Table IV shows the number obtained from correlation in original image of the horizontal, vertical, and diagonal, which are more than 0.9 which means the two neighboring pixels are strongly correlated to each other. In contrary from Table IV, in the encrypted image, the correlation coefficients for the horizontal, vertical, and diagonal directions are so close to zero. Tables V and VI compare the results of the correlation coefficients from Images 1 and 3 to those from other methods [25-28] to show that the suggested technique produces better correlation coefficient values, namely correlation coefficients that are ~ 0 in the vertical, horizontal, or diagonal directions, as required for security criteria.

TABLE IV
CORRELATION COEFFICIENTS OF IMAGES

Scheme	Horizontal	Vertical	Diagonal
Plain Image1	0.9285	0.9633	0.9001
Plain Image2	0.9345	0.9120	0.8730
Plain Image3	0.9822	0.9894	0.9723
Encrypted Image1	0.0026	0.0033	0.0021
Encrypted Image2	0.0026	0.0010	0.0050
Encrypted Image3	7.642e-04	5.929e-04	-0.0019

TABLE V
COMPARISON CORRELATION COEFFICIENT FOR IMAGE1

Scheme	Horizontal	Vertical	Diagonal
[26]	0.0206	0.0272	0.0037
[27]	0.007	0.0139	0.0063
[28]	0.0030	0.0002	0.0010
The proposed	0.0021	0.0033	0.0021

D. The Randomness

The concept of the randomness is generally measured by the Shannon entropy over the cipher image to test an image's encryption quality. It was first proposed by Claude Shannon in [29] and given by

TABLE VI
COMPARISON CORRELATION COEFFICIENT FOR IMAGE3

Scheme	Horizontal	Vertical	Diagonal
[26]	-0.0353	-0.0209	-0.0047
[28]	0.3361	0.3802	-0.2711
The proposed	7.642e-04	5.929e-04	-0.0019

TABLE VII
INFORMATION ENTROPY OF IMAGES

	Plain	Proposed
Image1	7.4421	7.9916
Image2	7.4284	7.9930
Image3	7.0753	7.9951

$$H(X)=-\sum_{i=1}^{n}\Pr(x_i)\log_2\Pr(x_i) \tag{14}$$

Where, X is the test random variable, x_i is the i_{th} potential value of X, and Pr (x_i) is the probability that X will have the value $X=x_i$. An ideal encrypted image is one in which the pixel values are consistently dispersed throughout. If an encrypted image is very random-like, it is thought that its entropy of information is quite similar to the theoretical maximum. Table VII shows the entropy of encrypted images; this is very similar to 8-bit image's theoretical upper bound on entropy.

E. Analysis for Differential Attack

The encrypted image should be commonly able to detect alterations in the original image. We significantly alter the plain image to check the encryption effect's security. Then, in order to protect against differential attacks, the encryption system can nonetheless make significant updates to the encrypted image. The encryption system's security is tested using two parameters. The first is the Number of Pixels Changes Rate (NPCR) and the second is the Unified Average Changing Intensity (UACI). Both of these variables are calculated by 15 and 16 as follows:

$$NCPR = \frac{\sum_{i,j} D(i,j)}{m*n} *100 \quad (15)$$

$$UACI = \frac{1}{m*n}\left[\sum_{i,j} \frac{|C_1(i,j) - C_2(i,j)|}{255}\right] *100 \quad (16)$$

Where, C_1 and C_2 are two obtained encrypted images after the plain image changes. If $C_1(i,j) \neq C_2(i,j)$, then D(i,j)=1, otherwise D(i,j)=0. In Table VIII, the Image1 is compared to other approaches [25, 26, 28, 30] . This shows how closely the NCPR and UACI reflects the standard value (NCPR > 99 and UACI > 33), which designates that the proposed approach can be more resistant to the original image attack.

TABLE VIII
COMPARISON OF THE OBTAINED NPCR AND UACI OF VALUES IMAGE1 TO OTHERS APPROACHES

Scheme	NPCR (%)	UACI (%)
[25]	99.53	33.61
[26]	99.62	33.33
[28]	99.63	33.47
[30]	99.60	33.48
The proposed	99.99	33.26

VI. CONCLUSION

This research paper proposes a method for image encryption based on two chaotic maps. The first chaotic map used in this approach is 2D-Henon map. The second chaotic sequences generated from 2D-Henon map are utilized to scramble the plain image according to our approach; this successfully achieves a good degree of visual confusion. In order to ensure a high level of security during the diffusion process, the Tent map is utilized to produce a chaotic sequence. This sequence is applied the Frobenius endomorphism to create key encryption. In the last one, we apply the XOR operation between the key encryption and the confused image to obtain the encrypted image. The experimental results and performance analysis based on histogram, randomness, entropy, PSNR and differential attack demonstrated that our proposed approach provided high security features. First, the suggested method can securely encrypt a variety of images. Second, the proposed approach avoids the correlation between original and encrypted image, owing to the new method proposed to scramble the plain image. Moreover, the encrypted image can be decrypted without loss.

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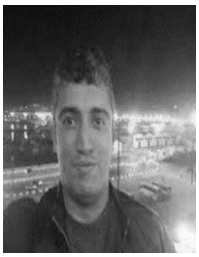

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