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The issue of personal safety on dolomite: A probability-based evaluation with respect to two- and three-storey residential units

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While single-storey (single-house) residential developments were considered in a previous paper, two- and three-storey (multiple house) residential developments, which result in significantly higher densities of land occupation, are considered in this paper.

The overall probability of injury for the occupants of two- and three-storey dwelling units is defined as co-dependent probabilities of sinkhole occurrence, coincidence of the sinkhole with a dwelling unit, structural collapse of the dwelling unit, occupancy of the dwelling unit, occupants in residence when the sinkhole occurs and fatal injury as a result of the event.

The probability of sinkhole occurrence is determined by the associated infiltration regime for residential development, and the geological ground profile. The probability of coincidence between a sinkhole and a dwelling unit is treated in terms of overlapping geometric shapes. The probabilities for the remaining events are subjectively assigned by engineering judgement. The resulting overall probability of injury enables the number of dwelling units and the associated population densities for each of the Inherent Hazard Classes to be determined.

It is found that the allowable population densities for two- and three-storey residential units amount to 890, 425, 170, 125, 40, 0, 0 and 0 people per hectare respectively for the eight Inherent Hazard Classes of dolomite land. This corresponds in principle with the allowable population densities for single-storey dwelling houses of 800, 400, 150, 100, 30, 0, 0 and 0 people per hectare respectively for the eight Inherent Hazard Classes.

INTRODUCTION

The purpose of the paper is to determine the permissible population densities for two- and three-storey dwelling units in townships on

dolomite land based on a residential infiltration regime as defined by Kirsten *et al* (2014).

The infiltration regime is a fundamental determinant in the evaluation of sinkhole



Figure 1 Representative two- and three-storey buildings

Table 1 Geological time-based assessment of sinkhole return period per 100 ha (years)

Class	Township	Sinkhole size (m)					
		Less than 2	2 – 5	5 – 15	15 – 25	25 – 40	Larger than 40
1	Natural	100 000	100 000	10 0000	100 000	100 000	500 000
	Residential	100	100	100	4 500	27 000	135 000
	City Centre	50	50	50	2000	12 000	60 000
2	Natural	100 000	10 0000	100 000	100 000	100 000	500 000
	Residential	45	45	45	2200	12 000	70 000
	City Centre	20	20	20	800	5 000	30 000
3	Natural	10 000	10 000	10 000	20 000	100 000	500 000
	Residential	25	25	25	800	6 000	30 000
	City Centre	10	10	10	300	2 000	12 000
4	Natural	10 000	10 000	10 000	20 000	100 000	500 000
	Residential	15	15	15	550	3 000	15 000
	City Centre	5	5	5	250	1 250	6 000
5	Natural	1 000	1 000	1 000	10 000	500 000	5 000 000
	Residential	5	5	5	150	2 500	25 000
	City Centre	1	1	2	10	100	1 000
6	Natural	1 000	1 000	1 000	10 000	200 000	1 000 000
	Residential	3	3	3	3	500	5 000
	City Centre	0.5	0.5	0.5	2	50	500
7	Natural	1 000	1 000	1 000	1 000	20 000	1 000 000
	Residential	3	3	3	3	500	5 000
	City Centre	0.5	0.5	0.5	0.5	50	500
8	Natural	800	800	800	800	10 000	100 000
	Residential	3	3	3	3	500	5 000
	City Centre	0.5	0.5	0.5	0.5	0.5	100

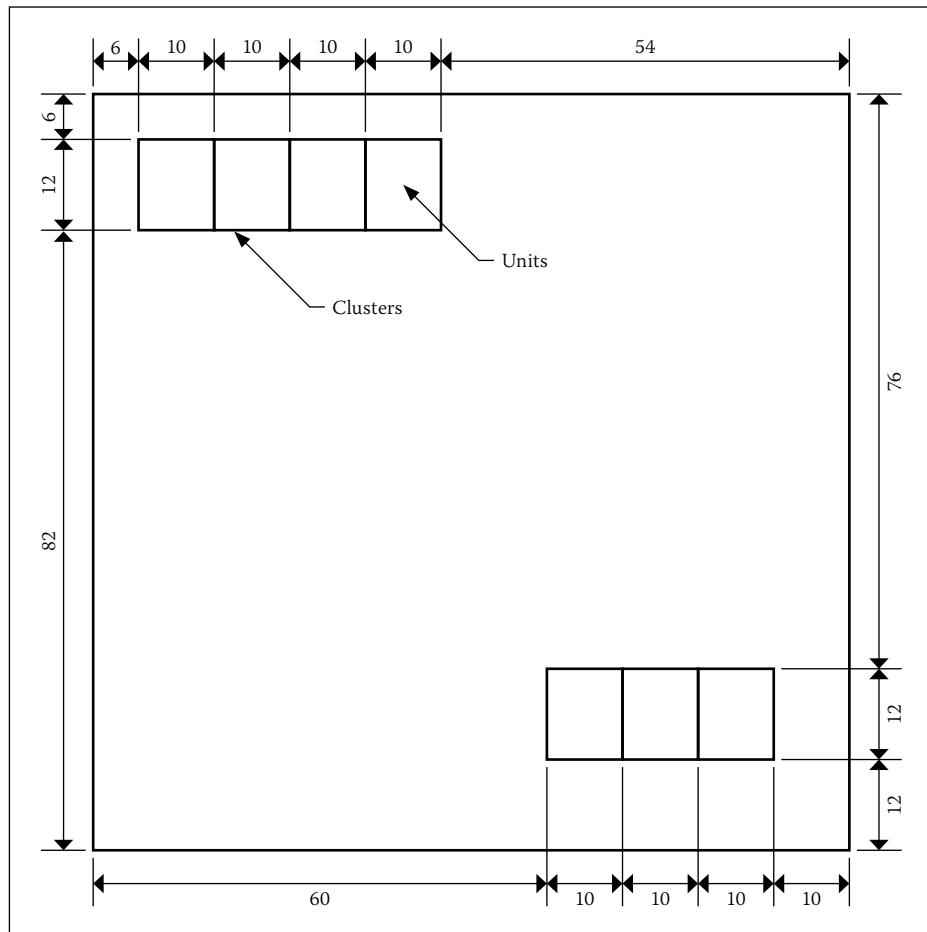


Figure 2 Initially developed layout of two- and three-storey dwelling units

occurrence. Kirsten *et al* (2014) distinguished between natural, residential and city centre infiltration regimes, as these are determined by the water-bearing services, measures to control stormwater, landscaping and irrigation provisions, occurrence of impermeable pavements and dewatering protocols characteristic to each regime.

The infiltration regimes are expressed in terms of potential sinkhole return periods for the eight Inherent Hazard Classes defined by Buttrick *et al* (2001) and the six sinkhole sizes as shown in Table 1. The two largest sinkhole sizes were added to the four sizes originally presented by Buttrick *et al* to extend their range. The research developments that preceded the definition of the return periods in Table 1 are presented by Kirsten *et al* (2009 and 2014).

Sinkhole occurrence is treated in the paper as a chance phenomenon, the qualifications for which are dealt with by Kirsten *et al* (2014). It mainly needs to be observed that, as a result thereof that sinkholes are caused by water-bearing services that tend to leak at isolated locations, only one sinkhole occurs at any point in time in a neighbourhood, and that, as a result of how the causes and the sinkhole itself are repaired, only that one sinkhole occurs at that location over a substantial period of time.

CURRENT APPROACH TO DOLOMITE RISK EVALUATION IN TWO- AND THREE-STORY RESIDENTIAL DEVELOPMENTS

Two- and three-storey developments are market driven options in which much higher densities of land occupation can be achieved than in conventional single-house type developments. Various layout options, footprint sizes and multi-storey options can be considered to achieve higher population densities. Representative two- and three-storey residential developments are shown in Figure 1.

Despite the fact that multi-house two- and three-storey developments may be more competent structurally and may have more robust wet services, a rigorous way of accounting for these advantages is not yet available. SANS 1936:2012 defines population density limits for the eight Inherent Hazard Classes in terms of land usage design specifications. However, it is not indicated how these limits that are higher than those for single-storey houses may be definitively determined in terms of the underlying parameters. The methodology developed by Kirsten *et al* (2014) for the determination of population density for single-storey residential houses was adapted to determine

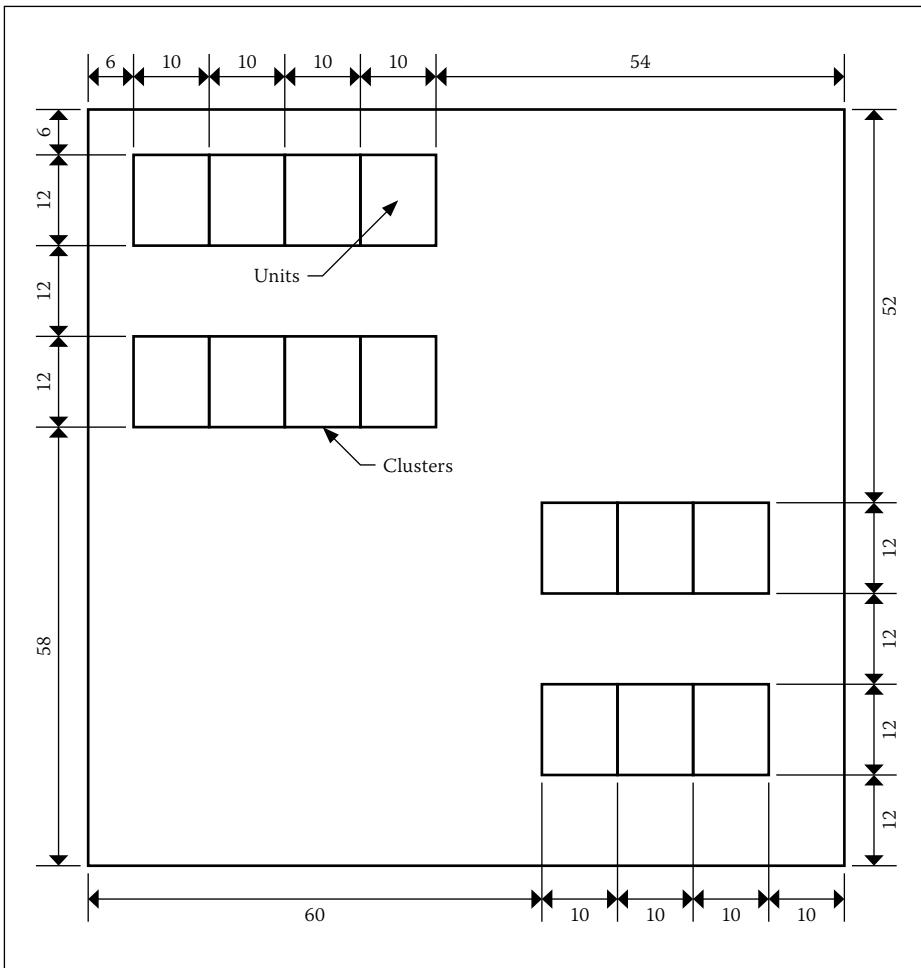


Figure 3 Half developed layout of two- and three-storey dwelling units

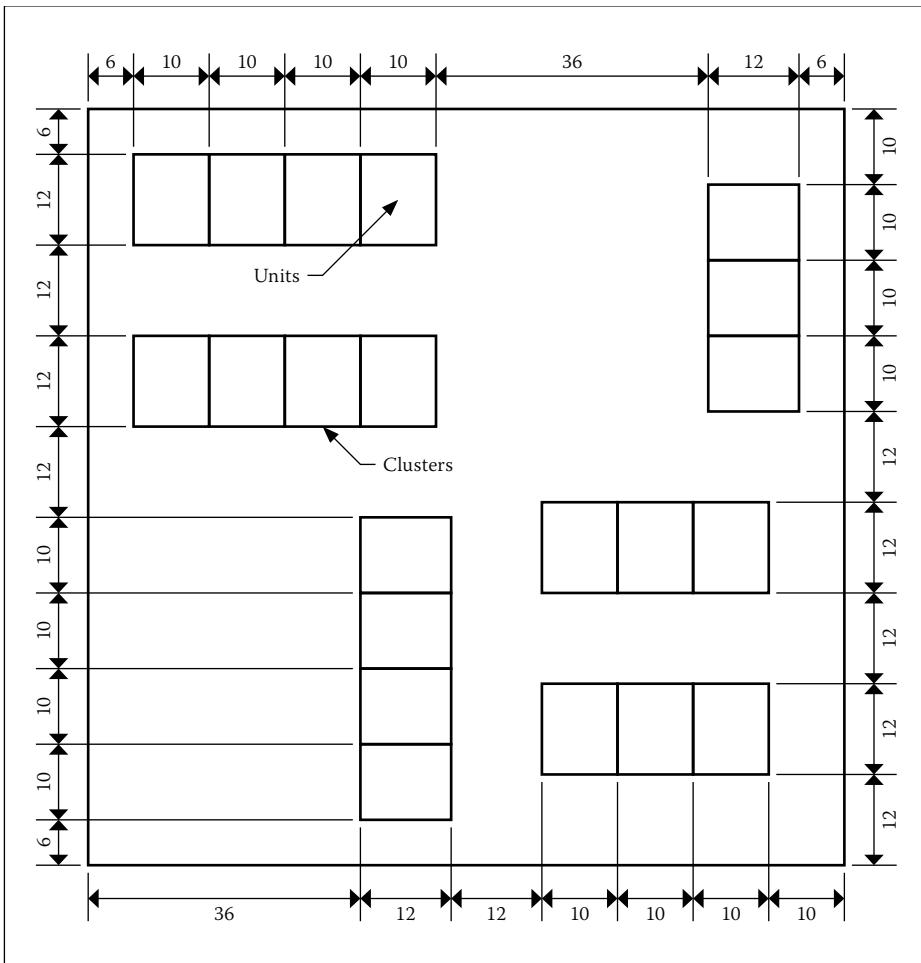


Figure 4 Nearly fully developed layout of two- and three-storey dwelling units

the permissible population densities for two- and three-storey dwelling units, accounting for the particular structural competence of such buildings and the associated robustness and more optimal use of wet services as presented in this paper.

LAYOUT OF LOW-RISE RESIDENTIAL ACCOMMODATION CONSIDERED

A dwelling unit denotes the living accommodation for a group of five people. A cluster denotes a single two- or three-storey building containing three or four dwelling units per floor. Dwelling units overlying one another on different floors are referred to as a stack of units. The low-rise multiple-house residential township considered in the paper is assumed to consist of one or more two- and three-storey clusters as shown in Figures 2 to 5 for various stages of development respectively.

PROBABILITY OF INJURY

Let the following symbols denote:

- P_s Probability of sinkhole occurrence
- P_c Probability of sinkhole coinciding with two- or three-storey stack
- P_f Probability of two- or three-storey stack collapsing when affected by a sinkhole
- P_r Probability of occupants in residence when affected by a sinkhole
- P_h Probability of stack occupied when affected by a sinkhole
- P_d Probability or relative number of fatal injuries of occupants of stack when affected by a sinkhole to provide for the case in which not all the occupants are fatally injured

The overall probability of injury, P_O , is then given by the following products of the above dependent component probabilities:

$$P_t = P_s P_c P_f P_r P_h P_d \quad (1)$$

PROBABILITY OF OCCURRENCE

An event of recurrence interval T years, i.e. a T -year event, is an event of such magnitude that the average time between events of larger magnitude is T years. This length of time is also referred to as the return period. The events considered in the paper refer to sinkholes in intervals of increasing diameter. Reference to a T -year event is therefore with regard to the occurrence of a size of sinkhole in a particular interval.

Let D denote the lifetime of a residential development. The probability that a T -year event will be exceeded at least once in the lifetime of the development is given by the expression in Equation 2. It is not necessary

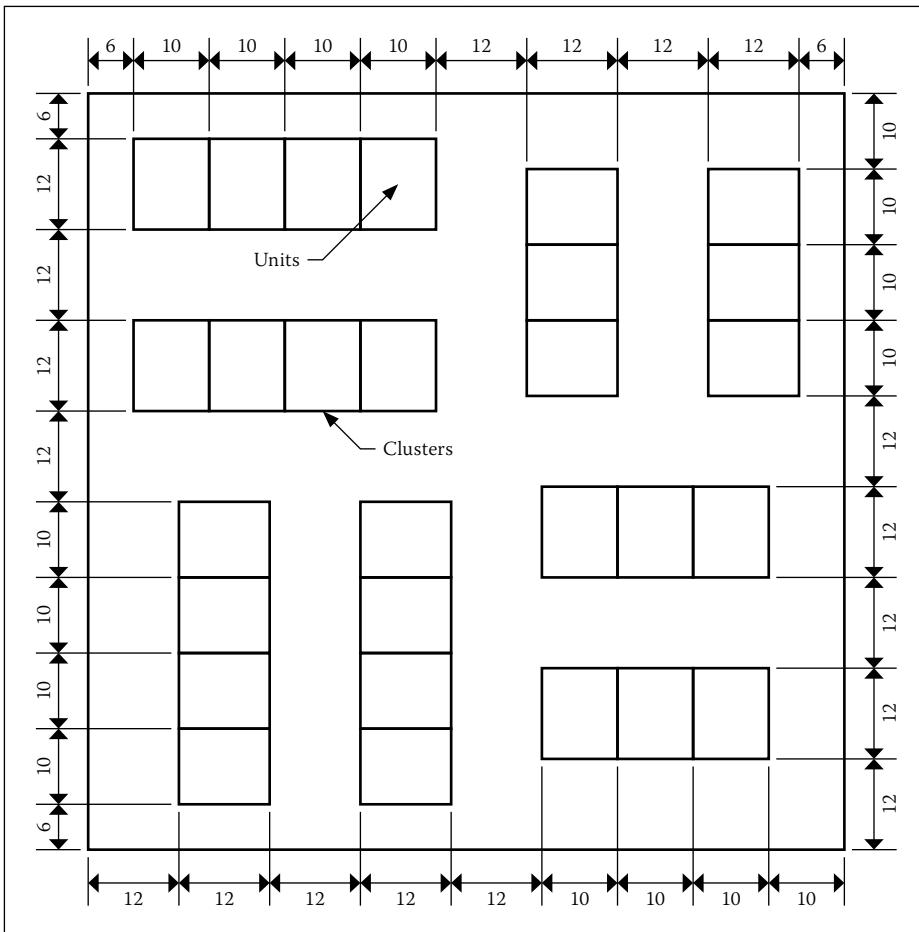


Figure 5 Fully developed layout of two- and three-storey dwelling units

Table 2 Sinkhole return period per hectare for residential infiltration regime

Inherent hazard class	Sinkhole size (m)					
	Less than 2	2 – 5	5 – 15	15 – 25	25 – 40	Larger than 40
1	10 000	10 000	10 000	450 000	2 700 000	13 500 000
2	4 500	4 500	4 500	220 000	1 200 000	7 000 000
3	2 500	2 500	2 500	80 000	600 000	3 000 000
4	1 500	1 500	1 500	55 000	300 000	1 500 000
5	500	500	500	15 000	250 000	2 500 000
6	300	300	300	300	50 000	500 000
7	300	300	300	300	50 000	500 000
8	300	300	300	300	50 000	500 000

to consider the occurrence of more than one sinkhole in a neighbourhood because of the way in which sinkholes occur and are repaired, as had been explained in the Introduction. A lifetime is defined as 70 years.

$$P_s = 1 - (1 - \frac{1}{T})^D \quad (2)$$

The sinkhole return period per 100 hectares is shown in Table 1 for three comparative regimes of infiltration, namely natural, residential and city centre, as presented by Kirsten *et al* (2014). The infiltration regime for a residential development represents the conditions around two- and three-storey dwelling units. The

corresponding return periods per hectare, T^2 , given in Table 2, were obtained by substituting the return periods T^1 from Table 1 in the expression $T^2 = N / [(1/100)N/T^1] = 100T^1$.

PROBABILITY OF COINCIDENCE

Let A, B, C and D denote the various stacks of units in a two-storey cluster, and E, F and G those in a three-storey cluster, as shown respectively in Figures 6 and 7. A sinkhole coincides with any stack of two- or three-storey units in a cluster if it lies within the hatched zones in Figures 8 and 9, representing the respective situations for the sinkhole less than and larger than the

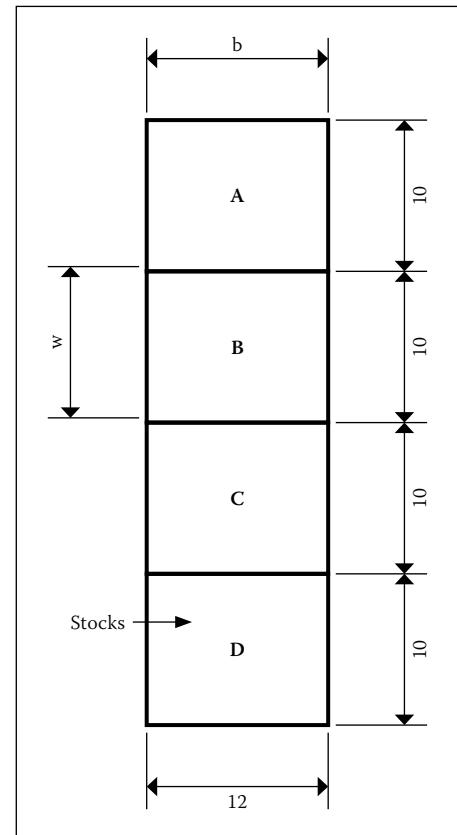


Figure 6 Designation of stacks in two-storey cluster

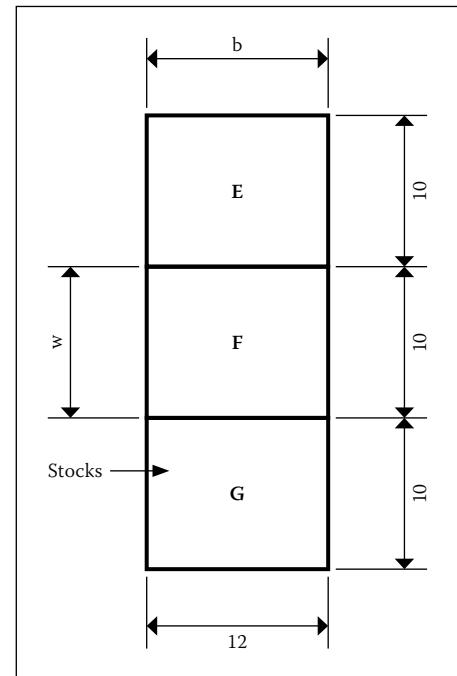


Figure 7 Designation of stacks in three-storey cluster

width of the unit w , as shown in Figures 6 and 7. Observe, as already defined above, that units that overlie one another on different floors are referred to as a stack of units. The probability of coincidence P_c is accordingly given by the larger value of the following two expressions:

$$P_c = \frac{w(d + b)}{10 000} \quad (3)$$

Table 3 Probability of coincidence for range of sinkhole size considered

Sinkhole size (m)					
Less than 2	2–5	5–15	15–25	25–40	Larger than 40
0.013	0.015	0.022	0.096	0.245	0.558

$$P_c = \frac{(2d - w)(b + d)}{10000} \quad (4)$$

where d denotes the sinkhole diameter and b the width of the cluster, as shown in Figures 8 and 9. The resultant probabilities of coincidence for the range of sinkhole sizes considered are shown in Table 3 for $b = 12$ m and $w = 10$ m.

PROBABILITY OF BUILDING COLLAPSE

A two-storey residential building is structurally damaged by a 15 m sinkhole, as shown for example in Figure 10. Consider the potential collapse of two- and three-storey stacks of dwelling units as follows.

Two-storey dwelling unit – Figure 6

Consider the designation of stacks of units for a two-storey cluster as defined in Figure 6 and let,

- P_{fAA} , P_{fAB} , P_{fAC} and P_{fAD} denote the probabilities of two-storey stack A collapsing when stacks A, B, C and D are respectively affected by a sinkhole;
- P_{fBA} , P_{fBB} , P_{fBC} and P_{fBD} denote the probabilities of two-storey stack B collapsing when stacks A, B, C and D are respectively affected by a sinkhole;
- P_{fCA} , P_{fCB} , P_{fCC} and P_{fCD} denote the probabilities of two-storey stack C collapsing when stacks A, B, C and D are respectively affected by a sinkhole; and
- P_{fDA} , P_{fDB} , P_{fDC} and P_{fDD} denote the probabilities of two-storey stack D collapsing when stacks A, B, C and D are respectively affected by a sinkhole.

Note that:

$$\begin{aligned} P_{fAA} &= P_{fDD}, P_{fAB} = P_{fDC}, P_{fAC} = P_{fDB}, \\ P_{fAD} &= P_{fDA}, P_{fBA} = P_{fCD}, P_{fBB} = P_{fCC}, \\ P_{fBC} &= P_{fCB} \text{ and } P_{fBD} = P_{fCA}. \end{aligned}$$

Values for these probabilities of two-storey dwelling stack, sinkhole-induced collapse may be assigned by subjective engineering judgement as given in Table 4.

Three-storey dwelling unit – Figure 7

Consider the designation of stacks of units for a three-storey cluster as defined in Figure 7 and let,

- P_{fEE} , P_{fEF} and P_{fEG} denote the probabilities of three-storey stack E collapsing

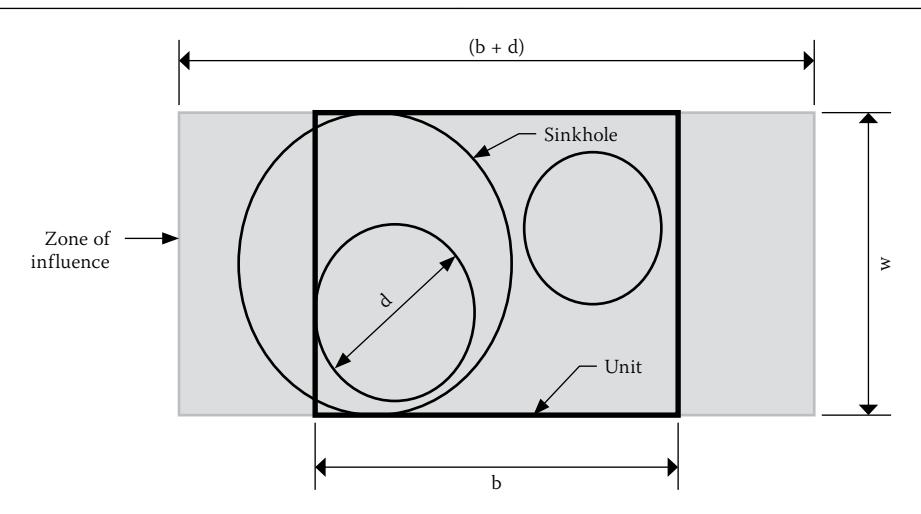


Figure 8 Zone of influence for sinkhole size \leq unit width

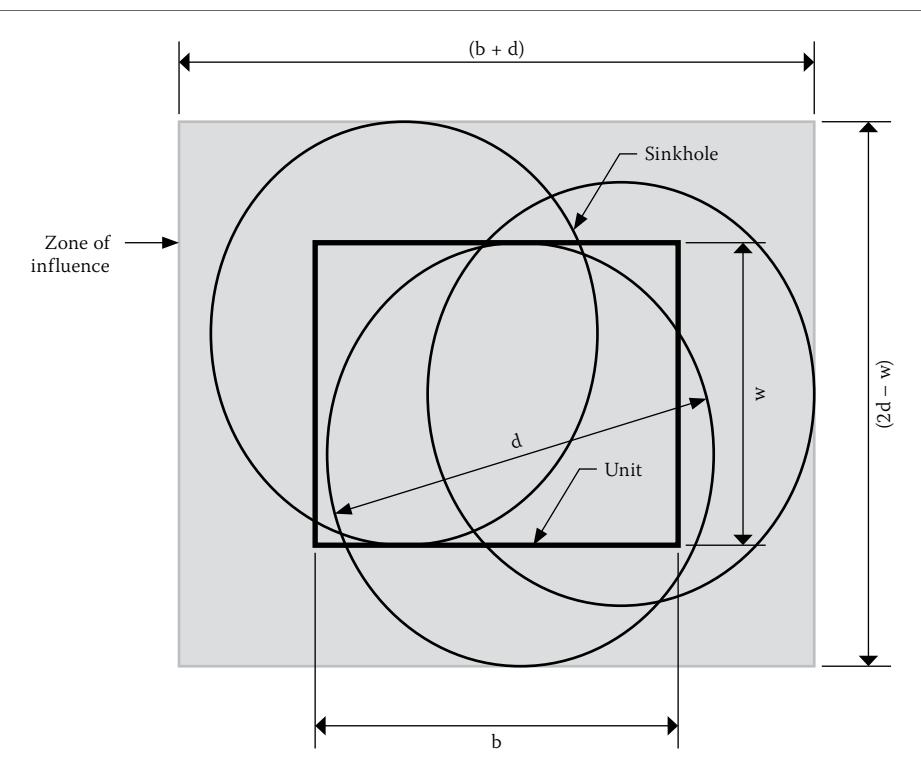


Figure 9 Zone of influence for sinkhole size $>$ unit width



Figure 10 Two-storey residential building structurally damaged by 15 m sinkhole

Table 4 Probability of two-storey unit collapsing when affected by a sinkhole

Sinkhole diameter < 1 m		Unit affected			
		A	B	C	D
Unit failing	A	0.0001	0.0001	0.00001	0.000001
	B	0.0001	0.0001	0.00001	0.000001
	C	0.000001	0.00001	0.0001	0.0001
	D	0.000001	0.00001	0.0001	0.0001
Sinkhole diameter 1 – 5 m		Unit affected			
Unit failing	A	0.01	0.01	0.001	0.0001
	B	0.01	0.01	0.001	0.0001
	C	0.0001	0.001	0.01	0.01
	D	0.0001	0.001	0.01	0.01
Sinkhole diameter 5 – 15 m		Unit affected			
Unit failing	A	1	1	0.1	0.01
	B	1	1	0.1	0.01
	C	0.01	0.1	1	1
	D	0.01	0.1	1	1
Sinkhole diameter 15 – 25 m		Unit affected			
Unit failing	A	1	1	1	0.1
	B	1	1	1	0.1
	C	0.1	1	1	1
	D	0.1	1	1	1
Sinkhole diameter 25 – 40 m		Unit affected			
Unit failing	A	1	1	1	1
	B	1	1	1	1
	C	1	1	1	1
	D	1	1	1	1
Sinkhole diameter > 40 m		Unit affected			
Unit failing	A	1	1	1	1
	B	1	1	1	1
	C	1	1	1	1
	D	1	1	1	1

when stacks E, F and G are respectively affected by a sinkhole;

- P_{fFE} , P_{FFF} and P_{fFG} denote the probabilities of three-storey stack F collapsing when stacks E, F and G are respectively affected by a sinkhole; and
- P_{fGE} , P_{fGF} and P_{fGG} denote the probabilities of three-storey stack G collapsing when stacks E, F and G are respectively affected by a sinkhole.

Note that:

$$P_{FEE} = P_{fFF}, P_{FEF} = P_{fGF}, P_{FEG} = P_{fGE} \text{ and } P_{FFE} = P_{fFG}.$$

Values for these probabilities of three-storey dwelling stack, sinkhole-induced collapse may be assigned by subjective engineering judgement as given in Table 5.

Table 5 Probability of three-storey unit collapsing when affected by a sinkhole

Sinkhole diameter < 1 m		Unit affected		
		E	F	G
Unit failing	E	0.0001	0.0001	0.00001
	F	0.0001	0.0001	0.0001
	G	0.00001	0.0001	0.0001
Sinkhole diameter 1 – 5 m		Unit struck		
Unit failing	E	0.01	0.01	0.001
	F	0.01	0.01	0.01
	G	0.001	0.01	0.01
Sinkhole diameter 5 – 15 m		Unit affected		
Unit failing	E	1	1	0.1
	F	1	1	1
	G	0.1	1	1
Sinkhole diameter 15 – 25 m		Unit affected		
Unit failing	E	1	1	1
	F	1	1	1
	G	1	1	1
Sinkhole diameter 25 – 40 m		Unit affected		
Unit failing	E	1	1	1
	F	1	1	1
	G	1	1	1
Sinkhole diameter > 40 m		Unit affected		
Unit failing	E	1	1	1
	F	1	1	1
	G	1	1	1

Table 6 Probability of fatal injuries in two- and three-storey clusters

Number of storeys	Sinkhole size (m)					
	< 1 m	1 – 5 m	5 – 15 m	15 – 25 m	25 – 40 m	> 40 m
2	0.0001	0.001	0.05	0.5	0.5	0.5
3	0.0001	0.001	0.075	0.75	0.75	0.75

PROBABILITIES OF OCCUPANTS AT HOME AND UNITS OCCUPIED

The probabilities of the occupants being at home and the dwelling stacks being occupied when affected by a sinkhole are assumed to be 0.5 and 1.0 respectively in all instances.

PROBABILITY OF FATAL INJURY

Let P_{d2} denote the probability of fatal injury of the occupants when any of the stacks in a

two-storey cluster are affected by a sinkhole. Likewise, let P_{d3} denote the probability of fatal injury of the occupants when any of the stacks in a three-storey cluster are affected by a sinkhole.

Values for these probabilities of fatal injury in sinkhole-induced collapse of two- and three-storey dwelling stacks may be assigned by subjective engineering judgement for various sizes of sinkhole as given in Table 6.

POPULATION DENSITY

Let j denote the average number of people per dwelling unit and N_2 and N_3 the potential numbers of fatal injuries in two- and three-storey stacks of units respectively. In this instance $j = 5$. By definition then:

$$P_{d2} = \frac{N_2}{2j}, \text{ and} \quad (5)$$

$$P_{d3} = \frac{N_3}{3j} \quad (6)$$

The number of people accommodated per hectare, h , may be expressed in terms of the average number of people per unit, j , and the respective numbers of two- and three-storey clusters per hectare, m and n , by:

$$h = 8jm + 9jn \quad (7)$$

THRESHOLD LEVELS FOR FATAL INJURY

Lifetime probability is defined as the probable unit number of times that a detrimental event could occur during the life of the person affected. A natural lifetime is on average 70 years. Parameter D in Expression [1] is therefore taken as 70.

Threshold levels for fatal injury may be presented after Whitman (1984), as shown in Figure 11, in terms of lifetime probability P_t and potential number of fatal injuries N in the units in two- or three-storey stacks. The ordinate axis may alternatively represent the relative lapse of time in anecdotal terms as shown. The relationship between lifetime probability of injury P_t and the potential number of fatal injuries per stack of units N for any particular risk level R is given in principle by Expression 8.

$$P_t = \frac{R}{N} \quad (8)$$

For a fully developed mixture of two- and three-storey clusters as shown in Figure 5, the weighted average potential number of fatalities per stack may be determined from Expression [8] by substituting N_2 and N_3 from Expressions [5] and [6], and observing that there are $4m$ stacks in total in the two-storey clusters and $3n$ stacks in total in the three-storey clusters:

$$P_t = \frac{4m(\frac{R}{2jP_{d2}}) + 3n(\frac{R}{3jP_{d3}})}{4m + 3n} \quad (9)$$

Thus, on simplification

$$P_t = \frac{R(\frac{2m}{P_{d2}} + \frac{n}{P_{d3}})}{j(4m + 3n)} \quad (10)$$

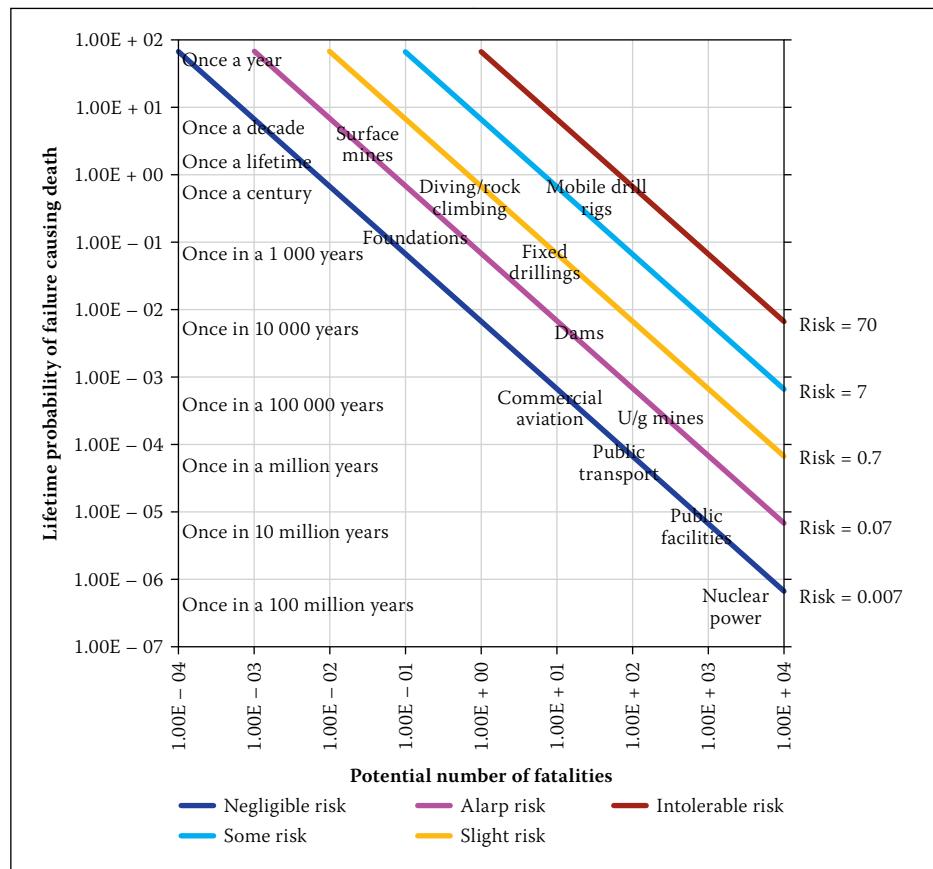


Figure 11 Risk criteria for fatal injury (after Whitman 1984)

COLLECTIVE PROBABILITY OF INJURY

Consider two- and three-storey dwelling units separately as follows for determining the collective probability of injury.

Two-storey dwelling unit – Figure 6

Let P_{AA} , P_{AB} , P_{AC} and P_{AD} denote the overall probabilities of injury in two-storey stack A when stacks A, B, C and D are respectively affected by a sinkhole. A number of events need to happen simultaneously for people in stack A to be fatally injured, namely, the sinkhole has to occur, the sinkhole has to coincide with the stack, the stack has to collapse catastrophically, the stack has to be occupied by people, the people have to be at home at the time, and people in the stack need to be fatally injured. The overall probabilities P_{AA} , P_{AB} , P_{AC} and P_{AD} can therefore be obtained as given in Expression [1] by multiplying the probabilities for the six underlying events as follows in terms of the parameters defined above:

$$P_{AA} = P_s P_c P_{fAA} P_r P_h P_{dAA} \quad (11)$$

$$P_{AB} = P_s P_c P_{fAB} P_r P_h P_{dAB} \quad (12)$$

$$P_{AC} = P_s P_c P_{fAC} P_r P_h P_{dAC} \quad (13)$$

$$P_{AD} = P_s P_c P_{fAD} P_r P_h P_{dAD} \quad (14)$$

The collective overall probability that people are injured in stack A is therefore given by:

$$P_A = 1 - (1 - P_{AA})(1 - P_{AB})(1 - P_{AC})(1 - P_{AD}) \quad (15)$$

The collective overall probabilities of injury in stacks B, C and D are given as follows by cyclic rotation of subscripts:

$$P_B = 1 - (1 - P_{BA})(1 - P_{BB})(1 - P_{BC})(1 - P_{BD}) \quad (16)$$

$$P_C = P_B, \text{ by symmetry} \quad (17)$$

$$P_D = P_A, \text{ by symmetry} \quad (18)$$

The collective overall probability of injury in a two-storey cluster is given by Expression 19:

$$P_{2 \text{ storey-cluster}} = 1 - (1 - P_A)^2 (1 - P_B)^2 \quad (19)$$

Three-storey dwelling unit – Figure 7

Let P_{EE} , P_{EF} and P_{EG} denote the overall probabilities of injury in three-storey stack E when stacks E, F and G are respectively affected by a sinkhole. A number of events need to happen simultaneously for people in stack E to be fatally injured, namely, the sinkhole has to occur, the sinkhole has to coincide with the stack, the stack has to collapse catastrophically, the stack has to be occupied by people, the people have to be at home at the time, and people in the stack need to be fatally injured. The overall probabilities P_{EE} , P_{EF} and P_{EG} can therefore be obtained as given in Expression [1] by multiplying the probabilities for the six underlying events as follows in terms of the parameters defined above:

$$P_{EE} = P_s P_c P_{fEE} P_r P_h P_{d3} \quad (20)$$

$$P_{EF} = P_s P_c P_{fEF} P_r P_h P_{d3} \quad (21)$$

$$P_{EG} = P_s P_c P_{fEG} P_r P_h P_{d3} \quad (22)$$

The collective overall probability that people are injured in stack E is therefore given by:

$$P_E = 1 - (1 - P_{EE})(1 - P_{EF})(1 - P_{EG}) \quad (23)$$

The overall probabilities of injury in three-storey stacks F and G are given as follows by cyclic rotation of subscripts:

$$P_F = 1 - (1 - P_{FE})^2(1 - P_{FF}) \quad (24)$$

$$P_G = P_F, \text{ by symmetry} \quad (25)$$

The collective overall probability of injury in a three-storey cluster is given by Expression 26:

$$P_{3 \text{ storey-cluster}} = 1 - (1 - P_E)^2(1 - P_F) \quad (26)$$

OVERALL PROBABILITY OF INJURY

The overall probability of injury per hectare P_O is given by Expression 27:

$$P_O = 1 - (1 - P_{2 \text{ storey-cluster}})^m (1 - P_{3 \text{ storey-cluster}})^n \quad (27)$$

The powers m and n in Expression [27] respectively represent the numbers of two- and three-storey clusters per hectare. For example, m = n = 1, 2, 3 and 4 respectively in Figures 2, 3, 4 and 5. Observe that m and n need not have the same value.

The overall probability of injury P_O should be $\leq P_t$ for the particular risk level considered. The number of people per hectare for which this condition is satisfied may be determined from Expression [28] by substituting P_t and P_O from Expressions [10] and [27] respectively:

$$F = \frac{P_t}{P_O} = \frac{R(\frac{2m}{P_{d2}} + \frac{n}{P_{d3}})}{j(4m + 3n)[1 - (1 - P_{2 \text{ storey-cluster}})^m (1 - P_{3 \text{ storey-cluster}})^n]} \geq 1.0 \quad (28)$$

EVALUATION OF PERMISSIBLE POPULATION DENSITIES

Based on the input parameter values presented in the preceding sections, values for ratio F are shown for ranges of population density and sinkhole size in Tables 7 to 14 for Inherent Hazard Classes 1 through 8

Table 7 Factor F for two- and three-storey residential buildings – inherent hazard Class 1

Number of clusters		No of people/ha	Sinkhole size (m)					
			1	3	10	20	32.5	50
0.1	0	4	v large	v large	4 323.37	302.44	551.59	1 209.68
0.5	0	20	v large	v large	864.68	60.49	110.32	241.94
1	0	40	v large	v large	432.34	30.24	55.16	120.97
2	1	125	v large	v large	111.86	8.31	16.46	36.09
2	2	170	v large	v large	72.25	5.52	11.40	24.99
4	3	295	v large	v large	44.13	3.33	6.76	14.82
5	5	425	v large	v large	28.90	2.21	4.56	10.00
8	7	635	v large	v large	19.89	1.51	3.09	6.78
11	10	890	v large	v large	14.08	1.07	2.20	4.82

Table 8 Factor F for two- and three-storey residential buildings – inherent hazard Class 2

Number of clusters		No of people/ha	Sinkhole size (m)					
			1	3	10	20	32.5	50
0.1	0	4	v large	v large	1 953.72	147.87	245.16	627.24
0.5	0	20	v large	v large	390.75	29.58	49.03	125.45
1	0	40	v large	v large	195.38	14.79	24.52	62.72
2	1	125	v large	v large	50.55	4.06	7.32	18.72
2	2	170	v large	v large	32.65	2.70	5.07	12.96
4	3	295	v large	v large	19.94	1.63	3.00	7.68
5	5	425	v large	v large	13.06	1.08	2.03	5.18
8	7	635	v large	v large	8.99	0.74	1.38	3.52
11	10	890	v large	v large	6.37	0.52	0.98	2.50

Table 9 Factor F for two- and three-storey residential buildings – inherent hazard Class 3

Number of clusters		No of people/ha	Sinkhole size (m)					
			1	3	10	20	32.5	50
0.1	0	4	v large	v large	1 092.06	53.79	122.58	268.82
0.5	0	20	v large	v large	218.42	10.76	24.52	53.76
1	0	40	v large	v large	109.21	5.38	12.26	26.88
2	1	125	v large	v large	28.26	1.48	3.66	8.02
2	2	170	v large	v large	18.25	0.98	2.53	5.55
4	3	295	v large	v large	11.15	0.59	1.50	3.29
5	5	425	v large	v large	7.31	0.39	1.01	2.22
8	7	635	v large	v large	5.03	0.27	0.69	1.51
11	10	890	v large	v large	3.56	0.19	0.49	1.07

Table 10 Factor F for two- and three-storey residential buildings – inherent hazard Class 4

Number of clusters		No of people/ha	Sinkhole size (m)					
			1	3	10	20	32.5	50
0.1	0	4	v large	v large	661.25	36.99	61.29	134.41
0.5	0	20	v large	v large	132.26	7.40	12.26	26.88
1	0	40	v large	v large	66.13	3.70	6.13	13.44
2	1	125	v large	v large	17.11	1.02	1.83	4.01
2	2	170	v large	v large	11.06	0.68	1.27	2.78
4	3	295	v large	v large	6.75	0.41	0.75	1.65
5	5	425	v large	v large	4.43	0.27	0.51	1.11
8	7	635	v large	v large	3.05	0.19	0.34	0.75
11	10	890	v large	v large	2.16	0.13	0.24	0.54

Table 11 Factor F for two- and three-storey residential buildings – inherent hazard Class 5

Number of clusters		No of people/ha	Sinkhole size (m)					
m	n		1	3	10	20	32.5	50
0.1	0	4	v large	v large	230.64	10.10	51.08	224.02
0.5	0	20	v large	v large	46.13	2.02	10.22	44.80
1	0	40	v large	v large	23.07	1.01	5.11	22.40
2	1	125	v large	v large	5.97	0.28	1.52	6.68
2	2	170	v large	v large	3.86	0.18	1.06	4.63
4	3	295	v large	v large	2.36	0.11	0.63	2.74
5	5	425	v large	v large	1.55	0.07	0.42	1.85
8	7	635	v large	v large	1.07	0.05	0.29	1.26
11	10	890	v large	v large	0.76	0.04	0.20	0.89

Table 12 Factor F for two- and three-storey residential buildings – inherent hazard Class 6

Number of clusters		No of people/ha	Sinkhole size (m)					
m	n		1	3	10	20	32.5	50
0.1	0	4	v large	v large	144.70	0.23	10.22	44.81
0.5	0	20	v large	v large	28.95	0.05	2.04	8.96
1	0	40	v large	v large	14.48	0.02	1.02	4.48
2	1	125	v large	v large	3.75	0.01	0.31	1.34
2	2	170	v large	v large	2.42	0.00	0.21	0.93
4	3	295	v large	v large	1.48	0.00	0.13	0.55
5	5	425	v large	v large	0.97	0.00	0.09	0.37
8	7	635	v large	v large	0.67	0.00	0.06	0.25
11	10	890	v large	v large	0.48	0.00	0.04	0.18

Table 13 Factor F for two- and three-storey residential buildings – inherent hazard Class 7

Number of clusters		No of people/ha	Sinkhole size (m)					
m	n		1	3	10	20	32.5	50
0.1	0	4	v large	v large	144.70	0.23	10.22	44.81
0.5	0	20	v large	v large	28.95	0.05	2.04	8.96
1	0	40	v large	v large	14.48	0.02	1.02	4.48
2	1	125	v large	v large	3.75	0.01	0.31	1.34
2	2	170	v large	v large	2.42	0.00	0.21	0.93
4	3	295	v large	v large	1.48	0.00	0.13	0.55
5	5	425	v large	v large	0.97	0.00	0.09	0.37
8	7	635	v large	v large	0.67	0.00	0.06	0.25
11	10	890	v large	v large	0.48	0.00	0.04	0.18

Table 14 Factor F for two- and three-storey residential buildings – inherent hazard Class 8

Number of clusters		No of people/ha	Sinkhole size (m)					
m	n		1	3	10	20	32.5	50
0.1	0	4	v large	v large	144.70	0.23	10.22	44.81
0.5	0	20	v large	v large	28.95	0.05	2.04	8.96
1	0	40	v large	v large	14.48	0.02	1.02	4.48
2	1	125	v large	v large	3.75	0.01	0.31	1.34
2	2	170	v large	v large	2.42	0.00	0.21	0.93
4	3	295	v large	v large	1.48	0.00	0.13	0.55
5	5	425	v large	v large	0.97	0.00	0.09	0.37
8	7	635	v large	v large	0.67	0.00	0.06	0.25
11	10	890	v large	v large	0.48	0.00	0.04	0.18

respectively. The values for F correspond to a “negligible risk level” as defined in Figure 11, i.e. to $R = 0.007$. The values for F in these tables have a minimum turning point character for particular population densities. The maximum population density for any Inherent Hazard Class corresponds to that value for which $F \geq 1.0$ for all sinkhole sizes.

The maximum permissible population densities determined, as described above for two- and three-storey residential buildings, are summarised in Table 15 in terms of the sinkhole size for which factor F is approximately equal to unity. Owing to the minimum turning point character of factor F, the maximum population densities in Table 15 apply to all sinkhole sizes.

The maximum permissible population densities for two- and three-storey developments correspond to 890, 425, 170, 125, 40, 0, 0 and 0 people/ha respectively for the eight Inherent Hazard Classes compared to 800, 400, 150, 100, 30, 0, 0 and 0 people/ha for single-storey dwelling house developments.

A fully developed township consisting of two- and three-storey clusters as shown in Figure 5 can at most accommodate 340 people per ha, which approaches the permissible number of 425 people per ha in Inherent Hazard Class 2, but which is still far below that permissible in Class 1 of 890 people per ha. A population density of 340 people per ha is as expected almost three times as much as the maximum number of 160 people per ha that can be accommodated in single-storey houses.

The most probable number of fatal injuries will be determined by the number of stacks affected by a 20 m sinkhole, since this size of sinkhole corresponds to the minimum turning point with regard to population density as shown in Table 15. A 20 m sinkhole can only affect two stacks in either a two- or a three-storey cluster. As a result the most probable number of fatalities will in principle be $0.5 \times 20 = 10$ for a two-storey cluster and $0.75 \times 30 = 22.5$ for a three-storey cluster.

It should be emphasised that the parameter values adopted are not definitive for all two- and three-storey residential developments, and were only considered to illustrate the principles presented in the paper. The parameter values should be verified for every specific application. The probability of structural failure should in particular be rigorously assessed.

EVALUATION OF SANS 1936:2012 LAND USAGE REQUIREMENTS

The permissible land usage requirements in terms of SANS 1936 – 1:2012 are compared with the population densities determined in

this paper in Table 16, which is an extract of the land usage requirements in the standard on two- and three-storey residences. The land usage requirements in SANS 1936 – 1:2012 are expressed in terms of maximum permissible population densities for four levels of precautionary measures represented by area designations D1, D2, D3 and D4 respectively, and are briefly defined as follows:

- D1 No precautionary measures considered
- D2 Precautionary measures prevent concentrated ingress of water into the ground
- D3 Additional precautionary measures to D2 requirements as provided for in the standard
- D4 Precautionary measures determined rationally and specifically for the particular site

It follows by inference that the maximum permissible population densities for land usage requirements D1, D2 and D3 by definition correspond to minimum population densities for land usage requirements D2, D3 and D4 respectively, i.e. for an area designation one level higher in each instance, hence the minimum inferred population densities denoted by superscript "1" in Table 16.

The permissible population densities determined in the paper are not subject to the implementation of any specified precautionary measures. SANS 1936:2012 does not allow any people to be accommodated on dolomite land unless precautionary measures as specified in Table 16 are provided. The population densities corresponding to area designation D1 in the table are accordingly zero.

SANS 1936:2012 land usage requirement D2 for Inherent Hazard Class 1 is more onerous with regard to population density than determined in the paper, but is sensible, because it is always good practice to prevent the concentrated ingress of water into dolomite ground.

SANS 1936:2012 land usage requirement D2 for Inherent Hazard Class 2 is less onerous with regard to population density than determined in the paper and is sensible, because it is always good practice to prevent concentrated ingress of water. Inferred land usage requirement D3 or D4 for Inherent Hazard Class 2, if appropriate, is sensible.

SANS 1936:2012 land usage requirement D3 + FP1 for Inherent Hazard Class 3 is less onerous than determined in the paper, except perhaps for the limitation on sinkhole size. Inferred land usage requirement D4 for Inherent Hazard Class 3 is sensible.

SANS 1936:2012 land usage requirement D3 + FP1 for Inherent Hazard Class 4 is sensible compared to that determined in the paper, except perhaps for the limitation on

Table 15 Summary of permissible population densities for two- and three-storey residential buildings

Hazard class	Number of clusters		No of people/ha	Sinkhole size (m)					
	m	n		1.0	3	10	20	32.5	50
1	11	10	890	v large	v large	14.08	1.07	2.20	4.82
2	5	5	425	v large	v large	13.06	1.08	2.03	5.18
3	2	2	170	v large	v large	18.25	0.98	2.53	5.55
4	2	1	125	v large	v large	17.11	1.02	1.83	4.01
5	1	0	40	v large	v large	23.07	1.01	5.11	22.40
6	<1	<1	0	v large	v large	144.7	0.23	10.22	44.81
7	<1	<1	0	v large	v large	144.7	0.23	10.22	44.81
8	<1	<1	0	v large	v large	144.7	0.23	10.22	44.81

Table 16 Permissible land usages for three-storey residential units

Inherent hazard class	Number of people/ha (units/ha)			Permissible land usage requirement					
				Sinkhole size (m)					
Table 18	SANS 1936:2012 (various alternatives)		2.0	5	10	20	32.5	50	
1	> 890	0	0						D1
		≤ 600 (80–120)	≤ 400 (≤ 80)						D2 + FP1
		$\leq 600^1$ (80–120)	$\leq 400^1$ (≤ 80)						D3 + FP1 or D4
2	425	0	0						D1
		≤ 600 (80–120)	≤ 400 (≤ 80)						D2 + FP1
		$\leq 600^1$ (80–120)	$\leq 400^1$ (≤ 80)						D3 or D4
3	170	0	0						D1 or D2
		-	≤ 400 (≤ 80)		D3 + FP1				-
		0	0		-				D3 + FP1
		≤ 600 (80–120)	$\leq 400^1$ (≤ 80)						D4
4	125	0	0						D1 or D2
		-	≤ 400 (≤ 80)		D3 + FP1				-
		0	0		-				D3 + FP1
		≤ 600 (80–120)	$\leq 400^1$ (≤ 80)						D4
5	40	0	0						D1 or D2
		-	≤ 400 (≤ 80)	D3 + FP1					-
		0	0	-					D3 + FP1
		≤ 600 (80–120)	$\leq 400^1$ (≤ 80)						D4
6, 7 & 8	0	0	0						D1, D2 & D3
		≤ 600 (80–120)	≤ 400 (≤ 80)						D4

Note 1: Minimum population densities inferred from SANS 1936-1:2012

sinkhole size. Inferred land usage requirement D4 for Inherent Hazard Class 4 is sensible.

SANS 1936:2012 land usage requirement D3 + FP1 for Inherent Hazard Class 5 is sensible compared to that determined in the paper, except perhaps for the limitation on

sinkhole size. Inferred land usage requirement D4 for Inherent Hazard Class 5 is sensible.

SANS 1936:2012 land usage requirements D1, D2 and D3 for Inherent Hazard Classes 6, 7 and 8 are sensible compared to that determined in the paper. Inferred land usage

requirement D4 for Inherent Hazard Classes 6, 7 and 8 is sensible.

CONCLUSION

The objective of the paper is to present a methodology in terms of which the various factors that determine the risk of fatal injury when a sinkhole gives rise to the collapse of a two- or three-storey residence, can be rigorously accounted for on the basis of probability theory and in terms of which it can in principle be shown how mathematical modelling can be applied to address some of the issues on personal safety that are involved. The values for the underlying parameters are estimates based on subjective engineering

judgement and can be adjusted considerably. However, the overall result of the proposed methodology, based on the parameter values considered, is in principle compatible with long-standing observation by the authors.

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