A comparison of limit equilibrium and numerical modelling approaches to risk analysis for open pit mining

by H.T. Chiwaye* and T.R. Stacey*

Synopsis

Risk analysis is an important step in the design of rock slopes in open pit mining. Risk is defined as the product of the probability of slope failure and the consequences of the failure, and is generally evaluated in terms of safety and economic risk. Most of the risk analysis done at present is based on the use of limit equilibrium (LE) techniques in evaluating the probability of failure (POF) of the slopes. The approach typically makes use of full Monte Carlo simulations of the limit equilibrium models, with all uncertain variables randomly varied. The number of required simulations is generally over a thousand, at times as high as 20 000, in order to produce statistically valid results of the POF. Such an approach is clearly not practical when using numerical modelling programs due to the high computational effort required. This paper explores the impact of using numerical modelling instead of the traditional LE techniques in evaluating the probability of slope failure. The difference in the overall assessed risk, in terms of economic impact, for the mining operation is then evaluated. With numerical models, approximate methods are used in the calculation of the probability of failure instead of full Monte Carlo simulations. This paper will use a method called the response surface methodology (RSM) for estimating the POF from numerical analyses. Simple slope models were used to verify the accuracy of the RSM method by comparing the results with those obtained from full Monte Carlo simulations. It is shown that there is good agreement between the POF values computed using full Monte Carlo simulation and those obtained using the RSM method. Finally, the use of numerical modelling in the assessment of risk is shown to bring a significant difference in the result compared with that from LE methods. One of the reasons for the difference is that LE models tend to underestimate the failure volumes and hence the consequences of slope failure.

Keywords

Risk, probability of failure, limit equilibrium, numerical modelling, Monte Carlo simulation, response surface methodology

Introduction

The determination of acceptable slope angles is a very important business planning parameter in open pit mining. However, uncertainties associated with the slope geometry, rock mass properties, loading conditions and model reliability complicate the process of choosing appropriate slope angles. Traditionally, assessments of the performance of open pit mine slopes have been made on the basis of the allowable factor of safety (FOS), which is the ratio of the nominal capacity and demand of the system. By definition, limiting equilibrium is attained when the FOS is equal to unity with lower FOS values signifying failure and higher values signifying stability. The uncertainties alluded to earlier are typically taken into consideration by adopting FOS values greater than unity as the acceptability criterion. However, the main disadvantage of the FOS approach for slope design is that the acceptability criterion is based on case studies and combines the effect of many factors that make it difficult to judge its applicability in a specific geomechanical environment. In other words, the acceptability criterion is empirical and might not be applicable to a geomechanical setting that is different from the ones used as case studies forming the basis of the criterion, Tapia *et al.* (2007).

An alternative to the FOS approach to slope design is the probabilistic method which is based on the calculation of the probability of failure (POF) of the slope. In this case the input parameters are described as probability distributions rather than point estimates of the mean values. By combining these distributions within the deterministic model used to calculate the FOS, the probability of failure of the slope can be estimated. Figure 1 shows the definition of POF and its relationship with FOS according to uncertainty magnitude. The illustration depicts two slopes with mean FOS values of 1.35 and 1.50, with the slope having the higher FOS 'unexpectedly' also having a higher POF as well. This simple illustration reveals the inadequacy of the FOS as a measure of stability due to its inability to take

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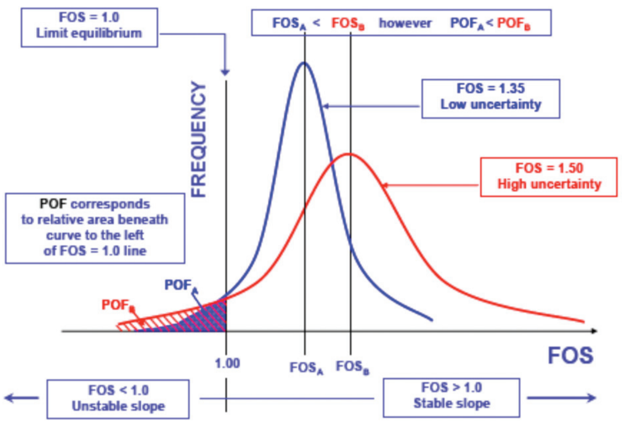

Figure 1—Definition of POF and its relationship with FOS according to uncertainty magnitude (after Tapia *et al.*, 2007)

uncertainty into consideration. The greatest attraction of the POF approach is the explicit representation of the uncertainties in the stability assessment. However, as with the FOS approach, difficulties arise when it comes to defining adequate acceptability criteria for design. As a result very few authors have attempted to define acceptable levels of POF for various engineering structures. Some of the notable contributions in this regard can be found in Priest and Brown (1983), Kirsten (1983), and Read and Stacey (2009).

A slope design approach that alleviates the shortcomings of both the FOS and POF approaches is the risk approach in which the consequences of the failure are taken into account. The risk associated with a slope failure is generally described as the product of the POF and the consequences of the slope failure. The risks associated with a major slope failure can be broadly categorized by the following consequences: injury to personnel, damage to equipment, economic impact on production, force majeure (a major economic impact), industrial action, and public relations (such as stakeholder resistance). Commercial risks quantified must be acceptable to the mine owners, and are related to the probability of failure. Hence the acceptability criterion for economic risk is set by mine management. Safety risks are usually regarded as being beyond management discretion with most companies seeking compliance with industry norms or other indicators of societal tolerance. In other words, the risk approach suggests that the stability of the slope is not the end objective, but rather that safety is not to be compromised as the economic impact of the chosen slope angles is optimized (Read and Stacey, 2009).

Before carrying out the risk analysis, the POF of the slope has to be evaluated. Most of the methods used to compute POF values for slopes cannot be used efficiently with numerical modelling codes; hence limit equilibrium (LE) programs have been the tool of choice in risk analysis work. Due to their much greater ability to handle problems with complex failure mechanisms, complex geometry and geology, and *in situ* stress conditions, numerical models have found increased usage among slope designers. Therefore it would be desirable to be able to practically incorporate numerical analysis into the risk analysis process. This paper will describe a method that can be used to incorporate numerical models in probabilistic analysis, the response surface methodology (RSM), and also compare the risk assessed using numerical tools with that obtained from LE analysis. It is, however, worthwhile to first give a brief overview of the methods currently used to compute POF values, before describing the RSM.

Methods of calculating probability of failure

The most common tool used in probabilistic analyses is Monte Carlo simulation. In this method the analyst creates a large number of sets of randomly generated values for the uncertain parameters and computes the performance function (FOS for instance) for each set. A simulation can typically involve over 20 000 evaluations of the model. By using random inputs, the deterministic model is essentially turned into a stochastic model. Some advantages of the Monte Carlo method are its simplicity, its flexibility in incorporating a wide variety of probability distributions without much approximation, and its ability to readily model correlations among variables (Hammah and Yacoub, 2009). Most limit equilibrium slope stability analysis packages use Monte Carlo simulation to compute probabilities of failure. Examples are SLIDE, RocPlane, and SWEDGE (RocScience Inc.). However, although straightforward, Monte Carlo simulation can be very expensive computationally, especially when multiple runs are required to calculate sensitivities and/or when low probabilities are needed (Baker and Cornell, 2003; Baecher and Christian, 2003). As a result, it has not been practically feasible to use Monte Carlo simulation to calculate POF values from numerical models due to the extensive runtimes required in numerical modelling.

An alternative method that has been widely used in civil engineering applications is the first order second moment method (FOSM). The FOSM is an approximate probabilistic method that is based on a Taylor series expansion of the performance function. All terms in the Taylor series that are of a higher order than one are assumed negligibly small and then discarded (Baecher and Christian, 2003). From the remaining first order terms, the probability that the performance function is less than any given value can be calculated. One of the great advantages of the FOSM method is that it reveals the relative contribution of each variable in a clear and easily tabulated manner (Morgan and Henrion, 1990; Baecher and Christian, 2003). This is very useful in deciding what factors need more investigation and also in revealing a factor whose contribution cannot be reduced by any realistic procedure. Many of the other reliability analysis methods do not provide this information. Kinzelbach and Siegfried (2002) used the FOSM method to quantify the uncertainty in groundwater modelling and the results compared well with Monte Carlo simulation results. However, since FOSM is based on a Taylor series expansion, the evaluation of partial derivatives or their numerical approximations is critical to the use of the method (Harr, 1987). The attainment of these partial derivates is generally very difficult and is impossible in some cases such as when the performance function is given implicitly in the form of design charts. This is the greatest limitation of the FOSM method. Another shortcoming of the FOSM approach is that the results depend on the particular values of the variables at which the partial derivatives are calculated (Christian, 2004). At present, no slope stability analysis program uses the FOSM method to calculate POF values. An extension of the FOSM called first order reliability method (FORM), also called the 'Hasofer-Lind' approach, has been used to overcome some of the weaknesses of FOSM. However, as in FOSM, the requirement to evaluate partial derivates makes the FORM approach impossible to use in some cases where the performance function is not explicitly given, (Baecher and Christian, 2003).

Another method that has enjoyed widespread use among the geotechnical community is the point estimate method (PEM) developed by Rosenblueth (Rosenblueth, 1975; Rosenblueth, 1981). The PEM uses a series of point-by-point evaluations (called point estimates) of the performance function at selected values (known as weighting points) of the input random variables to compute the statistical moments of the response variable. The method applies appropriate weights to each of the point estimates of the response variable to compute moments. Hoek (2006) noted that, while the PEM technique does not provide a full distribution of the output variable, as does the Monte Carlo method, it is very simple to use for problems with relatively few random variables and is useful when general trends are being investigated. When the probability distribution function for the output variable is known, for example, from previous Monte Carlo analyses, the mean and standard deviation values can be used to calculate the complete output distribution. Hammah and Yacoub (2009), quoting Christian and Baecher (2002), stated that, 'Although the method is very simple, it can be very accurate'. Harr (1987) also stated, 'The method is straightforward, is easy to use, and requires little knowledge of probability theory.' However, a great limitation of the original PEM for multiple variables is that it requires calculations at $2N$ points, where N is the number of uncertain variables. When N is greater than 5 or 6, the number of evaluations becomes too large for practical purposes, (Baecher and Christian, 2003). Two relatively simple methods for reducing the number of points in the general case to $2N$ or $2N + 1$ have been proposed (Harr 1987; Hong 1996, 1998). Unfortunately, these methods for reducing the number of points move weighing points further from mean values as the number of variables increases, and can lead to input values that extend beyond valid domains (Hammah and Yacoub, 2009). Since the number of uncertain parameters in slope stability analyses can be large, the PEM approach does not present a practical way of computing POF values with numerical methods.

A method that has been gaining attention in geotechnical engineering is the response surface methodology (RSM) which will be described in the next section.

Description of response surface methodology (RSM)

The RSM consists of three separate steps which are summarized in Figure 2. The first step involves the use of the stability model (for example SLIDE, Phase2, UDEC, etc.) to compute the FOS at various combinations of the input variables and then uses regression techniques to create a curve (or a surface in multi-dimensional space) that gives the response of the FOS to changes in the inputs. This curve is what is called the response surface. In the second step Monte Carlo Simulation is carried out, using the response surface instead of the stability model, in a spreadsheet package such as Microsoft Excel. The third step involves the statistical analysis of the distribution of FOS values generated by the Monte Carlo simulation in step 2. From this distribution the POF can then be calculated as the number of trials that gave a FOS less than one divided by the total number of trials. Since the Monte Carlo simulation is carried out in Excel making use of regression equations, the task takes very little time compared with traditional Monte Carlo simulation that uses the stability model. As an example, 50 000 trials will typically take between five and ten seconds on a 1GHz processor. This means that the only time that is needed in RSM analyses is the running of the stability models needed to create the response surface.

Several methods for selecting the small sample of scenarios to run and consequently create the response surface are in use. The most common method involves perturbing one input at a time, keeping the rest at their nominal values. Though this sampling approach is not very efficient computationally (Morgan and Henrion, 1990), it enjoys the benefit that it is simple and easy to implement in practice. Using this sampling approach the number of required runs is given by $2N + 1$, where N is the number of uncertain variables. The linear form of this relation means that the number of runs required remain within reasonable limits even for large numbers of variables e.g. a slope with 3 variables requires only 7 runs whereas 10 variables require 21 runs. As a result the RSM allows computationally expensive modelling tools to be incorporated into probabilistic analyses.

The basic mathematical theory behind the RSM method is outlined below, as adapted from Tapia *et al.* (2007). In the response surface method, with N uncertain variables contributing to the distribution of FOS, this distribution of FOS could be viewed as being represented by a function given in Equation [1]. In N-dimensional space, defined by variables x_1- x_N, values of R can be viewed as a surface (called the response surface). The response surface method assumes that the effects of each variable x_i on the FOS are independent of the other variables. Inaccuracies arise because this assumption is not true in general (Tapia *et al.*, 2007). However, these inaccuracies are minimized by calculating the best-estimate of FOS first using the best-estimates for each of the uncertainties x'_1, x_2',...,x'_N using Equation [2]. FOS_{bc} is the best-estimate (mean) value and is referred to as the base case.

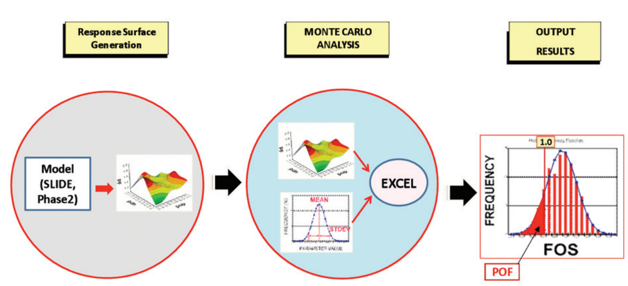

Figure 2—Diagrammatic representation of the response surface methodology

$$FOS = R(x_1, x_2, \ldots, x_N) \quad [1]$$

$$FOS_{bc} = R(x'_1, x'_2, \ldots, x'_N) \quad [2]$$

The sensitivity of the FOS to the various inputs is investigated by varying each variable while keeping the rest constant at their best estimate values as shown in Figure 3. A new FOS is then calculated using this new set of input variables with the aid of the relevant stability analysis tool e.g. SLIDE, Phase2, or FLAC. This sensitivity is quantified by a parameter termed the sensitivity parameter, β, defined as:

$$\beta(x_i - x'_i) = \frac{R(x_i, x'_2, \ldots, x'_N)}{FOS_{bc}} \quad [3]$$

The sensitivity parameter, β, is defined twice for each variable, one point on each side of the best estimate (referred to as the '-'and '+' case). β is plotted against the normalized variable and regression techniques used to fit a 2nd order polynomial that gives the value of β for any given value of the normalized variable. These regression lines can also be defined using piece-wise linear functions.

The points at which the '-' and '+' case are evaluated can be located anywhere in the region of interest. For most problems, points located at '-' and '+' one standard deviation give good results. When modelling a system with N variables, *N* 2nd order equations relating β to the variable are formed. For any random combination of the input variables, a value of the factor of safety (FOS_i) is obtained by:

$$FOS_i = FOS_{bc} \times \beta_{1i} \times \beta_{2i} \times \ldots \times \beta_{Ni} \quad [4]$$

Multiple Monte Carlo simulations (numbering up to 50 000) in a spreadsheet package such as Microsoft Excel, using the various probability distributions of the uncertainties, results in a distribution of FOS from which the POF can be calculated as:

$$POF = P(FOS < 1) \quad [5]$$

Figure 4 illustrates the definition of the probability of failure from Monte Carlo simulations. In essence, the RSM approach seeks to replace the slope stability analysis tool with a model constructed using only a few runs of the stability model and regression techniques. Full Monte Carlo simulations are then executed by randomly sampling the input variables and estimating the FOS from the RSM model and not from the actual stability model.

Using the RSM, it has been possible to use very sophisticated numerical modelling programs in evaluating probabilities of failure for complex geomechanical scenarios. For example, Tapia *et al.* (2007) describe an approach which uses Monte Carlo simulation together with the RSM to carry out a risk analysis for an open pit mine using the programs FLAC3D (Itasca Inc.), SLIDE and SWEDGE. The same approach was used by Contreras *et al.* (2006) in a risk analysis for Cerrejon mine. Mollon *et al.* (2009) used the RSM together with the FORM to carry out probabilistic analysis of a circular tunnel using the 3D numerical modelling code FLAC3D. The most extensive applications of RSM are in the particular situations where several input variables potentially influence some performance measure (factor of safety, for instance) or quality characteristic of the process (Carley *et al.*, 2004). Morgan and Henrion (1990) recommend the use of RSM for large computationally expensive models. One advantage of RSM is their flexibility in that they can handle any probability distribution as well as performance functions that are not given in explicit forms, for example, in design charts. Since no work has been carried out in attempting to validate the results from RSM computations in slope stability analyses, the following sections will give a more detailed description of the RSM, as well as some validation examples of the method in slope stability analyses.

Verification of RSM for slope stability analysis

The verification of the RSM formulation involved carrying out full Monte Carlo simulation in SLIDE (simply called SLIDE Monte Carlo in this section) and comparing the results with the RSM approximation. In other words, the first approach involved running 10 000 SLIDE models with randomly sampled strength parameters. The POF was then defined as the number of models that resulted in a FOS of less than one divided by the total number of models, which were 10 000 in this case. The RSM approximation involved running only a few SLIDE runs (5 or 13, depending on the slope geometry as described below) and using the results to create the response surface as explained in the previous section. Monte Carlo simulations were then carried out in an Excel spreadsheet to

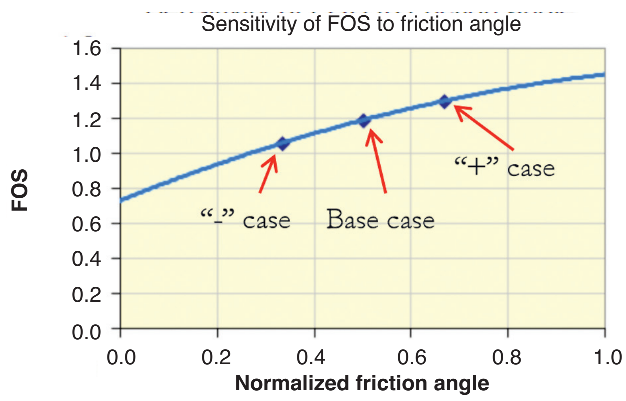

Figure 3—Sensitivity of FOS to friction angle showing the locations of the '-'case, the '+' case, and the base case

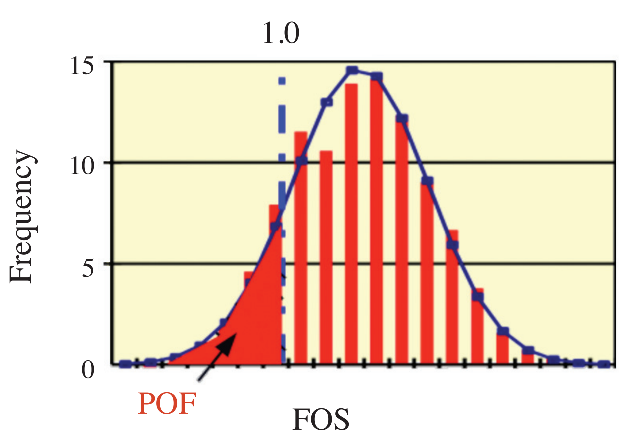

Figure 4—Graph showing the distribution of FOS from Monte Carlo simulations and definition of the probability of failure

determine the POF. The POF values obtained using the two different approaches were compared. Two different models were used for the analyses: a homogeneous slope and a slope consisting of 3 materials as shown in Figure 5. In the model with more than one rock type, the slope limits in the SLIDE analysis were located so as to force the slip surface to pass through all rock units. For the analyses, only the cohesion and friction angle were regarded as uncertain parameters.

Since some parameters used in slope stability analysis are known to be correlated in some way (e.g. cohesion and friction angle), the ability of the RSM formulation to handle correlated variables was investigated. To achieve this, correlation coefficients (CC) between the friction angle and the cohesion of 0.5, 0, and -0.5 were specified so as to cover conditions of positive correlation, independent variables, and negative correlation respectively. In addition to the CC, another parameter that was varied during the analyses is the coefficient of variation (CV). The CV is simply the ratio of the standard deviation to the mean and is a measure of the spread of the distribution. A low CV implies a distribution with the data points concentrated around the mean, whereas a high CV implies wide scatter among the data.

The normal distribution is an open distribution which allows the variable to assume negative variables. However, most of the input parameters in geotechnical analyses, such as cohesion and friction angle, cannot take on negative values and are better modelled using lognormal or beta distributions (Baecher and Christian, 2003). The ability of the RSM in handling non-normal variables was therefore studied by assuming lognormal distributions to the cohesion and friction angle in some of the homogeneous models. Table I gives a summary of the mean values of the cohesion and friction angle of the materials used in the RSM verification models. The material named 'homogeneous' refers to the material constituting the homogeneous model. Materials A to C refer to the materials making up the heterogeneous slope consisting of three materials as shown in Figure 5. Three different distributions were created by varying the CV from 0.1 to 0.3, keeping the mean values constant in all models. Since the normal distribution is an open distribution, the sampling was carried out in the region encompassing 3 standard deviations both sides of the mean (99.7% of the data in a normal distribution lie within ± three standard deviations from the mean).

The RSM was implemented using an Excel spreadsheet, together with a commercial add-in, Crystal Ball (Oracle Inc.). In all cases, the model was evaluated at two points placed at ±1 standard deviation from the mean, as described earlier. A piecewise linear interpolation/extrapolation scheme was used in the RSM. Monte Carlo simulation on the response surface was carried out using 50 000 trials, with a Latin Hypercube sampling scheme. Figure 6 shows the results of the analyses

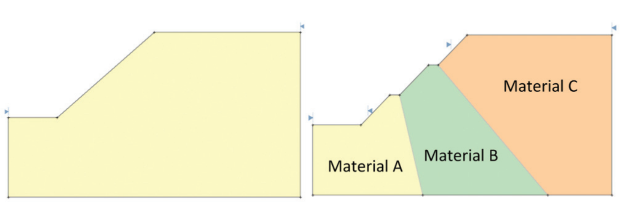

Figure 5—The slope geometries used for validating the RSM approach to probabilistic slope design: (a) homogeneous slope (b) heterogeneous slope

Table I

Strength parameters used for the SLIDE RSM verification models

Rock type	c (kPa)	ϕ (°)	γ (kN/m^3)
Homogeneous	500	30	27
Material A	35	22	27
Material B	85	27	27
Material C	120	25	27

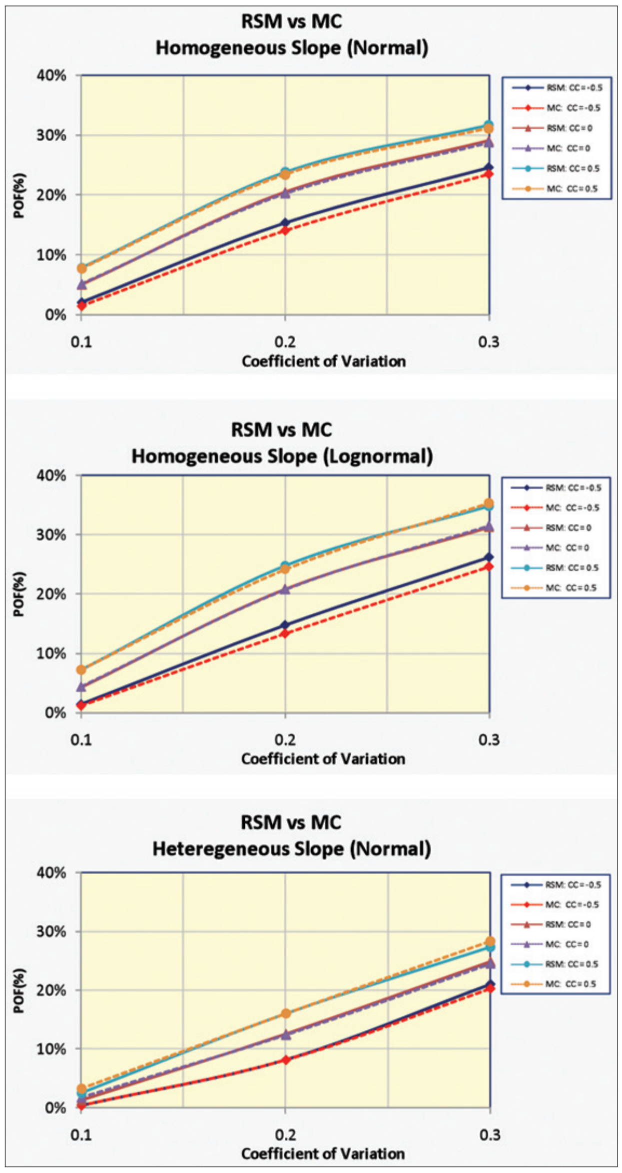

Figure 6—Comparison of POF values from Monte Carlo Simulation in SLIDE (MC) and the RSM method for different levels of correlation between *c* and ϕ, (a) Homogeneous slope, normally distributed variables (b) Homogeneous slope, lognormally distributed variables (c) Heterogeneous slope

for the homogeneous and heterogeneous slopes. In the graphs, the dotted curves marked MC represent the results of the full Monte Carlo simulations in SLIDE (10 000 SLIDE runs) and the solid curves marked RSM show the results of using the RSM approach to compute the POF (5 SLIDE runs for the homogeneous slope and 13 SLIDE runs for the heterogeneous slope). The results show good agreement between the SLIDE Monte Carlo simulation and the RSM formulation. The RSM results compared well with Monte Carlo simulations for both normal and non-normal variables as well as for the correlated and uncorrelated variables. From these results it can be concluded that the RSM can be used with confidence in probabilistic slope stability analysis. These results highlight the computational efficiency of the RSM approach, a feature that makes it particularly well suited for use with computationally expensive analysis tools such as numerical models.

POF comparison of LE and numerical models

The POF from LE models (computed using SLIDE) was compared with the POF from numerical models (computed using PHASE 2) for various geomechanical parameters. A homogeneous 600 m high slope, having a slope angle of 45° was used for the analyses, Figure 7. The POF values for all the analyses were calculated using the response surface methodology as described earlier.

The analyses made use of three different sets of strength parameters shown as model-1, model-2, and model-3 in Table II. As can be seen in the table, the cohesion and friction angle had the same mean values in all the three models while the dilation angle, k ratio, and locked in stresses (LIS) were different. Therefore, since LE analyses do not take into account the k ratio, dilation angle and LIS, only one SLIDE model was run. On the other hand, three PHASE 2 models were run using the different strength parameters given in Table II. The values in the table represent the mean values of the given parameters, with the variability represented by assuming a normal distribution and a coefficient of variation (CV) of 0.3 for each variable. The CV is simply the ratio of the standard deviation to the mean.

Only the friction angle and cohesion were considered as uncertain inputs while the rest of the strength parameters were considered deterministic. The results of the analyses are shown in Figure 8. The SLIDE result is similar to the Phase2 result for model-1 but significantly different from model-2 and model-3. Increasing the k ratio, dilation angle, and locked in stresses in model 2, had the effect of improving the stability of the slope. Consequently the POF values for model-2 are lower than the SLIDE result. Further increases in the dilation angle, k ratio, and LIS resulted in even lower POF values for model-3. Regardless of the correlation coefficient between the input parameters, the results show that parameters like the k ratio, the dilation angle, and the locked in stresses do have a substantial influence on the calculated probability of failure of a slope. Since these parameters are not included in an LE analysis, the consequence is that the LE and numerical modelling POF values for the same slope can differ significantly as indicated by Figure 8. An analysis of the FOS distributions showed that the mean FOS changes significantly from model-1 to model-3. It is this change in the mean FOS that results in the lower POF for model-2 and model-3.

Sensitivity analysis

In the previous results the only variable inputs were the cohesion and friction angle. Two more scenarios were considered to investigate the major contributors to the uncertainty in slope stability. In the first scenario, the dilation angle was added as a third variable and in the second scenario the k ratio was added as a fourth variable. A coefficient of variation of 0.3 was assumed for all input variables. The results for the analysis, using 'model-3', are shown in Figure 9.

The graph shows that increasing the number of variables has little effect on the POF. It is important to note that the contribution to the POF of an input parameter depends on

Table II

Uncertain input parameters for models used to compare POF values

Parameter	Model-1	Model-2	Model-3
Cohesion (kPa)	640	640	640
Friction angle (°)	30	30	30
Dilation angle (°)	0	7	15
k ratio	1	2	3
Locked-in stress (MPa)	0	1	3

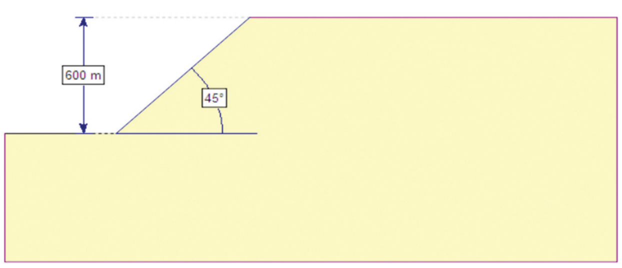

Figure 7—Slope geometry used for the comparison of SLIDE and Phase2 POF computations

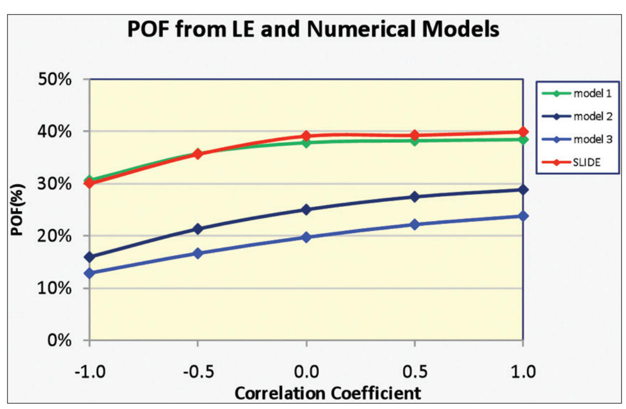

Figure 8—A comparison of the POF determined from SLIDE and three Phase2 models for different levels of correlation between the cohesion and friction angle

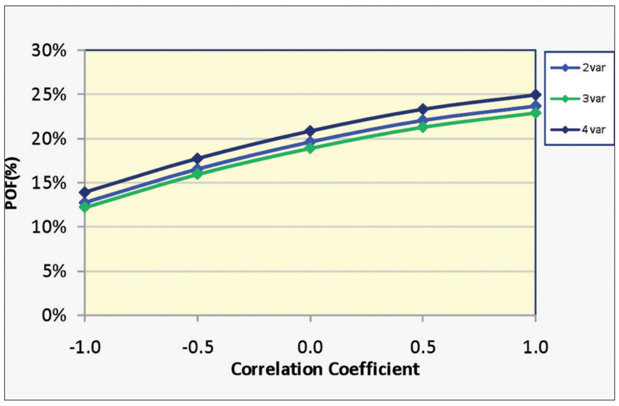

Figure 9—The effect on POF of increasing the number of uncertain variables from two to four

two things; the variability of the parameter (measured by the coefficient of variation) and the influence the parameter has on the stability of the slope. Results from a sensitivity analysis for the models show that the strength parameters, cohesion and friction angle, have a greater influence on the POF than the other variables such as k ratio and dilation angle. This then explains the fact that including the k ratio and dilation angle as uncertain variables does not significantly change the computed POF, (Figure 9). Increasing the variance of the other parameters like dilation angle will certainly increase their relative contribution to the POF, but the relative influence will still be small compared to the strength parameters like cohesion and friction angle. The implications of these results is that, when carrying out a probabilistic analysis with numerical models, the number of uncertain variables can be reduced by ignoring the variability in parameters like the k ratio and dilation angle. This means that fewer runs of the numerical models will be needed to create the response surface from which the POF is determined. It should be noted, however, that this conclusion is valid for cases in which shear failure of the rock is the failure mechanism at play. Assuming a different failure mechanism may result in different dominant parameters in the probabilistic analysis.

The plot in Figure 10 summarizes the findings from the comparison of the POF determined with LE and Phase2 models. The graph is for models assuming only two uncertain inputs, cohesion and friction angle, and complete independence between those inputs. When parameters not included in an LE analysis (like dilation angle) are set at values that eliminate their effect e.g. by setting dilation angle at zero (coinciding with model 1 in Table II), the POF from the Phase2 and SLIDE models will be the same. However, if these parameters take up other values, the POF from Phase2 will deviate significantly from the SLIDE result as shown in the case of model-2 and model-3 in Figure 10. These differences can be significant, as the result for model-3 shows a 40% POF from SLIDE compared with 20% from Phase2.

Failure volumes

The failure volumes that resulted when all input parameters were at their mean values were taken to be representative of the expected failure volume in the risk analysis. A comparison of the failure volumes for the three Phase2 models and the SLIDE models is shown in Figure 11.

In general, the failure volumes predicted by the LE models are lower than those from the numerical models. Figure 12 shows a typical comparison of the failure surfaces predicted by the two modelling approaches. The higher volumes from the numerical models are due to the fact the predicted failure surface extends further behind the crest in numerical models than in LE models.

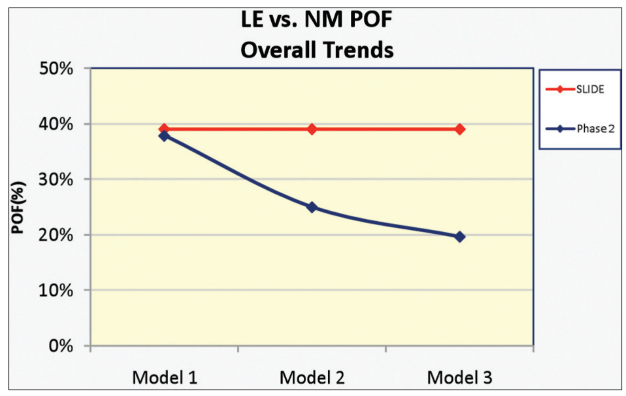

Figure 10—SLIDE and Phase2 POF results for models with 2 independent input variables

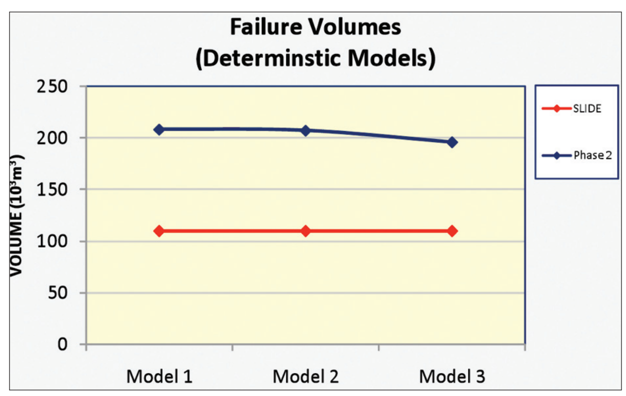

Figure 11—Failure volumes from SLIDE and Phase2 for scenarios where all inputs are at their mean values

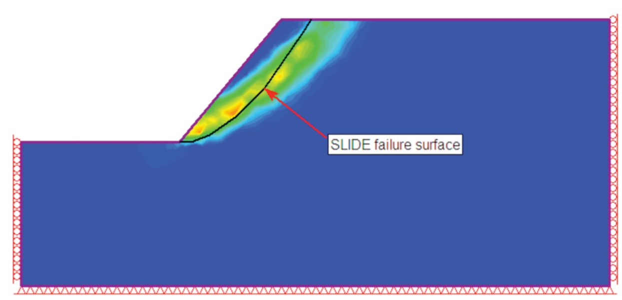

Figure 12—Phase2 result showing contours of maximum shear strain which depict the failure surface. The SLIDE failure surface has been superimposed onto the image

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Risk analysis comparison of LE and numerical models

The simplified event tree in Figure 13 was used to carry out the risk analysis. The probabilities associated with each branch of the event tree are given in Table III. These were hypothetical values adopted arbitrarily for the analyses in this paper. The probability of overall slope failure was taken from the results shown in Figure 10. Other sources of variability not accounted for in the stability analysis were ignored, e.g. mining methods, geology, etc. Of the three possible economic consequences in Figure 13, only the loss of profit was considered in this paper. The reason for this was to keep the work simple and tractable since the aim is to investigate trends rather than determining actual quantitative risk figures.

The risk analysis carried out considered only the economic impact of the slope failures, assuming that the probability of loss of profit is representative of all the economic consequences of the slope failure. For the purposes of the current investigation, the total loss of profit was assumed to be made up of two components: the cost of cleaning up after a failure and the cost due to production delays. The assumption is that the slope being modelled is just above an access ramp which happens to be the only access into the pit. Therefore, any slope failure will result in delays to production until the failed volume has been cleared. This was a hypothetical scenario in order to compare the risk determined from the two stability analysis methods without confusing the analysis with financial calculations that would cloud the main issue under investigation. Other contributors to loss of profits such as damage to equipment were not included in the analysis. The cost of clearing was assumed to be R500/m^3 and the cost of lost production was assumed to be R100 000/h. It is important to note that the choice of these costs has no bearing on the final comparison between the risk assessed using LE methods and that from numerical methods. However, the values have been chosen to reflect typical costs associated with open pit mining operations. The rate of clearing of the failed material was assumed to be 5 000 t/h.

The probabilities at each stage of the event tree in Figure 13 are accumulated to give the probability of total loss of profit, after the third stage. Once the total loss of profit was determined for any model, it was multiplied with the probability of loss of profits to come up with a risk quantity. It is this quantity which was used to compare the risk assessed by the different modelling tools. Technically speaking, this risk quantity is the expected loss of profits.

$$Risk = P(\text{Loss of profit}) \times \text{Loss of profit}$$

This risk quantity was considered to be representative of the economic risk associated with the mining operation.

Results

The POF results obtained in the models described above and reported in Figure 10 were input into the event tree of Figure 13 so as to determine the probability of incurring loss of profits. Since a slope failure can generally result in three different economic consequences (normal operating conditions, loss of profits, and force majeure), the probability of loss of profit is always less than or equal to the probability of overall slope failure. Figure 14 shows a comparison of the

Table III

Probability values used in the event tree used to determine probabilities of occurrence of various economic consequences

Likely event	Probability	
	Yes	No
Production affected?	0.7	0.3
Can contracts be met?	0.5	0.5
Additional costs?	0.2	0.8
Is cost prohibitive?	0.2	0.8
Production replaced by spot?	0.9	0.1

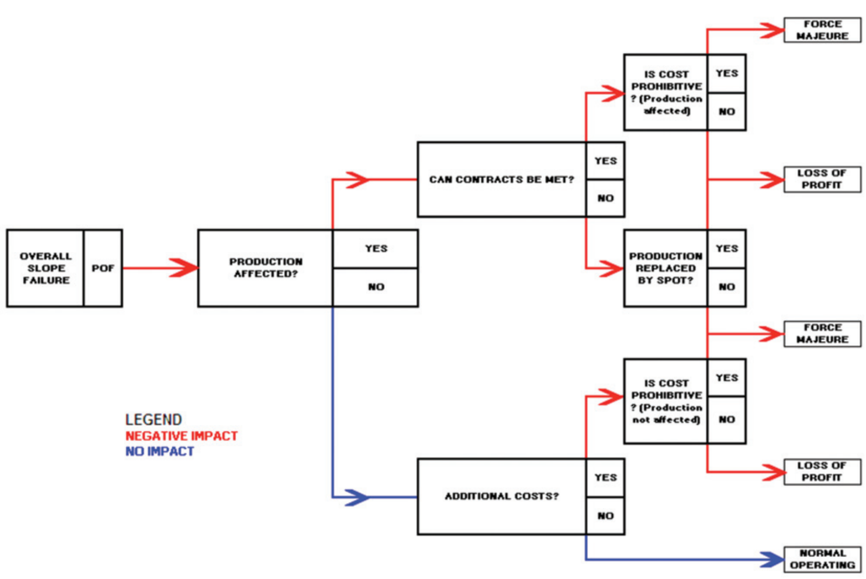

Figure 13—The event tree used for the evaluation of economic impact of slope failures (Steffen *et al.*, 2008)

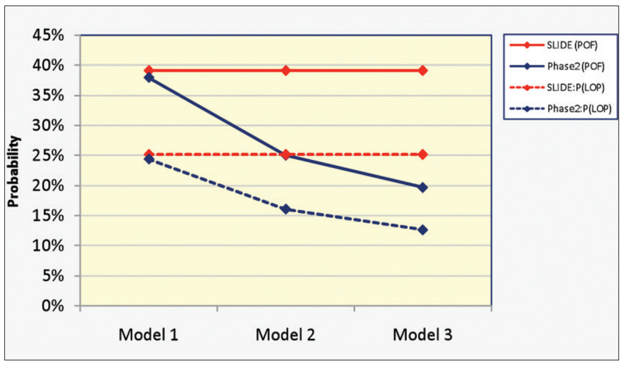

Figure 14—Adjustment of the probability of overall slope failure (POF) to the probability of loss of profit, P(LOP), for the SLIDE and Phase2 models

probability of overall slope failure and probability of loss of profits for the SLIDE and Phase2 models while Figure 15 shows the comparison of the assessed risk levels for the Phase2 and SLIDE models. The assessed risk from the Phase 2 analyses is larger than from SLIDE in model-1 and model-2, whereas in model-3, Phase2 gives a risk assessment slightly less than the SLIDE result. The observed trends in Figure 15 can best be described by comparing the SLIDE model with the three Phase2 models one by one.

Phase2 model-1 gave a POF value almost identical to the SLIDE model (Figure 10) but the failure volume predicted by the Phase 2 model is approximately double that predicted by the SLIDE model (Figure 11). As a result, the risk assessed using the SLIDE model is approximately half that predicted using the Phase 2 model, Figure 15. Thus, using a LE tool to assess the risk for this particular combination of strength parameters would result in a very optimistic value for the risk i.e. the expected loss of profits. For model-2, the assessed risk is almost the same with SLIDE and Phase2 models. However, it must be emphasized that this is simply a coincidence brought about by the fact that the POF from the Phase2 model-2 is much smaller than that from the SLIDE model whereas the failure volume from the Phase2 model is much higher. For model-3, the POF from the Phase2 analysis is sufficiently lower than the SLIDE POF to give risk levels slightly lower than the SLIDE estimates.

Summary of results

Response surface methodology verification

The calibration of the response surface methodology was carried out by comparing its results with those obtained from Monte Carlo simulations within the program SLIDE. The results showed very remarkable agreement, validating the use of RSM in probabilistic slope stability analysis. The main advantage of the method is the requirement for only a few model runs compared to the traditional Monte Carlo methods that require thousands of model runs. For example, in determining the POF for a homogeneous slope with two uncertain inputs, five SLIDE runs were used to create a response surface. The POF was then determined by carrying out Monte Carlo simulation using the response surface instead of the SLIDE model. This gave the same POF as 10 000 Monte Carlo SLIDE runs produced within the SLIDE program. This outstanding efficiency makes the RSM a very practical method of incorporating numerical models directly into probabilistic analysis. Other interesting conclusions from the calibration were that the RSM formulation can be used with correlated or independent variables, as well as with non-normally distributed variables.

Comparison of FOS from LE and numerical models

There are parameters not included in a limit equilibrium analysis which do have an effect on slope stability. Examples of these are the dilation angle, the horizontal to vertical stress ratio (k ratio), and the locked-in horizontal stresses. Generally these parameters have the tendency of improving the stability of the slope, thus making LE analyses somewhat conservative. Another interesting result was that the numerical models almost always predict greater failure volumes than the LE models. The failure surface indicated by Phase2 is always flatter at the top than the SLIDE surface. The reason for this result is probably that the limit equilibrium solution identifies only the onset of failure, whereas the numerical solution includes the effect of stress redistribution and progressive failure after movement has been initiated. Thus, numerical models are better tools for determining the failure surfaces/volumes for slope failures.

Comparison of POF from LE and numerical models

Those parameters, such as dilation angle, not accounted for in LE analyses tend to lower the probability of failure of slopes. This means that the probabilities of failure reported by LE programs are generally on the conservative side. However, the chief contributors to variance in the FOS results remain the strength parameters (c and ϕ in the Mohr Coulomb failure criterion; UCS, m, s, a in the Hoek-Brown failure criterion, etc.) and the other parameters (dilation angle, k, ratio etc.) contribute little uncertainty in the FOS. It is the effect of the 'other' parameters on the mean FOS that lowers the POF, not their variance. The implication of this observation is that the probabilistic analysis can be carried out with fewer variable parameters, by assuming inputs such as dilation angle and the k ratio to be constant, thereby reducing the number of runs required in the RSM implementation.

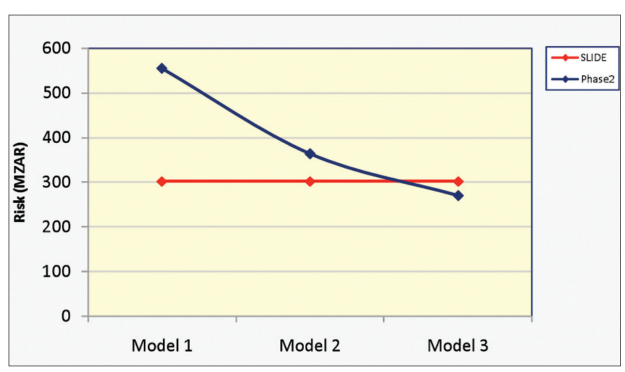

Figure 15—Overall risk associated with the slope determined using SLIDE and Phase2 stability models

Comparison of assessed risk from LE and numerical models

The assessed risk for a LE analysis is generally different from that obtained using numerical tools. The results show that for some models the risk from numerical models is greater than that from LE analyses whereas for other models the risk determined from numerical models is less than that from LE methods. The question of which method, LE or numerical, is more conservative in terms of risk assessment does not have a universal answer. Chiwaye (2010) also carried out risk analysis on a diamond open pit mine case study using LE and numerical modelling approaches, and reached similar conclusions.

Conclusions

The research described in this paper has demonstrated a practical way in which numerical analyses, and thus the benefits they add, can be easily incorporated into probability of failure determinations for rock slopes. The major conclusion from the analyses carried out in this paper is that using numerical models, instead of LE models, in carrying out risk analyses results in significant differences in the assessed risk. In some cases the LE models give a lower estimate of the risk than numerical models whereas in other cases they give higher estimates of the risk. These differences in the assessed risk are mainly due to two reasons:

- LE methods are simplistic in their approach and ignore some very important complex mechanisms of rock mass failure. Parameters such as the dilation angle and the horizontal to vertical stress ratio (k ratio), to name a few, are not included in an LE analysis, and yet they can have an effect on slope stability. The effect of these parameters tends to indicate greater stability than that given by a LE analysis. This means that, generally speaking, LE methods are conservative in terms of their stability (both FOS and POF) estimates. For some models the POF values from LE models were higher than those from numerical models by a factor of up to 2.
- The failure volumes (and hence the consequences of failure) predicted by LE models are almost always less than those from numerical models. Limit equilibrium solutions only identify the onset of failure, whereas numerical solutions include the effect of stress redistribution and progressive failure after movement has been initiated. Thus the failure surfaces from numerical models extend further behind the crest than in LE models.

For the above reasons, the risk assessed using numerical analyses is considered to be a more reliable estimate of the risk associated with a rock slope. Therefore, it is recommended that numerical modelling be incorporated into risk analyses in addition to LE models, possibly replacing them when more confidence has been gained. When such confidence has been gained, the outputs may allow steeper slopes to be designed for the same risk level, with corresponding economic benefits.

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