Statistics or geostatistics? Sampling error or nugget effect?

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Synopsis

What is a nugget effect? In the early development of geostatistics, the term 'nugget effect' was coined for the apparent discontinuity at the beginning of many semivariogram graphs. This name was chosen to reflect the large differences found between neighbouring samples in 'nuggety' mineralizations such as Wits gold reefs. Geostatistical theory assumes that the difference between a sampled value and a potential repeat sample at the same location is actually zero. Included in this 'nugget effect' would be true variation between contiguous samples due to the nature of the mineralization, micro-fracturing, nugget or crystal size, and so on. Also included in the nugget effect would be any 'random' sampling variation which might occur due to the method in which the sample was taken, the adequacy of the sample size, the assaying process, etc.

Arguments were put forward that 'sampling errors' actually exist at zero distance. Some geostatistical schools actually maintain that the 'nugget effect' is all sampling error. This would imply that 'perfect' sampling would eliminate the nugget effect entirely.

There is now a dichotomy both in the geostatistical world and in the software packages provided for geostatistical analyses. It may seem academic to argue over whether the semivariogram model should take a value of zero, a value equal to the nugget effect, or a partial value at distance zero. However, the decision can have a profound effect on both the estimated resource and in our confidence on that resource.

Whereas most geostatistical texts define the semivariogram model as taking the value of zero at zero distance, others imply that the full nugget effect should be used at zero distance. For example:

- The nugget effect refers to the nonzero intercept of the variogram and is an overall estimate of error caused by measurement inaccuracy and environmental variability occurring at fine enough scales to be unresolved by the sampling interval[3]
- Christensen[4] has shown that the 'nugget effect', or non-zero variance at the origin of the sernivariogram, can be reproduced by a measurement error model
- The nugget effect is considered random noise and may represent short-scale variability, measurement error, sample rate, etc.[5].

In many training texts and Web courses, the definition of the semivariogram is ambiguous as the formulae for semivariogram models is not actually specified at zero distance[6,7,8].

Geevor Tin Mine, Cornwall

Our discussion of sampling error versus spatial discontinuity is illustrated using a case study in which contiguous samples were taken along a vein. Each sample was then split and assayed four times.

Mining in Cornwall dates back to between 1 000 and 2 000 bc, when Cornwall is thought to have been visited by metal traders from the eastern Mediterranean. They even named Britain as the 'Cassiterides'—'Tin Islands'. Cornwall along with the far west of Devon provided the vast majority of the United Kingdom's tin and arsenic and most of its copper. Initially the tin was found as alluvial deposits in the gravels of stream beds, but before long some sort of underground working took place. In fact, where the tin veins outcropped on the cliffs, underground mines sprung up as early as the 16th century.

Some background on the project might be useful to understand our concerns. In West Penwith, tin occurs as 'black tin' SnO_2 in a hydrothermal vein which intruded into cracks in the granite rocks as they cooled. The Simms Vein, which was studied extensively, is almost vertical and averages around 23 inches over the study area.

Co-ordinates are in feet along section and elevation above an arbitrary base level. In length the study area is 1 500 feet and depth is from 600 feet below surface to 1 400 feet. Every 100 feet, a 'development drive' is driven horizontally along the length of the vein. Prior to 1972, the general sampling interval was 5 feet along the development drives with a change to 10 feet shortly before that date.

The thickness of the vein or 'lode' is measured to the nearest inch. Thicknesses as high as 127 inches and as low as 1 inch were

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encountered in the development drives in Simms Vein. Samples around 6 inches (15 cm) wide were chipped across the vein and bagged separately. The bagged sample was returned to surface for 'vanning' assay which produces a value for 'content of recoverable cassiterite'. This is expressed in pounds of cassiterite per ton of ore (lb/ton). The width of the vein is measured in inches on site.

In this case study, we discuss whether the nugget effect can be interpreted as sampling error or inherent geological variability (or both). The assaying technique which is used to produce the measured grades is examined in detail with a special sampling scheme.

Statistical analysis

Roughly 2 700 development samples were collected from the Simms Vein at Geevor Tin Mine in the early 1970s. Prior to March 1971 development drives were sampled at five foot intervals, but this was changed to ten feet, and in 1976 to three metres. In the study area all development except on 600 foot level and for minor westward extensions on the 1 000, 1 100 and 1 200 foot levels was completed prior to March 1971. Thus, virtually all drive sampling is available at five foot intervals. Figure 1 shows the study area. Each circle denotes a sample.

Statistical behaviour of the development data

A histogram of the grade data from the development drives is shown as Figure 2. The data are very highly skewed. Figure 3 shows the same data with the histogram plotted from the natural logarithm of the grades.

It is fairly obvious that the tin does not come from a simple lognormal distribution—even with an additive constant. After discussions with the mine personnel and the consulting geologist, we determined that this shape was caused by the fact that there were actually three hydrothermal 'surges' contributing to the final tin deposited. Geological studies were carried out by the mine to determine which of the statistical components was related to which phase of the mineralization process.

Geostatistical behapviour of the development data

The semivariogram is an essential tool in any geostatistical analysis. It provides a graphical and numerical measure of the 'continuity' of the mineral values within the deposit. Since most of our data are at five foot intervals horizontally, experimental values can be calculated for any multiple of five feet. A decision has to be made on how far to carry this calculation. For various reasons, the experimental semi-variograms were produced for distances up to 250 feet.

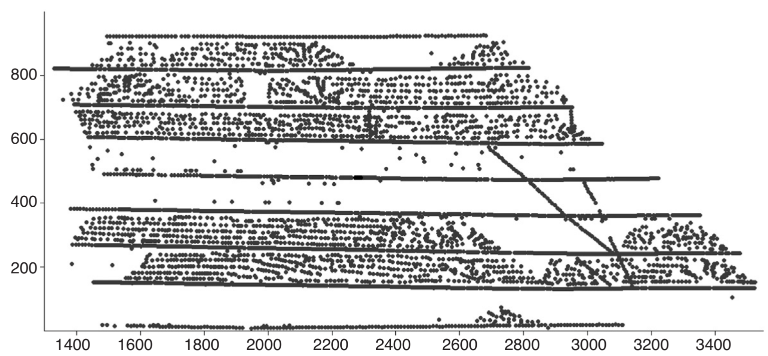

Figure 1—Post-plot of data used in case study, Geevor Tin Mine, Simms Lode

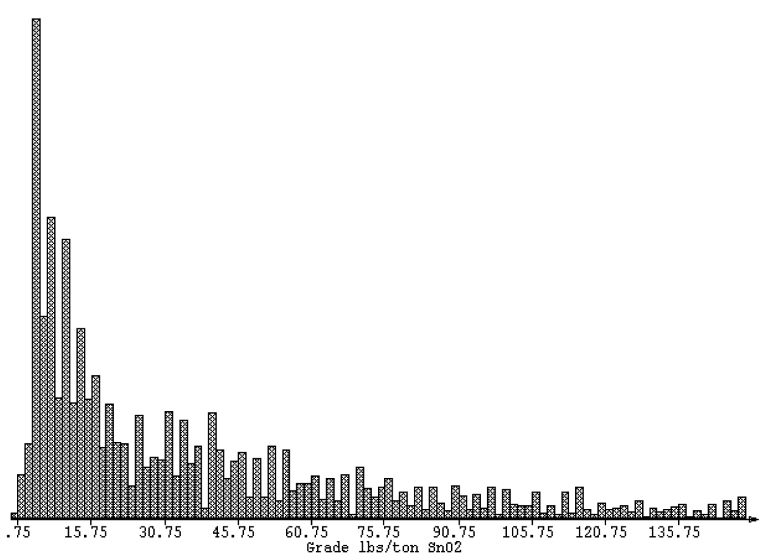

Figure 2—Histogram of tin grades in development drives, Simms Lode

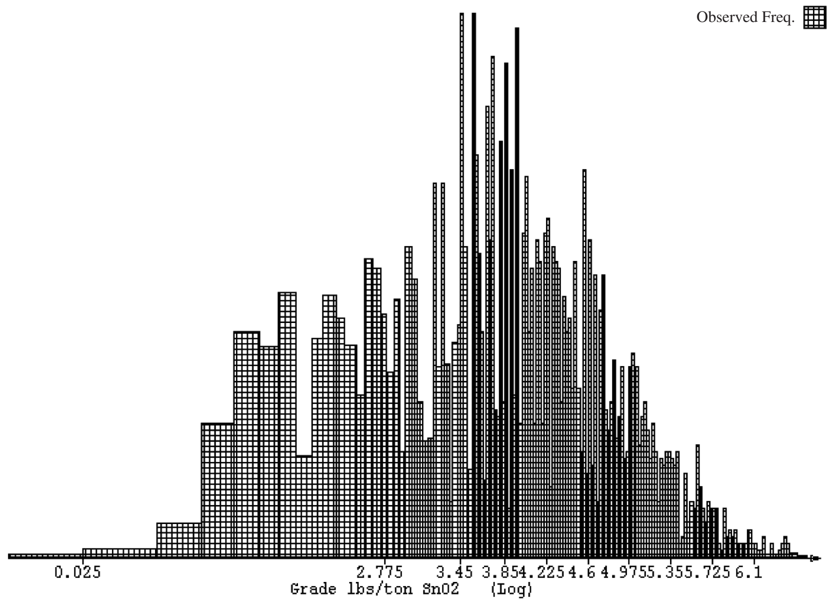

Figure 3—Histogram of logarithm of tin grades in development drives, Simms Lode

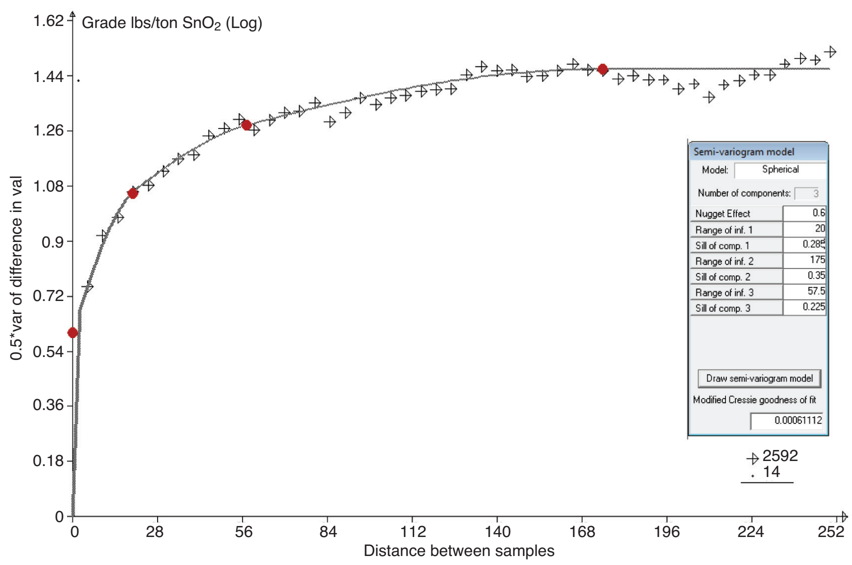

Figure 4—Semi-variogram for logarithm of tin grades in development drives, Simms Lode

The tin values at Geevor are highly skewed, although not exactly lognormal. Logarithms of the tin grades were calculated with no additive constant.

The experimental semivariogram is shown in Figure 4, together with a classic spherical type model. This model has three components, in addition to a substantial nugget effect at almost 40% of the total sill. The three ranges of influence are 20, 58 and 175 feet respectively. Very similar results can be obtained by using relative semivariograms instead of a logarithmic transform.

Nugget effect or sampling error?

There seem to be two schools of thought:

- There can be only one value at a sample site, therefore $\gamma(0)=0$. the nugget effect C_0 exists for all distances except exactly zero.
- The nugget effect reflects sampling error and, therefore, exists at zero distance: $\gamma(0)=C_0$.

Both commercial and public domain software packages vary according to which of the above philosophies is accepted.

The truth is probably somewhere between the two, with part of the nugget effect being random errors accumulated during sampling and part being some inherent short-scale variability in the phenomenon being studied. Golden Software's Surfer™ package, for example, allows for the nugget effect to be partitioned into the two possible parts.

It should be borne in mind that systematic errors—such as a consistent bias in the measurements—will not be part of the nugget effect since they will vanish when one sample value is subtracted from the other. For example, in the well-known Bre-X case, samples were (apparently) 'salted' by adding the same amount of gold to each sample. This factor would vanish when a semivariogram is calculated so that the 'salting' would not show up in the nugget effect. In our current case study, vein width was measured by tape and the width recorded to the nearest inch. During detailed analysis, it was discovered that this was true in the stopes but not in the development drive. In the development drive, the sampler tended to round up to the next inch in each case. This caused a consistent half-inch bias in the vein width in the development drives. This did not show up in the semi variogram analysis but did bias the estimation of width in the stoping panels.

During the author's studies for Geevor Tin Mines Ltd., she was able to commission a special sampling plan to study this problem in a limited manner.

The vanning assay used at Geevor replicates, on a miniature scale, the amount of 'black tin' SnO_2 which can be recovered in the traditional gravity concentration process (shaking tables). The bagged samples of around 2 lbs in weight were taken to the sampling shed where:

- Each sample was passed through a small 'rough' crusher to reduce larger fragments of rock to a more homogeneous size.
- This roughly crushed material was divided into four quarters.
- One quarter of the sample was crushed more finely to simulate the grinding which would occur in the concentrator.
- A standard (small) quantity of this material was weighed out on the pan of an electronic balance.
- This representative sub-sample was then brushed on to a 'vanning' shovel, using a rabbit's foot.
- The sampler swirled water across the vanning shovel, occasionally pouring water and barren sand off and taking more water—repeating this process until only tin remained.
- When only black tin remained on the shovel, the sampler placed this on a coal fire to dry.
- The dried material was carefully brushed back on to the pan (using the rabbit's foot) and reweighed.

The final measurement is expressed as 'pounds per ton of black tin' (lb/ton)—i.e. pounds of SnO_2 per ton of rock crushed. 1 Imperial pound is around 454 g; 1 Imperial ton is 2 240 pounds (slightly over 1 000 kg). 1 lb/ton represents just under 0.045%.

When the semivariogram was constructed for grades in the development drives (Figure 4), it was seen that the nugget effect constituted a significant proportion of the height of the semivariogram—36% of the logarithmic model. Having watched the assaying process, this author wondered how much of the 'short-scale variation' was actually due to the assaying process.

The mine agreed to carry out a special sampling scheme where contiguous samples were gathered. The first sample was taken as normal, 6 inches wide and shallowly chipped. The next sample was taken immediately adjacent to this sample, 6 inches wide and centred 6 inches (15 cm) away. In this manner 41 samples were taken over a 20 foot length of development drive.

For each sample the first quarter was assayed as normal. The grades of these samples are shown as a transect in Figure 5. The original 5 foot sampling is shown for comparison. It should be borne in mind that the new samples cannot be at exactly the same position as the original sampling but are (at best) an inch or so deeper into the vein.

The remainder of the rough crushed material was remixed, divided into four portions and rebagged. The bags were randomized so that the sampler could not identify which sample was being vanned. This gives 5 replicates for each sample.

As with the complete data-set, the grades follow a moderately skewed distribution. A probability plot of the first quarter is shown in Figure 6. The behaviour is close to lognormal with a downturn in the very highest values. Plotting all 5 replicates give a very similar probability plot with a more pronounced downturn in the upper tail.

A single semivariogram was calculated using all of the replicates, taking logarithms for a robust calculation. A linear semivariogram model was fitted to the experimental semivariogram graph (Figure 7). The apparent nugget effect parameter on the model fitted through this semi-variogram is around 0.2 ($\log_e$ lb/ton)².

There are two approaches to calculating the 'replication error' variance:

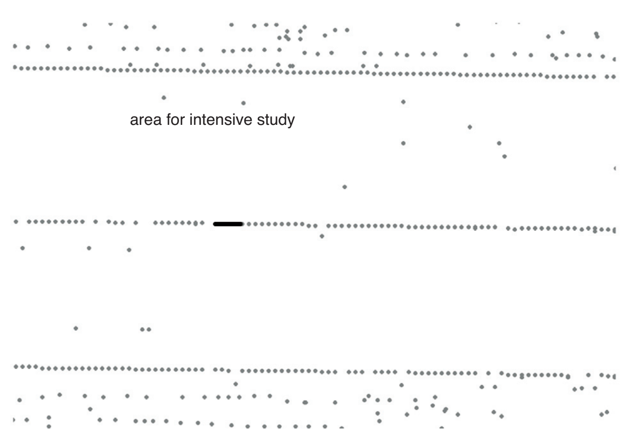

Figure 5—Development drive selected for contiguous sampling exercise

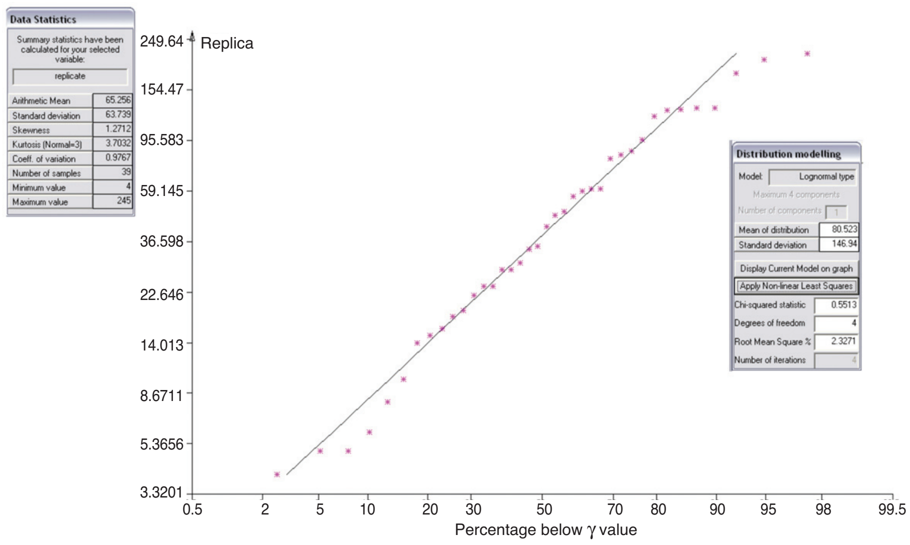

Figure 6—Probability plot of original 41 assays from contiguous sampling exercise

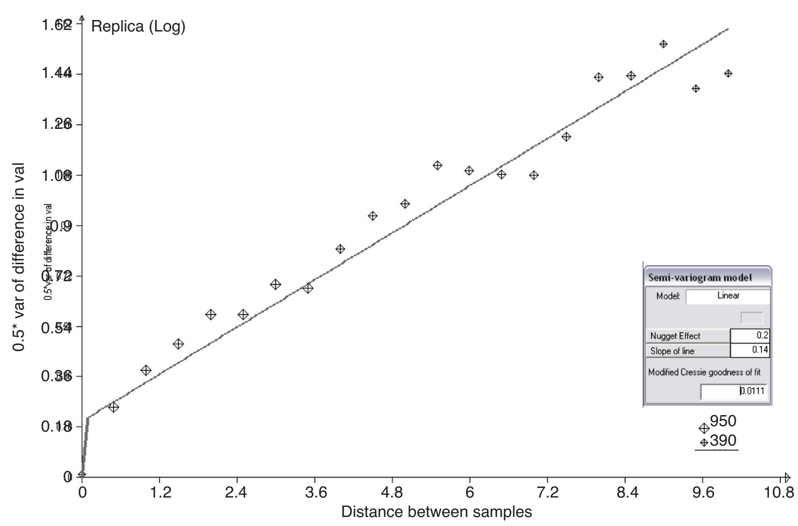

Figure 7—Semivariogram from replicated sampling, Simms Lode

The classical statistical analysis of variance

In classical statistics, the average of all replicates for the same sample is calculated. The difference between the replicate value and the 'local' average is calculated and squared. Since we have the same number of replicates for each sample, we can simply add all these squared 'deviations' and divide by 164.

The analysis of variance table calculated on logarithm of grade is shown in Table I, showing that the variation between samples is around 500 times higher than the variation between replicates within a sample. Of course, this type of analysis is predicated on normal sample values (hence the logarithmic transform) and on samples being taken randomly and independently. This latter assumption would seem to be severely compromised in such a study as we have described here.

Note: a similar calculation carried out on untransformed (raw) values gives an F ratio statistic of 567.

The semivariogram calculation for pairs of samples at zero distance

When a semivariogram is calculated, each sample is paired up with every other sample. The difference in value between them is calculated and squared. All of the pairs within a given distance 'interval' $\{h-\delta h, h+\delta h\}$ are identified and these

Table I

The analysis of variance table

Source of variation	Sum of squares	Degrees of freedom	Mean square	F-ratio
Between replicates	2.1782	164	0.0133	
Between samples	269.00	40	6.7251	506
Total variation	271.18	204		

squared differences are summed and averaged. In normal circumstances, no pairs are found at zero distance, so this interval {0, 0+δh} is generally not considered by most software packages. In this study, we had the opportunity to calculate pairs at zero distance, h=0. For each sample, we have 5 replicates, providing 10 distinct pairings for each of the 41 samples.

Using only unique pairs of replicates, our software calculated the point at zero distance. We also verified this by setting up a spreadsheet calculation with all the replicate pairs considered. Using logarithms of grades, we find that the semi-variance for replicate pairs at zero distance has a value:

$$\gamma^*(0) = \Sigma_{i=1}^{41} \Sigma_{j=1}^{5} \Sigma_{k=1}^{5} \{g_{ij} - g_{jk}\}^2 / 2n = 0.0133$$

where, in this context, g_{ij} denotes the jth replicate on the ith sample and N is the number of pairs found. This value is exactly the same as the mean square found in the statistical ANOVA table—without needing to assume independent random sampling.

It would seem that, no matter which way we calculate the replication error—the replication variance or the semivariance at distance zero—the answer is 0.0133 ($\log_e$lb/ton)².

Summary of nugget effect study

A special sampling scheme was carried out where samples were taken in a continuous fashion along a 20 foot section of development drive; 41 samples were available for this study. Five assays were obtained from each individual sample. The semi-variogram calculation and modelling suggests that the model fitted would intersect the axis at 0.2 ($\log_e$lb/ton)²—this is the definition of the nugget effect parameter.

The nugget effect should include any random errors incurred by sampling procedures as well as inherent variability of the ore deposition itself. Hence the term 'nugget effect' was coined when considering the likely difference between a (say) gold sample with a nugget in it and one immediately next to it with no nugget in it.

Using the replicates, we find that the 'assay error' associated with the vanning process gives rise to an actual nugget effect of 0.0133 ($\log_e$ lb/ton)²—around 6.6% of the total apparent nugget effect. In cruder terms, there is 15 times as much variance between two samples six inches apart than there is between replicate assays on the same sample.

Why should we bother?

Specifying a non-zero value for the semivariogram at zero distance has two major impacts on a geostatistical estimation exercise.

- ➤ In the kriging system, the equations each include a term for the semivariogram value between the sample and itself. In effect, the diagonal of the sample/sample matrix contains $\gamma(0)$. If we use zero here, we will get one set of 'optimal' weights. If we use the nugget effect, we get a different set of 'optimal' weights. Which set is really optimal? In addition, if we do not use zero, the kriging system will not honour the data values when it tries to estimate at a sampled location.
- ➤ The estimation variance for a linear geostatistical estimator is commonly expressed as:

$$\sigma_\varepsilon^2 = \Sigma_{i=1}^{k} w_i\gamma(g_i, A) - \Sigma_{i=1}^{k} \Sigma_{j=1}^{k} w_iw_j\gamma(g_i, g_j) - \gamma(A,A)$$

where g_i denotes sample i and A denotes the location at which an estimate is required. K is the number of samples used in the estimation. The terms affected by $\gamma(0)$ are:

$$- \Sigma_{i=1}^{k} \Sigma_{j=1}^{k} w_iw_j\gamma(g_i, g_j) - \gamma(A,A)$$

which reduces to

$$- \Sigma_{i=1}^{k} \Sigma_{j=1}^{k} w_iw_jC_0 - C_0 = -\{1+\Sigma_{i=1}^{k} \Sigma_{j=1}^{k} w_iw_j\}C_0$$

where C_0 denotes the nugget effect value. As a simple example, if all the weights were equal ($1/k$) the apparent estimation variance is reduced by: $(k+1)/k\ C_0$.

In plainer terms, if we acknowledge measurement and/or other sampling error in our data—i.e. $\gamma(0) = C_0$—we have more confidence in our estimator than if we trust our data completely—i.e. $\gamma(0) = 0$. In addition, the less we trust our data the more confident we get in the results.

Summary comments

We have discussed the concept of the nugget effect on the semivariogram and whether the semivariogram model should be set to zero at zero distance as opposed to some or all of the nugget effect.

A case study to determine the assay error has been described in detail. Standard statistical methods of analysis of variance produce identical results to experimental calculation of the nugget effect directly from the replicated sample values.

Finally we have shown that allowing the semivariogram model to intercept the axis rather than go to zero produces estimates which are apparently more reliable than assuming the data are accurate.

These results are completely counterintuitive and suggest that a more conservative measure of confidence is obtained if we trust our data.

References

1. Clark, I. Does geostatistics work?, *Proc. 16th APCOM*, Thomas J O'Neil, (ed.), Society of Mining Engineers of AIME Inc, New York, 1979. pp. 213–225.
2. Clark, I. and Garnett, R.H.T. Identification of multiple mineralisation phases by statistical methods, *Trans. Inst. Min. Metall.*, vol. 83, 1974. pp. A43–A52.
3. Fortin, M.-J. Spatial statistics in landscape ecology, *Landscape ecological analysis: issues and applications*. J.H. Klopatek and R.H. Gardner (eds.). Springer Verlag, New York. 1999. pp. 253–279.
4. Christensen, R. *Linear Models for Muitivariate, Time Series and Spatial Data*. Springer-Verlag, New York. 1991.
5. Chambers, R.L., Yarus, J.M., and Hird, K.B. Petroleum geostatistics for nongeostatisticians, Part 1, *The Leading Edge*, May 2000, vol. 19, no. 5, pp. 474–479;
6. Sinclair, A.J. and Blackwell, H. *Applied Mineral Inventory Estimation*, Cambridge University Press, 2002. 381 pp.
7. http://www.bioss.ac.uk/smart/unix/mvariog/slides/ sl07.htm, Biomathematics & Statistics Scotland, Edinburgh.
8. Samal, A. Basics of Variogram Analysis, *Pincock Perspectives,* Issue No. 84, May 2007, Pincock, Allen & Holt, Lakewood, Colorado. ◆