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Foundational mathematical knowledge of prospective teachers: Evidence from a professional development training

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This study examined the effect of a professional development training programme on 20 second-year preservice mathematics teachers' knowledge in foundational mathematical concepts at a rural university in South Africa. The training programme aimed to enhance preservice teachers' mathematical knowledge for teaching. An embedded mixed-methods case study design was employed. Baseline and endline assessments were administered before and after the training. A participant feedback survey was also administered after the training. Results showed that the training significantly improved the preservice teachers' understanding and confidence in the selected concepts, despite their low baseline scores. The participants also expressed satisfaction with the knowledge they gained and appreciated the integration of theory and practice in the training. These findings suggest the need for teacher training institutions to ensure that preservice teachers are well versed in both university-level and school-level mathematics. They also support the need for collaboration with other stakeholders to provide preservice teachers with relevant and engaging professional development opportunities that can enhance their mathematical knowledge for teaching.

Contribution: Findings of this study point to a renewed emphasis on the creation of greater collaborations between institutions of higher learning and other key stakeholders to promote the development of prospective teachers' knowledge of what they will be expected to teach.

Keywords: Foundational mathematical concepts; pedagogical skills, prospective mathematics teachers, professional development, subject matter knowledge.

Introduction

Foundational mathematical knowledge constitutes a deep and connected understanding of fundamental mathematical concepts. When considering prospective mathematics teachers, this knowledge extends beyond grasping mathematics solely at the student's level. It encompasses understanding how students learn, effective explanation of mathematical concepts, as well as addressing students' questions and misconceptions (Masingila et al., 2018). This implies that foundational mathematical knowledge is not only essential for understanding advanced mathematical concepts but is also pivotal for the effective teaching of such concepts. As a result, the necessity to enhance preservice teachers' foundational mathematical knowledge, accompanied by a concerted allocation of resources aimed at improving learning outcomes in school mathematics has been heightened in different settings (Askew, 2008; Bethell, 2016; Chikiwa & Graven, 2023; Spitzer & Phelps-Gregory, 2023). Despite these concerted efforts, national, regional, and international assessments consistently reveal suboptimal levels of achievement in mathematics (Mullis et al., 2012, 2020; Organisation for Economic Cooperation and Development [OECD], 2019).

In the specific context of sub-Saharan Africa, and particularly South Africa, the challenges appear more pronounced than in other global regions (Bethell, 2016). These challenges encompass inappropriate teaching methods that result in insufficient conceptual understanding and low achievement levels among learners, as well as deficiencies in teaching and learning materials. Additionally, subpar quality of preservice teachers, ineffective teacher trainers, and an inadequately designed teacher education curriculum contribute to the array of obstacles faced in the educational landscape, particularly mathematics education (Bethell, 2016; Luneta, 2022; Taylor, 2021). This disparity is evident in the findings of Reddy et al. (2019) and Osta et al. (2023), highlighting the unique difficulties faced by South Africa in addressing foundational mathematical knowledge. Jansen (2023) further supports this observation, demonstrating that South African primary school students consistently underperform in mathematics compared to their global counterparts, even those from countries with lower economic resources. This stark reality emphasises the urgent need for targeted interventions and comprehensive strategies to address this quality landscape.

One of the key interventions to undertake is the enhancement of prospective teachers' understanding of foundational mathematical concepts (Alex, 2019). Operations like addition, subtraction, multiplication, and division, as well as concepts like factors, multiples, and prime factorisation, are among foundational mathematical concepts that are pivotal. For instance, the significance of addition, subtraction, multiplication, and division lies in their fundamental nature as the four basic operations in mathematics (Hickendorff et al., 2019). These operations play a foundational role in various daily activities and lay the groundwork for comprehending more advanced topics like algebra and geometry. According to Paternoster and Bachman (2017), the study of statistics necessitates basic mathematical skills, such as addition, subtraction, multiplication, and division, which are also fundamental for solving mathematical expressions.

Factors, multiples, and prime factorisation are equally essential for grasping the numerical structure. According to Cambridge Mathematics (2022), 'breaking down numbers to explore their multiplicative structure can facilitate a versatile approach to problem-solving' (para. 3). Examining the characteristics of primes helps prevent misunderstandings about their size and prevalence as factors of other numbers. Proficiency in these concepts also aids in flexible reasoning regarding the divisibility of whole numbers. Ma (1999) argues that educators with a deep understanding of these fundamental mathematical concepts can effectively unveil and illustrate ideas and connections. Additionally, Hill et al. (2008) found that teachers with enhanced mathematical knowledge for teaching were more inclined to provide comprehensive mathematical explanations, employ better concrete models of mathematical processes, and adeptly translate between students' everyday language and mathematical terminology. Therefore, bolstering teachers' understanding of foundational mathematical concepts is pivotal for effective teaching and enhanced learning outcomes among students.

Despite the pressing need for intervention, inadequate mathematical knowledge for teaching foundational mathematical concepts among preservice teachers has been observed not only in South Africa but also in various educational settings (Ball, 1990; Bowie, 2014; Isiksal & Cakiroglu, 2011; Ma, 1999; Reid & Reid, 2017; Saili et al., 2023; Thanheiser et al., 2014). These shortcomings include a lack of conceptual understanding, procedural fluency, and problem-solving skills, as well as an inability to connect mathematics to real-world contexts and other disciplines (Mukuka et al., 2023). Additionally, there is a deficiency in the teachers' ability to use diverse representations and strategies for teaching mathematics, assess students' mathematical thinking, and provide effective feedback (Taylor, 2019). Lack of confidence and interest in mathematics and its teaching also contribute to these challenges (Niyukuri et al., 2020). A recent investigation by Taylor (2021) conducted at four South African universities also reveals these inadequacies. The study revealed that a significant number of preservice teachers struggled to accurately solve mathematical problems or articulate the associated concepts, emphasising the pressing need for comprehensive improvements in the training of mathematics teachers. While the contributions of prior studies, such as Fonseca and Petersen (2015) and Alex (2019), are recognised, there remains a critical need to augment the existing evidence concerning the foundational mathematical knowledge of prospective teachers in South Africa. Moreover, it is imperative to explore how this foundational knowledge can be effectively enhanced through targeted professional development training.

Arising from the above-stated problem, this article sought to present evidence of the foundational mathematical knowledge of prospective teachers at a rural university in South Africa. Additionally, the study evaluated the impact of professional development training on improving this knowledge among these preservice teachers. The following research questions were explored:

- What evidence exists to demonstrate the impact of professional development training on prospective teachers' knowledge of foundational mathematical concepts?
- How do prospective teachers perceive the role of professional development training in enhancing their subject matter knowledge in foundational mathematical concepts?

Foundational mathematical knowledge

Foundational mathematical knowledge, as defined by Newton (2018) and Yang et al. (2018), comprises the essential concepts and competencies that form the basis for a deeper understanding of mathematics. In the context of this study, we focus on some aspects of the foundational mathematical knowledge that prospective teachers should possess before entering the profession. This knowledge, which includes basic operations (such as addition, subtraction, division, and multiplication) and concepts such as prime numbers, factors, multiples, and prime factorisation, serves as the building blocks for more advanced teaching methods and strategies (Livy et al., 2019; Taylor, 2019; Superfine et al., 2013). It is important for teachers to have a solid grasp of these foundational concepts to effectively teach them to students and facilitate meaningful learning (Alex, 2019; Ball et al., 2008; Hill et al., 2008; Jacinto & Jakobsen, 2020; Prendergast et al., 2023).

This study examined a professional development training conducted by Numeric, a non-profit organisation, at a rural university in the Eastern Cape province of South Africa. Numeric, recognising the importance of a robust foundation in school mathematics, has been actively involved in designing and delivering training sessions that focus on fundamental mathematical concepts. Through partnerships with teacher training institutions across the country, Numeric provides prospective teachers with additional training in pedagogy and subject matter knowledge, preparing them for successful teaching careers. This initiative is a response to the documented need for enhancing foundational mathematical knowledge among preservice teachers. In the context of sub-Saharan Africa, the need for this focus is substantiated by the evidence presented in the existing literature (Bethell, 2016; Malambo et al., 2018; Saili et al., 2023; Venkat, 2019).

The training reported in this study covered specific mathematical concepts essential for the Senior Phase (Grade 7–9) as outlined in the Curriculum and Assessment Policy Statement (CAPS) by the Department of Basic Education (2011). Despite the fundamental nature of these concepts, research shows that many preservice teachers display low levels of achievement (Thanheiser et al., 2014; Li & Howe, 2021). For instance, Feldman and Roscoe (2018) found a significant lack of understanding among preservice teachers regarding multiplicative structure and divisibility. Similarly, a survey by Gürefe and Aktaş (2020) revealed substantial challenges in preservice teachers' understanding of prime numbers. These findings provide evidence of the need to strengthen preservice teachers' foundational mathematical knowledge to ensure effective teaching.

In light of the above highlighted challenges, there has been a persistent call within the realm of mathematics education research for targeted interventions aimed at enhancing the subject matter knowledge of prospective teachers in foundational mathematical concepts (Fonseca & Petersen, 2015; Saili et al., 2023; Taylor, 2019). This is attributed to the fact that discipline-specific expertise serves as the fundamental basis for all other forms of knowledge required for effective instruction (Golding, 2023; Taylor, 2019). As such, there is need for a more comprehensive and profound approach in cultivating prospective teachers' comprehension of foundational mathematical concepts, transcending mere algorithmic application.

This professional development training used interactive sessions, Khan Academy videos, and drills to demonstrate different methods and strategies for teaching basic operations such as addition, subtraction, division, and multiplication. For example, the training showed how to use manipulatives, models, and algorithms to perform and explain these operations. The training also emphasised the centrality of 'place value' for conceptual understanding and procedural fluency with regard to the basic operations. According to Hickendorff et al. (2018), place value can be understood as the specific value a digit holds, determined by its position within a number. For example, in the number 536, the digit 3 represents 3 tens, or 30, due to its position. However, in the number 398, the same digit 3 signifies 3 hundreds, or 300, because of its different position. Understanding place value helps students to perform operations with large numbers, decimals, and fractions, and to compare and order numbers.

Additionally, the training incorporated the notion of tree method and prime factorisation for understanding the concept of highest common factor (HCF) and lowest common multiple (LCM). According to Yiu-Kwong (2016), both the tree and prime factorisation methods involve breaking down a number into its prime factors by dividing it repeatedly by its smallest factor. The excerpt presented in Figure 6 shows a student's use of a combination of tree method and prime factorisation to find the HCF of 36 and 60. This demonstrates that the tree method and prime factorisation are not different methods, but rather the tree method is a way to perform prime factorisation. These mathematical procedures hold significant educational value as they equip students with the tools to determine the HCF and the LCM of two or more numbers. The application of the concepts and instructional methods employed during the training extends beyond mere arithmetic, proving instrumental in simplifying fractions, solving word problems, and identifying common denominators. Supporting the instructional efficacy of prime factorisation, a study conducted by Feldman and Roscoe (2018) revealed that preservice teachers, upon mastering this process, demonstrated enhanced abilities. They were proficient in discerning factors and nonfactors, constructing factor lists, and manipulating numbers with specific divisibility properties.

Theoretical framework

The framing of this study is located within mathematical knowledge for teaching (MKT), a model developed by Ball et al. (2008). A specialised form of knowledge, MKT is crucial for effective mathematics teaching. This framework recognises that being a proficient mathematician does not necessarily equate to being an effective mathematics teacher. Building on the work of Shulman (1986, 1987), Ball et al. (2008) identified two key areas that make up MKT: subject matter knowledge and pedagogical content knowledge.

The focus of this study is subject matter knowledge, which is further subdivided into three distinct categories: Common Content Knowledge (CCK), Specialised Content Knowledge (SCK), and Horizon Content Knowledge (HCK). The first of these, CCK, pertains to the mathematical understanding that is shared by both teachers and non-teachers. This type of knowledge is not exclusive to the realm of teaching but is also applicable in fields such as engineering, medicine, and marketing, among others. An example of CCK is the ability to perform basic arithmetic operations like addition, subtraction, division, and multiplication as illustrated in Table 1.

In contrast to CCK, SCK is unique to the teaching profession. This suggests that SCK is distinct and specific to teachers. It implies that this type of knowledge goes beyond simply

TABLE 1: Sample questions and solution techniques on the four basic operations.

Operation	Sample question	Sample technique/strategy
Addition	536 + 398	[1]3 [1]9 8 + 5 3 6 **9 3 4**
Subtraction	686 – 48	6 [7]~~8~~ [1]6 – 4 8 **6 3 8**
Multiplication	326 × 82	3 2 6 × 8 2 6 5 2 + 2 6 0 8 ↓ **2 6 7 3 2**
Division	64 ÷ 4	**16** 4⟌64 – 4↓ 24 – 24 0

knowing the subject matter (mathematics, in this case) and includes an understanding of how to teach that subject effectively. Teachers with strong SCK not only have a deep knowledge of the content but also possess pedagogical strategies and insights into how to convey the material in ways that are meaningful and comprehensible to students (Feldman & Roscoe, 2018). For instance, a teacher with strong SCK could evaluate the efficiency of the Euclidean algorithm for calculating the HCF and decide when it is appropriate to use.

In conventional practice (as illustrated in Table 1), the addition of 536 and 398 is performed by first adding the digits in the ones place (6 and 8), resulting in 14. The digit 4 is then written down, and 1 (representing 10) is carried over to the tens place. This carried 1 is added to the sum of the digits in the tens place (3 and 9), yielding 13. The digit 3 is written down, and 1 (representing 100) is carried over to the hundreds place. This carried 1 is added to the sum of the digits in the hundreds place (5 and 3), resulting in 9. This process gives the correct answer of 934, as demonstrated in Table 1. However, while this method is widely accepted, it does not foster a deep conceptual understanding as it overlooks the concept of place values. A teacher with adequate SCK would strive to explain the process of adding 536 and 398 in a way that highlights the significance of place values, as depicted in Figure 1.

In terms of prime numbers, a teacher with SCK would be able to define them, identify them, and explain their properties and significance in number theory. They could also use prime factorisation to decompose a number into its prime factors and explain the fundamental theorem of arithmetic. For HCF and LCM, a teacher with SCK would be able to calculate them using different methods, such as listing factors and multiples, prime factorisation, and Euclidean algorithm (Yiu-Kwong, 2016). They could also explain the relationship between HCF and LCM, and their applications in simplifying fractions, solving word problems, and finding common denominators or multiples.

__Step 1:__ Add the units (ones) place: The units place in 538 is 8 and in 396 is 6. Adding these gives 14. Write down the 4 in the units' place of the answer and carry over the 1 (which represents 10) to the tens place.

__Step 2:__ Add the tens place: The tens place in 538 is 3 (representing 30) and in 396 is 9 (representing 90). Adding these gives 12 tens (or 120), plus the 1 ten carried over from the units' place gives 13 tens (or 130). Write down the 3 in the tens place of the answer and carry over the 1 (which represents 100) to the hundreds place.

__Step 3:__ Add the hundreds place: The hundreds place in 538 is 5 (representing 500) and in 396 is 3 (representing 300). Adding these gives 8 hundreds (or 800), plus the 1 hundred carried over from the tens place gives 9 hundreds (or 900). Write down the 9 in the hundreds place of the answer.

So, 538 + 396 = 934 when using place value for addition.

FIGURE 1: Steps for adding two numbers using place values.

Finally, HCK encompasses the understanding of how mathematical topics interconnect and evolve (Ball et al., 2008). This type of knowledge is crucial for teachers as it guides them in leading their students towards more advanced concepts. For example, a teacher with a solid grasp of HCK would be able to explain how the method for determining the LCM of small numbers can also be applied to larger numbers. Furthermore, such teachers can ensure that students acquire the necessary skills and competencies for understanding more complex topics. A concrete example of this would be ensuring students have a thorough understanding of prime numbers, which would subsequently facilitate their comprehension of prime factorisation. In addition, a teacher with HCK would be able to see the bigger picture and understand how these basic operations and concepts fit into the broader landscape of mathematics. They could explain how the mastery of basic operations lays the foundation for learning more advanced topics, such as algebra, geometry, and calculus.

Scholars such as Ball et al. (2008) and Taylor (2019) advocate for teachers to possess SCK and HCK that go beyond commonly held understanding. They assert that these knowledge domains are essential for elucidating the intricacies of mathematical concepts to students effectively. Solutions presented in Table 1 demonstrate CCK primarily focusing on procedural steps without highlighting the underlying reasons. While this method may yield correct answers, it falls short in nurturing a deeper conceptual understanding among learners. Regrettably, this conventional approach to teaching, which prioritises procedural steps over conceptual understanding, is widespread and contributes to the misconception that anyone who knows these procedures can teach mathematics (Alex, 2019; Alex & Mukuka, 2024).

Therefore, it can be asserted that the MKT framework offers a thorough and nuanced understanding of fundamental mathematical knowledge. By distinguishing between CCK, SCK, and HCK, it highlights the complexity and depth of the knowledge that preservice teachers need to acquire and the challenges they face in their professional development. It is also worth pointing out that the MKT has undergone notable transformations since its inception. Initially stemming from Shulman's (1986) foundational model, MKT underwent refinement by Ball and her colleagues in 2008. Their practice-based theory of MKT stands out as a remarkably restructured framework within the mathematics education domain, recognised for its excellence in delineating teachers' pedagogical content knowledge. Despite its original design for assessing the specialised knowledge of elementary school mathematics teachers, the MKT model has found extensive application in appraising teacher learning within professional development programmes. It has also been instrumental in exploring the intricate interplay between teachers' knowledge and their instructional practices across diverse contexts (Alex, 2019; Livy et al., 2018; Moh'd et al., 2021; Ndlovu et al., 2017; Pournara et al., 2015; Scheiner et al., 2019).

Over time, the MKT model has evolved to include 'Mathematical Knowledge for Teaching Teachers (MKTT)' (Jankvist et al., 2020; Masingila et al., 2017), representing a significant advancement tailored to the specific requirements of mathematics teacher educators. In practical terms, the utilisation and refinement of MKTT occur as mathematics teacher educators collaborate and reflect on their teaching within a community of practice (Jankvist et al., 2020). This process encompasses activities such as formulating mathematical lesson goals, selecting and facilitating tasks, and employing questions to support the learning of prospective teachers and involve them in mathematical processes. The emerging theory of MKTT is particularly important as it explores the essential mathematical knowledge needed for teaching teachers, an area of research that warrants increased attention (Chapman, 2021). This expansion not only enhances the theoretical foundations of MKT but also strengthens its applicability by addressing the nuanced needs of those responsible for training teachers.

In this study, our primary focus is on both CCK and SCK, particularly how preservice teachers develop their mathematical knowledge for teaching as they build foundational mathematical knowledge. Building upon existing scholarly research, our hypothesis was that engaging in professional development training would serve as a highly effective strategy for enhancing preservice teachers' grasp of fundamental mathematical concepts. To support this assertion, we draw upon a comprehensive review of professional teacher development spanning a decade, as conducted by Avalos (2011). This research emphasised the significance of collaborative partnerships between university lecturers and external stakeholders, highlighting their superior effectiveness compared to the conventional 'master' role typically assumed by teacher educators and researchers (Mukuka & Alex, 2024). This illuminates a compelling example of how involving additional stakeholders can significantly contribute to the enhancement of preservice teachers' foundational mathematical knowledge.

Methods

Research design

This study utilised an embedded mixed-methods case study design. Drawing from Yin's (2009) framework for case study designs, a single case was examined in this research, which involved a group of second-year prospective mathematics teachers. The boundaries of the case were defined by the involvement of one group of participants at a rural university who underwent targeted professional development training aimed at enhancing their common content and specialised content knowledge of foundational mathematical concepts.

As an embedded mixed-methods study, both quantitative and qualitative data were collected simultaneously. In this context, qualitative data analysis was nested within the quantitative data analysis. This implies that the quantitative data were employed to test a hypothesis – that professional development training enhances prospective teachers' foundational mathematical knowledge – while the qualitative data served to provide context or background to the quantitative data (Creswell, 2014; George, 2021; Muhammad, 2023). The qualitative data further assisted in explaining or interpreting the quantitative findings, thereby offering a more comprehensive understanding of the impact of the professional development training. This mixed-methods approach allowed for a more comprehensive understanding of the research problem by offsetting the weaknesses of both quantitative and qualitative research. As such, the study not only quantified the improvement in foundational mathematical knowledge (through test scores) but also qualitatively captured the participants' perceptions and experiences (through a feedback questionnaire). Notwithstanding some weakness of the intervention setup (as specified in the study limitations), we believe that this approach led to more robust conclusions and recommendations for future practice.

The intervention setup

As already indicated, Numeric developed this training programme based on a needs analysis that the organisation has been carrying out throughout its over 10 years of existence. The training took place from 8:00 AM to 5:00 PM every day from 12 June through to 15 June 2023, for a total of four days. The development of prospective teachers' knowledge of foundational mathematical concepts was the focus of the training. Prospective teachers were given the chance to learn some fundamental mathematical concepts at the Grade 7 level of the South African school curriculum. Table 3 lists the topics that were taught as well as the resources used in the classroom. Even though baseline and endline tests were conducted, this intervention did not meet the criteria to be categorised as an experimental study because there was no control group and no random participant selection. As such, this research is classified as an embedded mixed-methods case study design (Creswell, 2014; George, 2021; Muhammad, 2023; Yin, 2009).

Participants

The study involved a sample of 20 preservice mathematics teachers drawn from a population of students enrolled at a rural university in the Eastern Cape province of South Africa. These students were pursuing a four-year Bachelor of Education (BEd) programme with a specialisation in Mathematics and Sciences Senior and Further Education and Training (FET) Phase teaching. The Senior and FET Phases pertain to Grades 8–12 in the South African school curriculum. Among the 20 prospective teachers in the sample, seven were female, and the remaining were male. The selection of participants was based on their voluntary participation and their readiness for school-based experience (SBE), commonly referred to as teaching practice. During the four years in the programme, student teachers are exposed to two weeks of observation (observing mathematics and science teaching) in the first year, three weeks of teaching Senior and FET Phase Mathematics in the second year, five weeks of teaching FET Phase Mathematics in the third year and 10 weeks of teaching FET Mathematics in the fourth year. The 20 second-year students who were voluntarily registered for the Numeric training programme had no previous teaching experience in the schools as part of the BEd programme. The training by the Numeric team was one of the additional supports given to the student teachers through the Mathematics Education and Research Centre established at the university.

Data collection tools

This programme included a baseline test and an endline test that assessed students' proficiency with foundational mathematical concepts using the material appropriate for Grade 7, which is part of the Senior Phase (Department of Basic Education, 2011). The contents of the baseline and endline assessments were the same. Respondents were expected to answer all the questions within 60 minutes (1 hour). The question paper had spaces where candidates were expected to show their working. Calculators were not allowed as all the questions involved basic arithmetic on place values and rounding off, addition and subtraction, multiplication, division and divisibility rules, word problems, factors and prime factorisation, HCF, and LCM. While place values and rounding off were not the primary focus of the intervention, these elements were incorporated to enhance the prospective teachers' SCK. It is essential for a mathematics teacher to possess this type of knowledge to foster conceptual understanding of the four basic operations (addition, subtraction, multiplication, and division) among learners. Furthermore, the test included certain questions that required the use of specific techniques for solution derivation. This approach was employed as a means of assessing both CCK and SCK, thereby providing a more comprehensive evaluation of the teachers' mathematical proficiency.

In addition to the baseline and endline assessments, a participant feedback survey was given to prospective teachers to ascertain their opinions on the efficacy of the training they received. The feedback semi-structured questionnaire was administered online via Google Forms immediately after the end of the training. Participants were given 30 minutes to complete the questionnaire online before leaving the training venue. All the 20 second-year prospective teachers managed to complete the feedback questionnaire. While the contents of the questionnaire included items on respondents' views of different aspects of the training, this article reports on subject-matter knowledge. The lesson sessions on subject matter knowledge involved addition and subtraction, multiplication, division, prime numbers, factors, and prime factorisation, HCF, and LCM. Before holding lesson sessions on the above-listed concepts, participants were exposed to place values and rounding off numbers as prerequisites. The questionnaire also required the respondents to comment on the effectiveness and usefulness of classroom tools used, the most and least enjoyable aspects of the training, and suggestions of what ought to be done to improve and sustain such an initiative.

Data analysis

In our data analysis, we employed Shapiro's (1987) intervention evaluation criteria, aligning with the approach taken by previous studies (Fonseca & Petersen, 2015; Prendergast et al., 2023). Shapiro's criteria offer a comprehensive framework for systematically assessing various facets of an intervention, encompassing its design, implementation, and impact. The utilisation of these criteria was driven by the aim of facilitating informed decision-making among stakeholders regarding the continuation, modification, or termination of the intervention, grounded in empirical evidence of its effectiveness and other pertinent factors. Within the context of this study, we focused on four components of Shapiro's evaluation criteria:

- Intervention Effectiveness Evaluation: This component involved gauging the professional development training's efficacy. This was achieved by assessing the shift in preservice teachers' performance from the baseline test to the endline test.
- Intervention Integrity Assessment: To ensure replicability and consistency, we examined the extent to which the professional development training adhered to the ideal scenario. This aspect encompassed participants' evaluations of facilitation skills, accessibility and approachability, and the preparedness of the facilitators.
- Social Validity Analysis: We scrutinised participants' viewpoints regarding the effectiveness of the training they underwent, aiming to understand its perceived value and relevance.
- Intervention Acceptability Evaluation: We assessed participants' satisfaction with the procedures and activities incorporated into the professional development training, aiming to gain insights into its acceptability.

Our data analysis approach encompassed both qualitative and quantitative methods. Quantitative analyses involved descriptive statistics, such as mean, standard deviation, and a bar chart, as well as inferential statistical analysis, employing a paired samples *t*-test. All statistical analyses were conducted using Statistical Package for Social Sciences (SPSS) version 27.

In the qualitative part of our analysis, we employed both thematic and content analysis to interpret the data. We analysed the open-ended responses from participants' feedback survey using the thematic analysis procedure proposed by Braun and Clarke (2006). This process began with the researchers familiarising themselves with the written responses, which involved multiple readings to ensure a thorough understanding. Following this, we coded the data to highlight significant features, which served as a precursor to generating themes. These themes were then linked to specific portions of the quantitative data analysis to provide a richer context and more detailed explanations in response to our research questions. It is also important to note that our approach to thematic analysis was iterative, not linear, meaning we moved back and forth between phases to ensure comprehensive information extraction. Guided by Luo (2019), we also conducted a content analysis of test scripts, examining students' problem-solving skills and their understanding of the foundational mathematical concepts being tested. This method proved useful in identifying common misconceptions, errors, and areas where participants required additional support or instruction. Selected excerpts from respondents' answer scripts were then extracted to furnish additional context and elucidation to the outcomes of the quantitative analysis. The MKT framework informed our analysis by providing a theoretical basis for understanding the different types of knowledge that teachers need to effectively teach mathematics. By aligning our analysis with the MKT framework, we were able to provide a more holistic response to our research questions.

Nonetheless, it is worth noting that, while the professional development training was designed to enhance prospective teachers' CCK and SCK in relation to selected foundational mathematical concepts, our analysis did not differentiate between these two knowledge domains of the MKT model. However, its application in our analysis was adapted to suit the specific context and objectives of our research. Rather than explicitly distinguishing between CCK and SCK, we inferred these aspects from the solutions that preservice teachers provided to the test questions. For instance, correct use of the place value method when answering questions on basic operations indicated adequate SCK in relation to those concepts. Similarly, correct use of the tree method or prime factorisation in determining the HCF and LCM suggested sufficient levels of SCK. This approach allowed us to gain a more general insight into their mathematical proficiency as reported in the results section. Regarding the expected student teachers' achievement level, we adopted a minimal mastery level of 60% set by Venkat and Spaull (2015) in the Primary Teacher Education (PrimTEd) project.

Roles of researchers and facilitators

The research team's responsibility was to keep an eye on the training's activities as well as the study's inception, data analysis, article writing, and other administrative tasks. Two of the Numeric staff members (referred to as facilitators in this article) prepared the training and oversaw its facilitation. The test questions were also created by the facilitators, who administered them to the participants before and after the training. The research team and the facilitators worked together in developing the feedback questionnaire.

Ethical considerations

The Directorate of Research and Innovations at Walter Sisulu University provided ethical and gate keeper approval for the intervention programmes in the Mathematics Education and Research Centre (No. FEDSRECC001-06-21). Each prospective teacher who had been invited to participate in the training programme gave their consent to be a trainee. By doing so, they gave consent to participate in the study, to have their comments recorded, and to allow the publication of their responses. In adherence to ethical standards, participants' information has been kept confidential with assurance that the contents and findings of this research will not harm them in any way.

Results

Results are presented according to Shapiro's (1987) intervention evaluation criteria. As stated earlier, one of the key reasons for using this approach is that it provides a well-established and comprehensive framework for assessing the effectiveness of interventions, such as the professional development training that we administered to a group of preservice teachers. These criteria have been widely recognised and utilised in the field of programme evaluation and intervention research. By following Shapiro's criteria, we followed a structured and systematic approach to evaluating our intervention, which enhances the rigour and validity of our study. This framework also provided opportunities for us to assess various aspects of the professional development training, including its design, implementation, and impact, providing a robust foundation for drawing meaningful conclusions about the outcomes and its potential implications for practice and policy.

Intervention effectiveness

A paired samples *t*-test was performed to determine the significance of the improvement in preservice teachers' comprehension of selected basic mathematical concepts between the baseline and endline assessments. Given that the sample size was rather small, it was thought important to determine whether the normality assumption was met before performing this statistical test. The score differences of the paired values between the baseline and endline tests were subjected to the Kolmogorov-Smirnov (K-S) and Shapiro-Wilk (S-W) normality tests in accordance with the recommended procedures (Field, 2013). The distribution of the score shifts between the two tests was not significantly different from normal, according to the results of both the K-S [$D(20) = 0.171$, $p = 0.128$] and the S-W [$D(20) = 0.940$, $p = 0.237$] normality tests. This means that the distribution of the score differences was normal. Table 2 and Table 3 display the descriptive and inferential statistics that were generated by the SPSS software.

In the context of the initial assessment, results displayed in Table 2 show that preservice teachers exhibited an average proficiency level of 35.6% regarding foundational mathematical concepts. A further analysis revealed that merely two among the preservice teachers achieved scores surpassing the 50% threshold on the baseline assessment. Results displayed in Table 2 also reflect that the baseline assessment possessed a minimum score of 14% and a maximum score of 70%. Specifically, one participant obtained a score of 51%, while another attained the highest score of 70%. On the other hand, six preservice teachers yielded scores falling below the 30% mark, with seven participants securing scores that ranged between 30% and 39%. Notably, only five preservice teachers managed to secure scores within the range of 40% to 49%. This is a clear indication that preservice teachers had very limited knowledge of foundational mathematical concepts before exposure to the intervention.

However, after their active engagement in a professional development training programme, there was a positive shift in preservice teachers' knowledge of foundational mathematical concepts. This transformation is demonstrated by the outcomes presented in Table 2, which reveal an elevated average performance of 80.3%, yielding a minimum score of 47% and a maximum score of 97%. A further analysis of data revealed that only one participant failed to attain a score exceeding the 50% threshold in the endline assessment. This marked enhancement in performance is justified by the results of a paired samples *t*-test (Table 3), which affirms the statistically significant improvement in preservice teacher proficiency levels after their participation in the professional development intervention.

Based on the results presented in Table 2 and Table 3, prospective teachers fared significantly better on the endline test ($M = 80.3$, $SD = 13.9$) compared to the baseline test ($M = 35.6$, $SD = 13.4$), $t(19) = 14.5$, $p < 0.001$. An additional analysis revealed that the average improvement score for prospective teachers from the baseline to the endline test was 44.8 (95% confidence interval [38.3, 51.2]). The Cohen's *d* value of 3.24 is not only significant but also equates to a large effect size according to the standards for effect sizes. This demonstrates clearly that the intervention had a beneficial effect on preservice teachers' comprehension of the foundational mathematical concepts.

TABLE 2: Paired samples descriptive statistics ($n = 20$).

Measure	Minimum	Maximum	Mean	Standard deviation
Baseline test	14	70	35.6	13.4
Endline test	47	97	80.3	13.9

Intervention integrity

In the context of this study, preservice teachers were asked to rate, on a scale of 1 (lowest) to 5 (highest), the level of facilitation skills, accessibility and approachability, and preparedness of the facilitators. Results show that almost all the respondents rated the facilitators very highly. Facilitators' preparedness ($M = 5$, $SD = 0$) was the highest as it reflects no variations in preservice teachers' responses, followed by facilitators' accessibility and approachability ($M = 4.7$, $SD = 0.57$), and their facilitation skills ($M = 4.65$, $SD = 0.93$). This implies that participants were satisfied with the way the training was carried out by facilitators, a clear demonstration of why the improvement in performance after the training was significantly large.

Social validity

Overall, all respondents conveyed a high level of satisfaction with the training programme, emphasising its value and relevance in enhancing their grasp of foundational mathematical concepts. The following excerpts from selected participants' responses further illustrate this consensus:

> 'I found the training quite beneficial, especially that I was exposed to new ways of finding HCF and LCM.' (Respondent 4)
>
> 'I had to interact with people and our facilitators who are experienced and shared their great experiences.' (Respondent 12)
>
> 'I enjoyed it as I was looking forward to seeing what was missing and the lesson was actually delivered in a way that I can never forget. For example, I wasn't actually that good in prime factorisation and to also mention multiplication. Overall, of it all the lessons were enjoyable, and it was a productive training for me as I learnt new and useful things.' (Respondent 15)
>
> 'I used to struggle in my earlier grades on topics like long division, HCF and LCM but now I am not struggling after the Numeric training.' (Respondent 19)

The words chosen by respondents when asked to sum up the training experience in one word were another sign of how enthusiastic they were about how good the training was. Words like *awesome*, *empowering*, *excellent*, *tremendous*, *great*, *helpful*, *wonderful*, *powerful*, *best*, *fantastic*, and *manipulative*, among others, were used by respondents to describe how impactful the training was to their future careers in teaching mathematics.

TABLE 3: Paired samples *t*-test results.

Variable	Mean	Standard deviation	95% confidence interval of the difference		*t*	*df*	Significance
			Lower	Upper			
Endline - baseline	44.8	13.9	38.3	51.2	14.5	19	0.000

Intervention acceptability

The findings of this study revealed that prospective teachers highly enjoyed the training programme, as evidenced by their consistent attendance. Notably, every preservice teacher exhibited exemplary attendance, participating diligently in every training session held from 8:00 AM to 5:00 PM throughout the four-day programme. Their punctuality further affirms their commitment to adhering to the provided guidance. Furthermore, to gauge the utility of the training's lesson sessions and classroom tools, preservice teachers were asked to rate their perceived usefulness on a scale ranging from 1 (not useful at all) to 5 (extremely useful). This evaluation, illustrated in Table 4, clearly reflects the acceptance of the training. Participants' positive perceptions regarding the lesson sessions on foundational mathematical concepts and the effectiveness of various teaching resources affirm their endorsement of the programme's value and relevance.

Regarding the lesson sessions, the data displayed in Table 4 demonstrate that each participant held a highly favourable view of the training activities, perceiving them as notably beneficial and thoroughly enjoyable. Nevertheless, it is equally important to consider that when participants were prompted to identify their least enjoyable or least favourite lessons, a discernible trend emerged. The majority of respondents, as evidenced in Figure 2, consistently pointed to lessons centred on addition and subtraction, as well as those addressing the concepts of HCF and LCM. These findings suggest areas that may require further attention or modification to enhance participants' engagement and satisfaction with specific content within the training programme.

Prospective teachers' perception of the concept of addition as excessively simplistic, unchallenging, and lacking in inherent appeal contributed to its classification as one of the less favoured lessons. Conversely, for those who did not derive enjoyment from the session on HCF and LCM, the primary contention was the perceived inadequacy of the allocated time for comprehensive understanding of the concepts. Upon comparing the solutions provided by the same respondent in both the baseline (Figure 3) and endline (Figure 4), it becomes evident that the respondent successfully represented 4 and 10 in terms of their prime factors following the training workshop. However, instead of finding the LCM, the respondent proceeded to determine the HCF. This error may arise from either a simple oversight or a student's misunderstanding of the distinction between HCF and LCM.

TABLE 4: Preservice teachers' perceived usefulness of the training activities.

Activities	Minimum	Maximum	Mean	Standard deviation
Lesson sessions				
Addition and subtraction	4	5	4.90	0.308
Multiplication	4	5	4.95	0.224
Division	4	5	4.90	0.308
Factorisation	4	5	4.85	0.366
Prime numbers	4	5	4.90	0.308
HCF and LCM	2	5	4.50	0.889
Classroom tools				
Robot cards	2	5	4.55	0.826
Ice-cream sticks	1	5	4.70	0.923
Flash cards	3	5	4.70	0.571
Drills	3	5	4.90	0.447
Maths 24 games	3	5	4.60	0.681
Khan Academy	3	5	4.80	0.523
Claps and energisers	3	5	4.60	0.598

Excerpts shown in Figure 3 and Figure 4 are a clear demonstration of what some respondents reported with regard to HCF and LCM. The majority of the respondents, on the other hand, claimed to have a better understanding of the two concepts and would no longer mistake HCF for LCM, as they had done before the training. The use of the tree method for prime factorisation, which was very useful in determining the HCF and LCM, was appreciated by the majority of the participants.

A further analysis of respondents' test scripts showed that almost all the preservice teachers who attempted to determine the HCF and LCM using the tree method managed to get a correct answer. Excerpts of the solution by Respondent 12 in the baseline test (Figure 5) and the endline test (Figure 6)

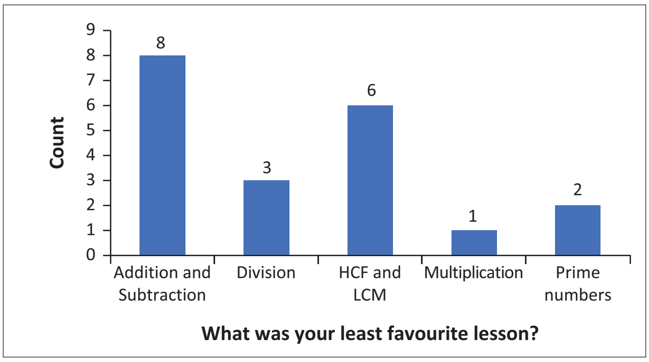

FIGURE 2: Prospective teachers' perceptions on their least favourite lessons.

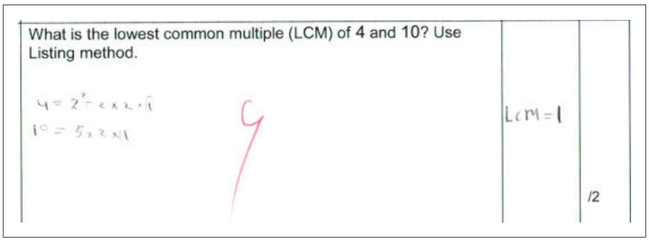

FIGURE 3: Solution by Respondent 17 in the baseline test.

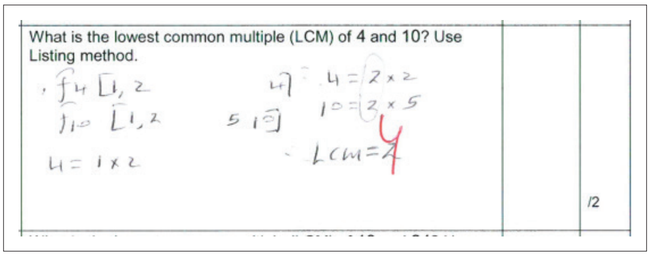

FIGURE 4: Solution by Respondent 17 in the endline test.

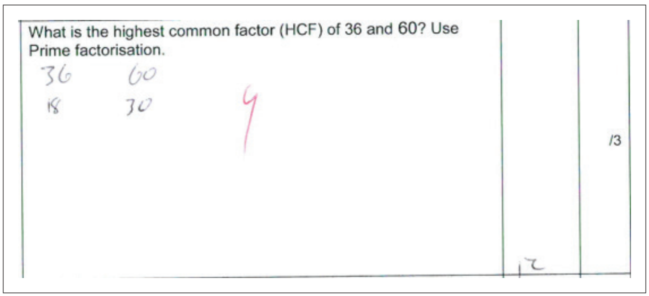

FIGURE 5*:* Solution by Respondent 12 in the baseline test.

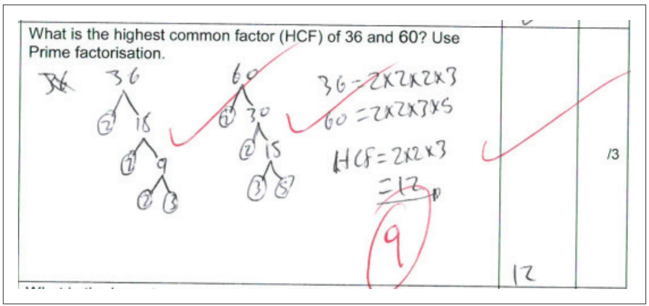

FIGURE 6: Solution by Respondent 12 in the endline test.

reflect this kind of improvement. In Figure 6, the excerpt pertaining to Respondent 12 displays the accurate determination of the HCF for 36 and 60. However, it is important to highlight an error where the respondent mistakenly represented 9 as a product of 2 and 3, leading to the incorrect prime factorisation of 36 as 2 × 2 × 2 × 3 instead of the correct representation 36 = 2 × 2 × 3 × 3. Such errors are common and may not necessarily indicate a fundamental misunderstanding of the concept, but rather a simple oversight. It is also possible that this oversight was either overlooked or went undetected by the marker.

The respondent whose solution excerpts are shown in Figure 3 and Figure 4 overall had a notable improvement from 14% in the baseline test to 77% in the endline test. A considerable improvement was also seen in the respondent whose solution snippets are shown in Figure 5 and Figure 6, going from 20% in the baseline assessment to 91% in the final assessment. It is a real reflection that none of the participants left the training facility the same way as they entered, with regard to the comprehension of selected school mathematics concepts. This is why most of the participants got excited about the opportunity and felt that such an initiative could be extended to other prospective teachers of mathematics within the university and beyond.

Discussion

The findings of this study emphasise the need for improvement in prospective teachers' proficiency levels regarding foundational mathematical concepts, particularly considering the baseline test scores. This study has shown that the average proficiency of prospective teachers in the tested foundational mathematical concepts was notably low, with a mean score of 35.6% and a standard deviation of 13.4%. This level of achievement was significantly below the minimum prospective teachers' masterly level of school mathematics set by Venkat and Spaull (2015) in their PrimTEd project. These results are not confined to the South African context or the current situation. Similar outcomes have been documented both within and outside of South Africa. For instance, a study conducted by Fonseca and Petersen (2015) on similar concepts found that preservice teachers' pretest achievement levels were quite low, with scores ranging from 17% to 73%, resulting in an average score of 37%. This pattern is consistent with findings from other studies (Alex, 2019; Alex & Roberts, 2019), clearly emphasising a widespread trend in the academic landscape. Moreover, studies from other countries have reported similar observations (Malambo et al., 2018; Mays, 2005; Meany & Lange, 2012; Niyukuri et al., 2020, Tabakamulamu et al., 2007). These scholars have consistently emphasised that possessing a high level of proficiency in university-level mathematics does not automatically translate into sufficient knowledge in teaching school-level mathematics. Considering these observations, it is imperative for teacher training institutions to exercise due diligence and prudence in their preparation of future teachers, ensuring that they are not merely acquainted with but deeply immersed in the subject matter they will ultimately impart to their students.

Despite the initial low achievement levels, this study provides compelling evidence that preservice teachers' understanding of foundational mathematical concepts can be significantly enhanced through targeted training. The substantial increase in average scores from the baseline test (*M* = 35.6, *SD* = 13.4) to the endline test (*M* = 80.3, *SD* = 13.9) not only surpassed the minimum mastery level of 60% set initially, but also demonstrated a statistically significant improvement in their knowledge of the tested mathematical concepts. Furthermore, the qualitative analysis of open-ended questionnaire items revealed that participants expressed a high level of satisfaction with the knowledge they acquired from the training. They specifically highlighted how the training effectively clarified their misconceptions, particularly in relation to the concepts of HCF and LCM. This demonstrates the transformative impact of targeted professional development trainings on prospective teachers' conceptual understanding and pedagogical skills. Consistent with calls by other scholars in the field (e.g., Bowie et al., 2019; Fonseca & Petersen, 2015; Malambo et al., 2018; Prendergast et al., 2023), findings of this study strongly advocate for the incorporation of such trainings in teacher education programmes to enhance preservice teachers' mathematical knowledge for teaching.

Findings of this study challenge the notion held by some participants that concepts of addition and subtraction were too simplistic for advanced learners. Our content analysis of pretest answer scripts paints a contrasting picture. While prospective teachers demonstrated proficiency in CCK, they exhibited deficiencies in SCK, particularly in understanding the concept of place value, and its application in teaching addition, subtraction, multiplication, and division.

This finding is consistent with previous studies conducted in various contexts (Li & Howe, 2021; Thanheiser et al., 2014), which established that preservice teachers often relied on standard algorithms but struggled to articulate the underlying rationale in the areas of addition, subtraction, multiplication, and division. Likewise, an investigation conducted by Gürefe and Aktaş (2020) brought to light significant difficulties in the comprehension of prime numbers among preservice teachers. This understanding is pivotal for the proficient application of prime factorisation in determining both the HCF and the LCM.

These discrepancies highlight the importance of not underestimating the complexity of foundational mathematical concepts and the need for comprehensive training in these areas for prospective teachers.

In line with calls for prospective teachers to possess adequate CCK and SCK, Scheiner et al. (2019) pointed out that these two types of knowledge are not mutually exclusive but rather complementary. This suggests that a deep understanding of mathematical concepts (SCK) is as important as the ability to perform mathematical operations (CCK). Our findings lend support to this argument, as we observed significant improvements in SCK, particularly in understanding the concept of place value and prime factorisation, following the training. Expanding on Scheiner et al. (2019), other scholars have contributed to the theoretical debates surrounding the necessity of both CCK and SCK in the MKT model. For example, a study by Chinnappan and White (2015) explored a strand of SCK among preservice teachers in the domain of proportional reasoning and their knowledge of evaluating the plausibility of students' claims and errors. The study found that preservice teachers, as a group, had developed a sense of student error but faced challenges in explaining the source of these errors, indicating a gap in their SCK. The authors recommended that preservice teachers needed more opportunities to develop this aspect of their knowledge through exposure to authentic student work and feedback. Similarly, Spitzer and Phelps-Gregory (2023) discovered that prospective teachers who could conceptually unpack a learning goal into subconstructs demonstrated higher-quality interpretations of student thinking. The authors argued that the skill of decomposing learning goals allows preservice teachers to apply their mathematical knowledge successfully to interpret student work. This highlights the importance of both CCK and SCK in the MKT model, emphasising their interconnected role in shaping teacher noticing.

This study provides empirical evidence to support the claim by Qian and Youngs (2016) that the quality of mathematics courses in teacher education programmes is more important than the quantity. Our findings demonstrate the positive impact of collaboration with organisations like Numeric on enhancing preservice teachers' mathematical knowledge for teaching, especially in the areas of CCK and SCK. We argue that changing the culture of mathematics education requires not only providing preservice teachers with the necessary knowledge, skills, and resources, but also fostering their confidence, motivation, and interest in mathematics. Therefore, we recommend that teacher education programmes incorporate targeted professional development training that focuses on both the content and the pedagogy of mathematics, as well as the affective aspects of teaching and learning mathematics.

Study limitations and future directions

We are aware of some limitations associated with this study, despite the success of the provided intervention. First, the facilitators were obligated to compress the course of action because the training period was quite short. Participants' answers to the feedback survey echoed this restriction. Most of them offered the following suggestions when asked to list some future changes they would like to see:

- An increase in the duration of the training as that would provide for a reduction in the number of hours per day.
- More trainers or facilitators so that more students could be incorporated.
- Inclusion of more technology in the training.

Second, the size of the trainee cohort in our study was relatively small when compared to the larger pool of preservice teachers who did not partake in the training programme. This issue has been a recurring concern in previous investigations in other settings, as evident in a study conducted by Prendergast et al. (2023). Unfortunately, due to constraints stemming from inadequate funding, addressing this issue within the scope of our current study was not feasible.

The third limitation was the absence of a comparison group, which could have been utilised to assess the effectiveness of the intervention. As Fonseca and Petersen (2015) noted, this absence of a comparison group is one of the factors that preclude studies of this nature from being categorised as true experimental research. Nevertheless, recognising the importance of evaluating the impact of such interventions, we provided an opportunity for prospective teachers to voice their suggestions for future improvements in similar interventions.

It is worth emphasising that while we value the input from prospective teachers regarding potential improvements, many of the suggestions put forth would necessitate additional financial resources for implementation. Consequently, if we aspire to significantly enhance the quality of the learning environment and the performance of students, it becomes imperative for key stakeholders to come together and commit resources to support the training of mathematics teachers.

Conclusion

The purpose of this study was to examine the foundational mathematical knowledge of prospective teachers at a rural

South African university and assess the effectiveness of professional development training in enhancing this knowledge. The study provides encouraging evidence that targeted training can significantly improve preservice teachers' understanding of foundational mathematical concepts. The significant increase in average scores from the baseline to the endline test demonstrates the potential of such training programmes in enhancing preservice teachers' foundational mathematical knowledge. The positive feedback from participants further stresses the effectiveness of these trainings in clarifying misconceptions and improving both the CCK and SCK among prospective teachers. Interestingly, the study revealed that even seemingly simple concepts like addition and subtraction can pose challenges for advanced learners, particularly in the context of SCK. This stresses the need for teacher education programmes to ensure a comprehensive understanding of all mathematical concepts, regardless of their perceived simplicity.

Furthermore, this study suggests a need for more research on how to design, implement, and evaluate such professional development initiatives in different contexts and settings. Specifically, future research should incorporate both experimental and comparison groups, with relatively longer professional development trainings to cater for more advanced mathematical concepts. On a practical level, the findings of this study have significant implications for teachers, schools, and education policy more broadly. For teachers, the results emphasise the importance of continuous professional development in enhancing their mathematical knowledge. For schools, the findings suggest the need to support such professional development opportunities for their teachers. At the policy level, the results advocate for the integration of such professional development trainings in teacher education programmes. By applying these findings in the classroom and at the policy level, we can ensure improved learning outcomes for students.

In all, this study stresses the importance of targeted professional development trainings in enhancing the mathematical competency of future teachers. It advocates for the incorporation of such trainings in teacher education programmes, echoing similar calls by other scholars in the field. By doing so, we can ensure that our future teachers are not just familiar with but have a deep understanding of the mathematical concepts they will be teaching, ultimately leading to improved learning outcomes for their students. The improvement that was seen after exposure to the training programme also reiterates a need for collaboration with organisations like Numeric to provide preservice teachers with relevant and engaging professional development opportunities that can enhance their mathematical knowledge for teaching.

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Competing interests

The authors affirm that neither their personal nor financial relationships might have intentionally influenced the development of this work.

Authors' contributions

Both authors, A.M. and J.K.A., contributed equally to the development of this manuscript.

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Data availability

The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy and ethical restrictions.

Disclaimer

The views and opinions expressed in this article are those of the authors and do not necessarily reflect the official policy or position of any affiliated agency of the authors.

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