Optimization of pipe networks using standardized pipes

E.H. Mathews[1] and **H.F. Brenkman** and **P.A.J. Köhler**

(First received December 1993; Final version June 1994)

Abstract

Large pipe networks are frequently encountered in engineering applications. Numerical optimization of these designs is therefore relevant. As pipes are usually only available in standard sizes, the optimization must be done for discrete pipe sizes. In this paper a discrete coordinate descent and a continuous gradient search technique are combined to solve this discrete non-linear optimization problem.

Nomenclature

a_i	Pressure head coefficient
b_i	Flow coefficient
C	Inequality error function
C_c	Capital cost ($)
C_d	Energy demand rate ($/kW)
C_e	Energy rate ($/kWh)
C_i	Pipe cost factor ($/kg)
C_p	Pump energy cost ($)
C_t	Total cost ($)
D	Pipe diameter (m)
D_p	Pump rotor diameter (m)
d	Design life time (years)
f	Friction factor
g	Gravitational constant (9.81 m/s^2)
$H(x)$	Error flow function in x
h	Static head (m)
I	Interest rate (%/100)
K_f	Dimensionless flow coefficient
K_h	Dimensionless pressure head coefficient (calculated from empirical data)
k	Damper coefficient
k_x	Arbitrary constant
L	Pipe length (m)
$L(\bar{x})$	Penalty function in x
M_i	Mass of pipe section (kg)
m	Pump mass flow (kg/s)
n_p	Pump speed (rpm)
P	Pressure (Pa)
P_p	Pressure drop over pump (Pa)
$PWEF$	Present worth escalation factor
Q	Volume flow rate (m^3/s)
Q_p	Pump volumetric flow rate (m^{3}W/s)
e_p	Pump efficiency
e_m	Pump motor efficiency
Po_p	Pump energy (kW)
Re_d	Reynolds number for pipes
T	Total operating hours in a year
V	Fluid speed (m/s)
x	Variable representing pipe diameter (m)
Δh	Head loss (m)
ΔP	Pressure drop (Pa)
ϵ	Interior roughness of pipe (m)
ρ	Fluid density (kg/m^3)

[1]Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria, 0002 Republic of South Africa and concultants to Transfer of Energy, Momentum and Mass International (Pty) Ltd

Introduction

Pipe networks are common in all engineering fields. For example, every town council has to provide water and a sewerage system for its residents. The petroleum and power generation industries use complex pipe networks.[1] In the field of mechanical engineering, pipe networks are found *inter alia* in the aircraft and car industries. Applications here vary from fuel distribution networks to hydraulic systems.[2]

Many of the above networks can be expensive to build and operate.[3] The most prominent cost for the developer is usually the capital invested to build the pipe network. However, it should be the objective of any designer to minimize not only the capital cost but also the operating cost of a pipe network.

Pipe networks are designed to conform to certain specifications. For instance, the outlet flows are often specified, as well as the bounds for the fluid speed, absolute pressure and diameters.[3; 4] The fluid speed usually has an upper bound to prevent excessive noise. A lower bound is only necessary when slurry is pumped, e.g. for a coal pipeline. This is to prevent settling of the mixture. The absolute pressure is often limited to a certain value to prevent cavitation, while an upper bound prevents the pipe from bursting. The bounds for the diameters are set according to space requirements and availability. Along with all these complications, pipes are usually made in standardized diameters. All these aspects make the design process difficult and time-consuming.

To obtain a workable design the engineer continually has to match compromises and decisions. Furthermore, the efforts of the engineer often do not guarantee a cost-effective design. A practical optimization and design tool is therefore a necessity.[3] The objective of this paper was to develop a computer algorithm for the discrete optimization of pipe networks. With the help of this optimization tool the engineer can produce a practical and cost-effective solution for the design problem.

Several algorithms can be used to solve the discrete non-linear optimization problem.[5] One possibility is to calculate the network cost and flow conditions for every possible combination of pipe diameters. This method has the advantage that it always finds the global minimum and is relatively easy to implement. It is, however, computer intensive and can restrict the size of network that can be optimized.

An alternative is to simply round off the solution obtained from a non-discrete optimization technique. This method does not guarantee an optimum working solution.[6] For most network design problems, however, the rounded off solution will in all probability not differ very much from the optimum solution. A $\frac{1}{3}$ boundary rounding procedure has been recommended for the discrete optimization of air-conditioning duct networks. By this procedure, the lower nominal size is selected when the initial size is close to the lower size at $\frac{1}{3}$ the range between available sizes. Otherwise, the upper nominal size should be selected.

The 'Branch and Bound' method, too, was examined. It is described in detail in Wismer & Chattergy.[6] Unfortunately it is difficult to program this method. Another drawback is that it requires a non-discrete optimization algorithm to find a discrete solution.[6] This makes the method time-consuming. The amount of computer memory needed is also not desirable. Furthermore it was unsuccessful when applied to optimizing a five-pipe network and also abandoned.

In this article a combination of a continuous gradient search technique and discrete coordinate descent technique was employed to solve the pipe network design problem.

Element theory

A pipe network usually consists of a number of pipes and at least one pump. The behaviour of the elements and the topology of a network must be evaluated before the network can be simulated.

The flow of a fluid through a pipe is described by Bernoulli's equation

$$\frac{P_{\mathrm{enter}}}{\rho g} + \frac{V_{\mathrm{enter}}{}^2}{2g} + h_{\mathrm{enter}} = \frac{P_{\mathrm{exit}}}{\rho g} + \frac{V_{\mathrm{exit}}{}^2}{2g} + h_{\mathrm{exit}} + \Delta h \quad (1)$$

For incompressible flow and a constant diameter the equation can be simplified to:

$$\frac{P_{\mathrm{enter}} - P_{\mathrm{exit}}}{\rho g} + (h_{\mathrm{enter}} - h_{\mathrm{exit}}) = \Delta h \quad (2)$$

The pipe friction losses can be calculated from equation (3):

$$\Delta h = \frac{V^2}{2g}\left(\frac{fL}{D}\right) \quad (3)$$

The friction factor in equation (3) is given by Colebrook's equation [7]

$$\frac{1}{f} = \sqrt{-2.0 \log\left(\frac{\epsilon}{3.7d} + \frac{2.51}{Re_{\mathrm{d}}\sqrt{f}}\right)} \quad (4)$$

These equations describe the flow through a single pipe.

The pressure increase over a pump is given by

$$P = K_{\mathrm{h}} \rho n_{\mathrm{p}}^2 D_{\mathrm{p}}^2 \quad (5)$$

and the dimensionless pressure head coefficient is calculated from

$$K_{\mathrm{h}} = a_0 + a_1 K_{\mathrm{f}} + a_2 K_{\mathrm{f}}^2 + \ldots \quad (6)$$

The coefficients in equation (6) are obtained by fitting a polynomial function to empirical data for the specific pump.[4]

The flow coefficient in equation (6) is defined as

$$K_{\mathrm{f}} = \frac{m}{\rho n_{\mathrm{p}} D_{\mathrm{p}}^3} \quad (7)$$

Equations (5), (6) and (7) now characterize the pump element.

Network theory

A relationship between the element and network properties must be found before a pipe network can be simulated. This is necessary to define the governing equations describing the behaviour of the elements in a network.

Networks are governed by certain laws. Kirchoff [4] defined these laws as

- the node law: The sum of the flow through any node in any network is zero.
- the loop law: The sum of all the pressure changes in any loop of any network must be zero.
- the element law: The flow through any network element in any network must have a relationship with the pressure difference over that element.

The first two laws describe the topological features of a network and the last the physical properties of an element in a network. These laws must be used to compile the necessary equations to solve the design problem. With the help of graph theory these laws were effectively linked together.[8]

Simulation and optimization

To calculate the operating cost of a network it is necessary to simulate it. An existing computer algorithm for the simulation of pipe networks [8] is used in this study. This computer algorithm uses a proven method in an iterative process for the calculation of the flow in each element of the network.

When optimizing pipe networks the most prominent problem is that of constraints. A constraint is a certain condition to which the solution of a system must comply. For practical reasons constraints are necessary. The following types of boundary conditions are usually used: specified flow rates at the outlets, minimum pressure in any element of the network and the maximum allowable fluid velocity. The optimization method must be able to optimize within these given constraints.

Some of these constraints are equality constraints and others inequality constraints. The difference will be discussed in the following paragraphs. The optimization method must be able to discern the difference between these two types of constraints.

Equality and inequality constraints

When a pipe network is designed, one of the objects might be to design it so that the cost is as small as possible, but with the provision that the specified outlet flow rates ($Q_{\text{specified}}$) are certain given values. The flow rate forms an equality constraint and it can be expressed as

$$H_{\text{flow}}(x) = Q_{\text{specified}} - Q_{\text{simulated}}(x) = 0 \tag{8}$$

where x is the diameter. An additional objective in the design process is to minimize the flow rate error $H(\mathrm{x})$ in the equality constraint equation.

Let us now consider inequality constraints. Each pipe has a minimum and maximum allowable diameter. The same principle applies for the absolute pressure in a pipe and the fluid velocity in a pipe. Usually only an upper or lower boundary is considered.

These boundaries can be defined as:

$$\begin{aligned} d_{\text{min}} &\leq x \leq d_{\text{max}} \\ P_{\text{min}} &\leq P \leq P_{\text{max}} \\ V_{\text{min}} &\leq V \leq V_{\text{max}} \end{aligned} \tag{9}$$

which are then the inequality constraints.

Cost

The principal objective in the optimization process is to minimize the cost subject to the equality and inequality constraints.

The cost of a pipe network consists of the capital investment and the energy cost to operate the system over a number of years.

A possible equation for the capital cost ($C_{\text{c}}(\mathrm{x})$) of the network can be expressed as

$$C_{\text{c}}(\bar{x}) = \sum_{i=1}^{n} C_{\text{i}} M_{\text{i}}(x_{\text{i}}) \tag{10}$$

where

$$\bar{x} = [x_1, x_2, x_3, ..., x_n]^T \tag{11}$$

The operating cost of the network consists mainly of the energy cost (C_{p}) of the pumps. In present worth terms it can be presented as in Mathews & Köhler.[8]

The present worth escalation factor ($PWEF$) [8] can be calculated from

$$C_{\text{p}} = (PWEF)(C_{\text{d}} + C_{\text{e}}T)\, Po_{\text{p}} \tag{12}$$

$$(PWEF) = \frac{(1+I)^d - 1}{I(1+I)^d} \tag{13}$$

The pump energy is calculated by

$$Po_{\text{p}} = \frac{P_{\text{p}} Q_{\text{p}}}{e_{\text{p}} e_{\text{m}}} \tag{14}$$

where

$$e_{\text{p}} = b_0 + b_1 K_{\text{h}} + b_2 K_{\text{h}}^2 + \ldots \tag{15}$$

The objective function (total cost ($C_{\text{t}}(\mathrm{x})$) is the sum of the capital cost ($C_{\text{c}}(\mathrm{x})$) and the pump energy cost (C_{p}).

$$C_{\text{t}}(\bar{x}) = C_{\text{c}}(\bar{x}) + C_{\text{p}} \tag{16}$$

It is the objective of our optimization to minimize the total cost function ($C_{\text{t}}(\mathrm{x})$).

Discrete optimization

The object of the cost optimization process is to find a discrete solution for the pipe network subject to certain constraints. In this process there is no guarantee that a combination of standardized pipes can be found having the same outlet flow rate for the network as specified. The use of dampers in the outlet pipes to control the outlet flow rates is therefore a requirement.

This optimization procedure comprises the following:

- Optimize the outlet pipes non-discretely
- Optimize the interior pipes discretely
- Replace each outlet pipe with a standardized pipe and a damper

This procedure will ensure that the outlet flow rates are accurate and that a local minimum is reached.

When optimizing a network the interior and outlet pipes are treated differently. The diameters of the interior pipes can only have discrete values. The outlet pipes can have any size diameter. The results for the outlet pipes are then given as a standardized pipe diameter plus a damper. The computer procedure automatically takes care of this.

To arrive at this optimum solution a penalty function is used in the discrete optimization procedure. A penalty function is a combination of the objective function and the error functions. In this case the objective function is the total cost. The error function of importance here is the error flow rate function. A penalty function is used to optimize a function which is bounded by constraints.

The penalty function is thus given by

$$L(\bar{x}) = C_{\text{t}}(\bar{x}) + \sum_{i=1}^{n} Pe_{\text{i}}^2 H_{\text{i}}^2(x_{\text{i}}) \tag{17}$$

The cost term in the penalty function is given by

$$C_t(\bar{x}) \text{ where } \bar{x} = [x_1, x_2, ..., x_n]^T \tag{18}$$

and the penalty term (error function) by

$$\sum_{i=1}^{n} Pe_i^2 H_i^2(x_i) \tag{19}$$

Pe_i is the penalty constant and $H_i(x_i)$ is the error flow through pipe 'i' with diameter x_i. The penalty term forms a constraint which has to be minimized along with the cost.

To understand the optimization procedure it is best to examine Figure 1. This figure represents a two-dimensional system. The function to be optimized depends on two variables. Variable x_1 is defined as

$$x_1 \epsilon [d_{\min}, d_1, d_2, ...d_n, d_{n+1}, d_{\max}] \tag{20}$$

and variable x_2 is defined as

$$x_2 \epsilon \{x_{\min} \leq x_2 \leq x_{\max} \mid R^1\} \tag{21}$$

This system is a typical example of a two-pipe network where the pipes are connected in series. Variable x_1 represents the interior pipe and variable x_2 the outlet pipe with its damper.

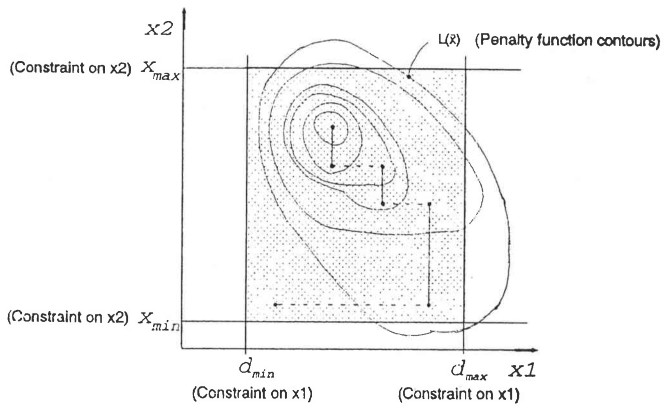

Figure 1 Optimization of a two-dimensional system.

Let us first shift variable x_1 in the direction in which the penalty function $L(x)$ decreases until a minimum is reached. After a minimum is reached, variable x_2 is manipulated in a similar way. These two (x_1 and x_2) are again manipulated until a local minimum is reached. Notice that the optimization process takes place in the shaded area formed by the constraints.

When a local optimum is reached, all the outlet pipes are replaced with a standardized pipe and a damper. The damping coefficient k_d for the outlet pipes can be calculated from equation (22). This will ensure that the outlet flow rates are as specified for the problem. The subscripts e and p refer to the non-discrete and discrete diameters, respectively.

$$\frac{f_e LV^2}{D_e} = k_d V^2 + \frac{f_p LV^2}{D_p} \tag{22}$$

Figure 2 shows a flow diagram for the above optimization procedure. Note the feedback loops. They ensure that the variables stay within the specified boundaries. The technique is a combination of a discrete coordinate technique for the interior pipes and a continuous gradient search technique for the outlet pipes.

Any set of diameters can be used as a starting point for the optimization proocedure. The only requirement is that they comply with the specified constraints. This is to get a realistic first simulation.

Case studies

Two case studies were carried out to verify the discrete optimization procedure. One of the case studies is a pipe network with no loops. The other one is a simple network having one loop.

Case study 1

The network for the first case study is given in Figure 3. It consists of 14 interior pipes and 8 outlet pipes. Specifications for this case study are given in Table 1. Two calculations were performed. Firstly, the discrete pipe diameters were calculated that will satisfy the flow requirements but not necessarily minimize the life cycle cost of the network. Secondly, the pipe diameters were calculated that satisfy the flow constraints as well as minimize the life cycle cost. The results of these calculations are given in Table 2. The savings obtained by minimizing the life cycle cost for this study was 28% compared to when only the flow requirements were taken into account.

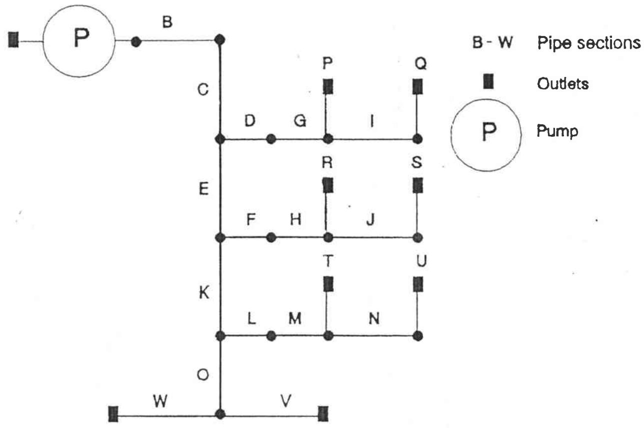

Figure 3 Pipe network for Case study 1.

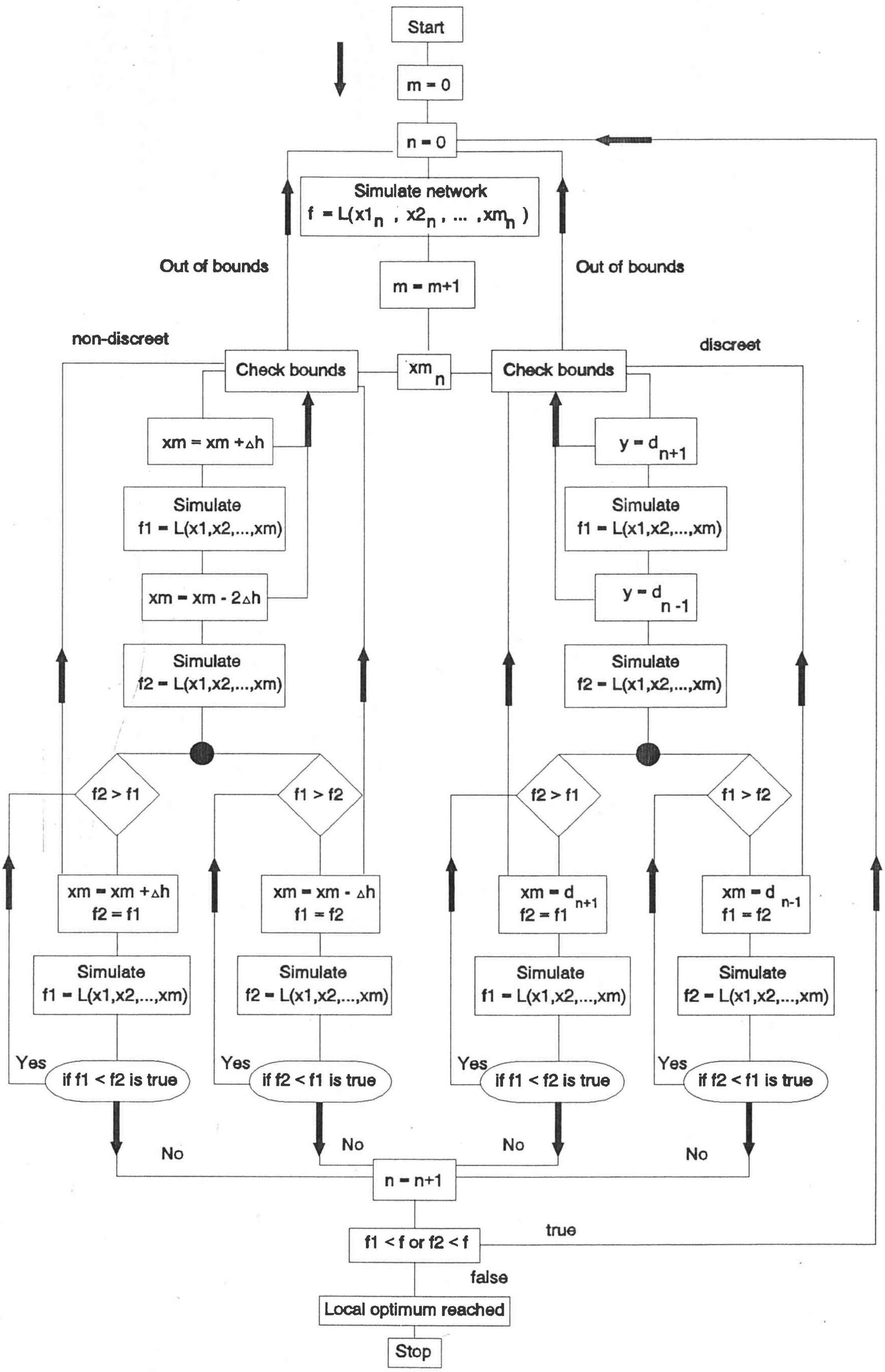

Figure 2 Flow diagram depicting the optimization procedure.

Table 1 Design data for Case study 1

Design specifications : Required flow rates in pipe sections (P,Q,R,S,T,U,V,W) are 0.15 kg/s

Fluid	Density	998 kg/m^3
(Water)	Viscosity	0.00101 Ns/m^2
Economic data	Interest rates (I)	15%
	Design life time (d)	20 years
	Energy rate (C_e)	8.62 c/Kwh
	Energy demand rate (D_d)	\$28/Kw/month
	Operating hours (T)	6 750 hours/year
	Pipe material cost factor (C_i)	\$20/kg
Pipe data	Pipe material	Brass tubing
	Surface roughness	0.0015 m
	Pipe thickness	5 mm
	Pipe material density	7 000 kg/m^3
	Interior pipe lengths	10 m
	Outlet pipe lengths	5 m
Pump data	Pump speed (n_p)	1 600 r.p.m.
	Pump rotor diameter	0.2
	Flow coefficients	$b_0 = 0.59336$
		$b_1 = -5956.741$
		$b_2 = 94802776.0$
		$b_3 = -430096466310.0$
	Pressure head coefficients	$a_0 = 0.00158$
		$a_1 = -1.09156$
		$a_2 = -2001.318$
		$a_3 = -30281303$
	Pump motor efficiency	0.7

Table 2 Results of calculations

	Non-optimized network		Discretely optimized network		
Pipe	Diam. (mm)	Flow (kg/s)	Diam. (mm)	Flow (kg/s)	k_d
B	22	1.20	60	1.178	none
C	22	1.20	20	1.178	none
D	40	0.30	25	0.283	none
E	60	0.90	60	0.895	none
F	46	0.30	30	0.299	none
G	40	0.30	20	0.283	none
H	46	0.30	20	0.299	none
I	51	0.15	40	0.143	none
J	52	0.15	40	0.151	none
K	59	0.60	40	0.596	none
L	54	0.30	45	0.298	none
M	54	0.30	20	0.298	none
N	53	0.15	60	0.149	none
O	46	0.30	20	0.299	none
P	45	0.15	25	0.140	10.5
Q	51	0.15	25	0.143	10.3
R	45	0.15	25	0.148	10.5
S	52	0.15	25	0.151	10.3
T	46	0.15	20	0.149	8.3
U	53	0.15	20	0.149	8.3
V	50	0.15	20	0.150	8.3
W	50	0.15	20	0.149	8.4
Economic data					
Energy cost (20 year)	\$1 330			\$1 009	
Capital cost	\$11 384			\$8 082	
Total cost	\$12 714			\$9 091	

Case study 2

In this case a simple theoretical network with a single loop is examined. Although it is not an existing network against which we could compare the optimization results, it was chosen to show that networks with loops can be optimized. The required outlet flow rates in branches F and G are 0.5 kg/s and 0.25 kg/s, respectively. The other input data are the same as for the previous case study. Figure 4 shows a diagram representing the network. The results of Case study 2 are presented in Table 3.

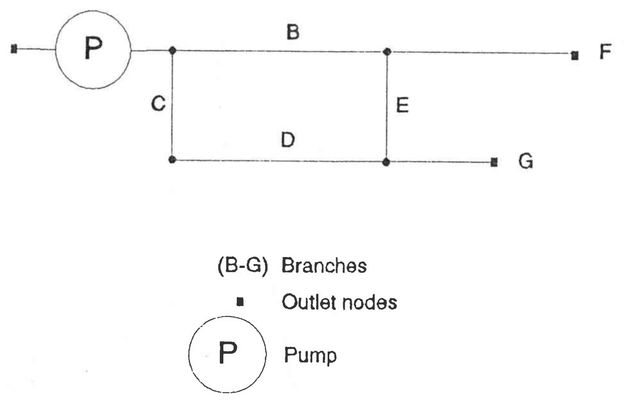

Figure 4 Network diagram for Case study 2.

Table 3 Results of Case study 2

Pipe	Optimized diameters (mm)
B	15
C	15
D	15
E	15
F	25
G	15
Economic data	
C_p	\$1 047
C_c	\$2 034
C_t	\$3 081
Mass flow rate in	
F	0.493 kg/s
G	0.244 kg/s

Conclusions

A procedure for the discrete optimization of pipe networks was developed. The procedure was implemented in a user-friendly computer program. The constrained non-linear optimization problem is modified to an unconstrained optimization problem using penalty functions. A combination of a discrete coordinate descent and continuous gradient search technique is then used to solve the unconstrained optimization problem.

Due to the complexity of the optimization problem the modified objective function will contain a large number of local minima. The only way to determine when the global minimum has been reached is to locate all the local minima and then choose the best one of them. The fact that multiple minima exist has been confirmed by using different starting values to the problems. In each case the solutions did not exactly converge to the same values. For each of these solutions the costs were, however, within reasonable limits approximately the same.

Further improvements to the program may include the optimization of networks with valves and reservoirs. The ultimate objective of our work is the development of a user-friendly computer program for application in such diverse fields as water management, the chemical industry, air conditioning, etc.

References

[1] Lathey Jr & Moody FJ. *The thermal-hydraulics of a boiling water nuclear reactor*, 3rd edn. American Nuclear Society, 1984.

[2] Green WL. *Aircraft hydraulic systems, An introduction to the analysis of systems and components.* John Wiley & Sons, 1985.

[3] Stephenson D. *Developments in water science, Pipeline design for water engineers*, 2nd edn. Elsevier Scientific Publishing Company, 1981.

[4] Daugherty RL, Franzini JB & Finnemore EJ. *Fluid mechanics with engineering applications*, Metric edn. McGraw-Hill, 1989.

[5] Rand Afrikaans University. *Optimization theory and applications.* A Halsted Press Book, 1979.

[6] Wismer DA & Chattergy R. *Introduction to nonlinear optimization.* North-Holland, New York, 1987.

[7] White FM. *Fluid mechanics.* McGraw-Hill, 1989.

[8] Mathews EH & Köhler PAJ. A numerical optimization procedure for complex pipe network design. *International Journal of Numerical Methods for Heat and Fluid Flow*, in press.