Optimization of a vibratory conveyor for reduced support reaction forces

A.J. van Wyk,[1] **J.A. Snyman**[2] and **P.S. Heyns**[2]

Abstract

Simple vibratory conveyors, consisting of a horizontal table supported by vertical helical springs, are of the most versatile material handling equipment in industry. A disadvantage of these conveyors is the transmission of dynamic forces to the supporting structure. A design of such a conveyor may be optimized with respect to the transmission of dynamic forces through the adjustment of various design parameters. In this study the positions at which the springs are attached to the table are taken as the design variables. The optimization procedure involves the mathematical modelling of the dynamic behaviour of the system, which allows for the computation of a so-called transmission function which is a measure of the dynamic forces transmitted, and which may be minimized with respect to the design variables. The computed optimum design gives reductions of more than 20% in the value of the transmission function, when compared to that of two arbitrary initial designs.

Nomenclature

a, b, h	geometrical design variables as defined in Figure 2
θ	angle of conveyor table to the horizontal
x_g, y_g	coordinates of centre of gravity of table
ℓ_1, ℓ_2	respective spring lengths at the feed and discharge ends of the table
α, α'	respective top angles
β, β'	respective base angles
$q_i, i = 1, ..., 9$	generalized notation for the nine state variables immediately above
$\mathbf{q}$	vector form of above state variables
ℓ_1^0, ℓ_2^0	respective undeformed lengths of the springs
m	mass of table
I_g	moment of inertia of table about axis through centre of mass and perpendicular to plane
$\Delta\ell_1, \Delta\ell_2$	respective linear deformations of springs
g	gravitational acceleration
$\mathcal{L}$	Lagrange function
T	kinetic energy
V	potential energy
k_1, k_2	torsional spring constants at feed end
k_3, k_4	torsional spring constants at discharge end
k_5, k_6	respective axial spring constants at feed and discharge ends
$g_j, j = 1, ..., 6$	expressions for geometrical constraints
$\lambda_i, i = 1, ..., 6$	Lagrange multipliers
$\boldsymbol{\lambda}$	multiplier vector
t	time
ω	angular frequency of harmonic exciting force F_0
μ	total eccentric mass
r	eccentricity
$\boldsymbol{\rho}$	vector position of action of harmonic force F_0
ρ	radial position of excitor
τ	angular position of excitor
σ	excitation angle relative to table
c_{xx}	damping constant in x direction
c_{yy}	damping constant in y direction
$c_{\theta\theta}$	rotational damping constant
R, S	respective reactive forces at feed and discharge ends
$x_i, i = 1, ..., n$	n general design variables
$\mathbf{x}$	vector form of design variables
d	objective function

[1] M Eng student, Department of Mechanical and Aeronautical Engineering, University of Pretoria, Pretoria, 0002 Republic of South Africa

[2] Professors, Department of Mechanical and Aeronautical Engineering, University of Pretoria

Introduction

Horizontal vibratory screens and conveyors are of the most common and versatile material handling equipment used in mining, in particular, and industry, in general.[1;2;3] They are capable of conveying a wide variety of materials economically and effectively over long and short distances. In addition to the transport function they are also used as screens to separate, dry, and grade materials.

A schematic representation of the design of the most basic horizontal vibratory conveyor is given in Figure 1. Typically it consists of a horizontal trough or table supported by four vertically placed helical springs. Motion is induced by a vibratory excitor mounted on the table. The direction of the stroke action may be adjusted depending on the type of material motion required. Although more complicated vibratory conveyors have been designed for specialized applications, most of the conveyors in use are of the above relatively simple design. It is indeed its simplicity and ease of operation that have made the basic vibratory conveyor an economically attractive machine.

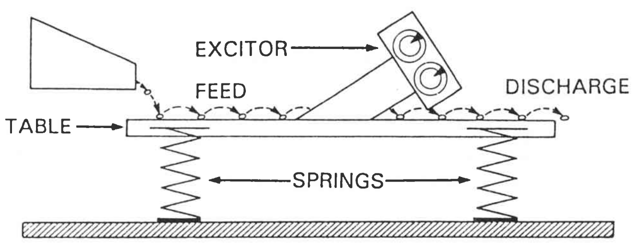

Figure 1 The basic horizontal vibratory conveyor

A disadvantage of the basic machine is the transmission of dynamic forces to the supporting structure and environment. It may therefore be important to be able to minimize this transmission, through the adjustment of design parameters within constraints imposed by the simplicity and the effective functioning of the machine. For example, it has been found that for the effective sorting and transportation of a wide range of materials a stroke of between 3 and 5 mm is required [4] when the table vibrates at a frequency of 16 Hz.

As far as can be ascertained from the literature very little work, and then with limited success, has been done with regard to the optimization of vibratory conveyors. The failure is mainly due to an over-simplified and unrealistic approach to the mathematical modelling of the dynamic behaviour of the conveyors.[5]

In a recent paper, Snyman & Vermeulen [6] analysed a detailed and realistic model of a vibratory conveyor mounted on the more complicated ROSTA suspension units. Although different configurations were modelled in order to select the most satisfactory design from those analysed, no systematic optimization was carried out with respect to design variables to determine the best or optimum design. In this study the modelling is restricted to that of the basic conveyor depicted in Figure 1, but the optimization is carried out in a systematic and economic way by means of a mathematical programming technique to yield the optimum design.

Optimization of the conveyor may be performed with respect to any number of material, operational and geometrical parameters. This initial study was restricted to the influence of geometrical design parameters. In particular, only the positions at which the supporting springs are attached to a standard conveyor table are taken as design variables, with standard prescribed operating conditions and component materials and dimensions being assumed. The optimization exercise requires, first of all, the successful mathematical modelling of the dynamic behaviour of the system. Having done so, a so-called transmission function, which is a measure of the dynamic forces transmitted to the supporting structure, may be computed. The transmission function may then be minimized, with respect to the geometrical design variables, by means of a suitable numerical optimization technique.

All the steps in the above optimization procedure were carried out successfully. A realistically detailed mathematical model was constructed and experimentally verified. This enabled the definition and computation of an objective function which accurately reflects the transmission of dynamic forces to the supporting foundation. Some difficulties were encountered with the application of standard 'off-the-shelf' optimization codes. These problems are ascribed to the existence of 'noise' in the computed objective function. The optimization was finally successfully done by means of Snyman's [7] dynamical and heuristic LFOP1(b) algorithm.

The computed optimum design gives significant reductions of more than 20% in the value of the transmission function when compared to that of two acceptable but arbitrarily chosen initial designs.

Mathematical model

System description

Since the conveyor has a symmetry plane and the harmonic force of the excitor acts in this plane, all motion is parallel to it, and use may therefore be made of a planar model. A schematic representation of the model is presented in Figure 2. The symmetry allows for the four symmetrically placed supporting springs to be consolidated into two for the purpose of the modelling. The table DC is attached to a rigid foundation by means of the two vertical helical springs. Assume that in the initial undeformed state, with the springs vertical and the table horizontal, the ends of the springs are rigidly attached to the base and to the table, respectively. This introduces torsional springs at the points of attachment. Linear elastic restoring forces and moments are assumed for which the spring constants may be obtained experimentally. The axial spring constants are denoted by k_5 and k_6 and the respective torsional spring constants by k_1, k_2, k_3, and k_4 as indicated in Figure 2.

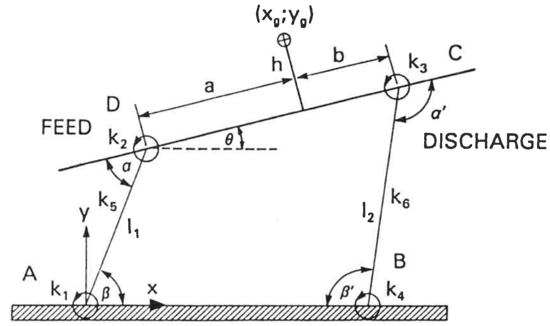

Figure 2 Schematic representation of a planar model of the conveyor

The *state variables* giving a detailed description of the system and also indicated in Figure 2 are: the angle θ to the horizontal, the coordinates of the centre of gravity of the table (x_g, y_g), the respective base angle, spring length and top angle of the two individual springs: $\beta, \ell_1, \alpha, \beta', \ell_2$, and α'. For convenience these nine state variables, in the order in which they were introduced, will also be referred to by $q_1, q_2, ..., q_9$. The undeformed natural lengths of the two springs are denoted by ℓ_1^0 and ℓ_2^0, respectively. In the case of the horizontal conveyor of interest here, the initial undeformed system corresponds to the state where $\theta = 0$, $\beta = \alpha = \beta' = \alpha' = \frac{\pi}{2}$ and $\ell_1 = \ell_1^0 = \ell_2^0 = \ell_2$.

The geometrical *design variables*, of interest in this optimization study, are the positions of attachment of the vertical springs relative to the centre of mass of the table. They are denoted by the distances a, b, and h, as indicated in Figure 2.

The system clearly has only three degrees of freedom. Having introduced nine state variables for convenience of description implies the existence of six *constraint equations*. These equations define the relationship and interdependence between the variables and are of the general form

$$g_j(q_1, q_2, ..., q_9) = g_j(\mathbf{q}) = 0, \quad j = 1, 2, ..., 6 \quad (1)$$

where $\mathbf{q} = (q_1, q_2, ..., q_9)^T$

By inspection of Figure 2, the following six geometrical and holonomic constraint equations may be identified:

$$g_1 = \ell_1 \cos\beta + (a+b)\cos\theta + \ell_2 \cos\beta' - (a+b) = 0 \quad (2)$$

$$g_2 = \ell_1 \sin\beta + (a+b)\sin\theta - \ell_2 \sin\beta' = 0 \quad (3)$$

$$g_3 = \beta + \beta' - \alpha - \alpha' = 0 \quad (4)$$

$$g_4 = \ell_1 \cos\beta + a\cos\theta - h\sin\theta - x_g = 0 \quad (5)$$

$$g_5 = y_g - \ell_1 \sin\beta - a\sin\theta - h\cos\theta = 0 \quad (6)$$

$$g_6 = \theta + \alpha - \beta = 0 \quad (7)$$

Lagrange equations of motion

With the system described in terms of generalized coordinates and constraint equations, the natural way to analyse the motion of the system is by means of Lagrange mechanics. Of particular importance for this study is that the Lagrange approach allows for the easy determination of the reaction forces, at the spring attachments, in terms of the so-called Lagrange multipliers associated with the constraint equations. We briefly sketch the derivation of the relevant equations of motion. For the moment the shaking force as well as the damping forces are ignored. They will be introduced later.

The Lagrange function is defined as

$$\mathcal{L} = T(\mathbf{q}, \dot{\mathbf{q}}) - V(\mathbf{q}) \quad (8)$$

where T denotes the kinetic energy and V the potential energy of the system at any instant t. For the conveyor system considered here equation (8) may be written as

$$\mathcal{L} = \frac{1}{2}\left\{ \begin{array}{c} m\dot{x}_g^2 + m\dot{y}_g^2 + I_g\dot{\theta}^2 - k_1\left(\beta - \frac{\pi}{2}\right)^2 \\ -k_2\left(\alpha - \frac{\pi}{2}\right)^2 - k_3\left(\alpha' - \frac{\pi}{2}\right)^2 \\ -k_4\left(\beta' - \frac{\pi}{2}\right)^2 - k_5\Delta\ell_1^2 - k_6\Delta\ell_2^2 - 2mgy_g \end{array} \right\} \quad (9)$$

where m denotes the mass, I_g the moment of inertia about the axis through the centre of mass and perpendicular to the plane, and $\Delta\ell_1$ and $\Delta\ell_2$ the respective linear deformations of the springs.

The Lagrange equations for the dynamical system with the six prescribed constraint equations (1) may now be written in terms of the Lagrange multipliers $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_6)^T$ as :[8]

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{q}_k}\right) - \frac{\partial\mathcal{L}}{\partial q_k} = \sum_{j=1}^{6}\lambda_j\frac{\partial g_j}{\partial q_k}, \quad k = 1, 2, ..., 9 \quad (10)$$

Applying (10) to (9) for $k = 1, 2$ and 3 yields the equations of motion:

$$\begin{array}{c} I_g\ddot{\theta} = f_1 = -\lambda_1(a+b)\sin\theta + \lambda_2(a+b)\cos\theta \\ +\lambda_4(a\sin\theta + h\cos\theta) + \lambda_5(h\sin\theta - a\cos\theta) + \lambda_6 \\ m\ddot{x}_g = f_2 = \lambda_4 \\ m\ddot{y}_g = f_3 = \lambda_5 - mg \end{array} \quad (11)$$

Since $\dot{q}_k$ for $k = 4, ..., 9$ does not explicitly appear in (10), it is important to realize that for any instant t, with $\mathbf{q}$ known, we may solve the six remaining equations in (10) to yield $\boldsymbol{\lambda}$, which of course, in general, also varies with time.

By defining new variables $q_{10} = \dot{\theta} = \dot{q}_1$; $q_{11} = \dot{x}_g = \dot{q}_2$ and $q_{12} = \dot{y}_g = \dot{q}_3$ we have, together with equations (11), six first order differential equations in the twelve dependent variables $q_1, q_2, ..., q_{12}$. A further six first order differential equations can be obtained by transforming the six constraint equations (1) by differentiation to the form:

$$\sum_{k=4}^{9}\left(\frac{\partial g_j}{\partial q_k}\right)\dot{q}_k = \sum_{k=1}^{3}\left(\frac{\partial g_j}{\partial q_k}\right)\dot{q}_k = \sum_{k=1}^{3}\left(\frac{\partial g_j}{\partial g_k}\right)q_{9+k} = \acute{c}_j, \quad j = 1, 2, ..., 6 \quad (12)$$

Combining these six equations with the previous six results in a system of twelve coupled first order differential equations in the twelve dependent variables $q_1, q_2, ..., q_{12}$. In matrix form the system may be written as

$$A\dot{\mathbf{q}} = \mathbf{c} \quad (13)$$

where more detailed information concerning the structure of the matrix A and the vector $\mathbf{c}$ is given in the thesis by Van Wyk.[9]

Harmonic disturbing force, damping forces and reactions

The forced response of the conveyor is due to a harmonic force generated by two synchronised and contra rotating eccentric masses. If the masses rotate at angular frequency ω the expression for the harmonic force becomes

$$F_0 = \mu r \omega^2 \cos(\omega t) \qquad (14)$$

where μ is the total eccentric mass and r the eccentricity. The harmonic force F_0 acts at a vector distance $\boldsymbol{\rho}$ from the centre of mass and at an angle σ with respect to the horizontal table (see Figure 3). It follows that the equivalent force and moment components acting at and about the centre of mass is given by

$$F_x = F_0 \cos(\sigma+\theta); \; F_y = F_0 \sin(\sigma+\theta); \; F_\theta = F_0 \rho \sin(\sigma-\tau) \qquad (15)$$

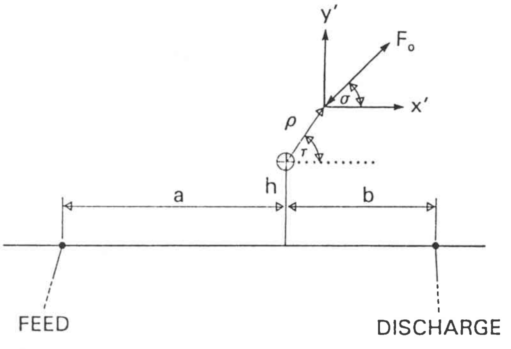

Figure 3 Position and action of excitor

A simple viscous and uncoupled damping model is assumed in which the damping force and moment components are given by

$$T_x = -c_{xx} m \dot{x}_g; \; T_y = -c_{yy} m \dot{y}_g; \; T_\theta = -c_{\theta\theta} I_g \dot{\theta} \qquad (16)$$

where c_{xx}, c_{yy} and $c_{\theta\theta}$ denote the damping constants. We simplify the model further by assuming $c_{\theta\theta} = 0$. The determination of c_{xx} and c_{yy} are dealt with in the next section.

The respective force and moment components in (15) and (16) may now be added to the equations of motion, (11) and (13), to complete the mathematical description of the system.

In order to compute a transmission function, required for the later optimization exercise, it is necessary to determine the time variational change of the reaction forces at the support points on the foundation. Assuming that the masses of the springs are negligible compared to the mass of the whole system, the reaction forces and moments at the supporting base may be taken as equal to that at the points of attachment of the springs to the table. The components R_x, R_y and S_x, S_y of the reaction forces R and S, respectively, as well as the moments M_S and M_R (see Figure 4) may be determined in terms of the computed Lagrange multipliers λ. Comparing the Newton equations of motion for the system in Figure 4 with the Lagrange equations (11) yields:[9]

$$\begin{aligned} S_x = \lambda_1; \; S_y = \lambda_2; \; M_S = \lambda_3 \\ R_x = \lambda_4 - \lambda_1; \; R_y = \lambda_5 - \lambda_2; \; M_R = \lambda_6 - \lambda_3 \end{aligned} \qquad (17)$$

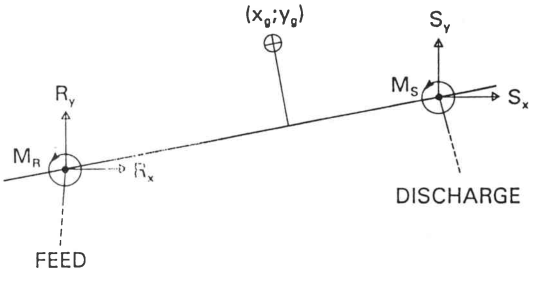

Figure 4 Reaction forces and moments acting at positions of attachment

Solution of the system equations and verification of the model

With the initial conditions $\mathbf{q}(0)$ known the system of differential equations (13) may be solved numerically to give $\mathbf{q}$ at any instant t. Notice that this solution also yields $\boldsymbol{\lambda}$ (see the paragraph following equations (11)), and consequently through (17) the reactions at the base. Here the initial value problem is solved numerically by means of the Runge-Kutta-Verner method as embodied in the IMSL subroutine IVPRK.[10]

Before commencing with the routine solution of (13) in an extensive optimization procedure, it is necessary to determine representative values for the damping constants c_{xx} and c_{yy} and to verify the model experimentally. For a fixed setting of the design variables $[a, b, h]$, the free response of a standard experimental system was studied. In agreement with the assumption that the respective damping forces in the x and y directions are uncoupled, the motions in these two directions were studied separately and compared with that predicted by the mathematical model. The values of the damping constants c_{xx} and c_{yy} in the model were systematically varied until excellent agreement between the experimental and predicted behaviour was obtained. These values were used throughout in the simulations that follow. For the given system, operating under standard conditions, the measured time-dependent behaviour of R_y and S_y and of the stroke, were compared to that predicted by the model. Excellent agreement was obtained.[9]

The most important geometrical, material and operational system parameters are listed in Appendix A. A more detailed exposition of the parameters and a description of the experimental determination of some of these geometrical and material parameters may be found in the thesis by Van Wyk.[9]

Optimization

Optimization of the system with respect to any set of design variables $\mathbf{x} = (x_1, x_2, ..., x_n)^T$ requires the definition of a suitable objective function. Since our objective is the minimization of the transmission of vibration to the supporting foundation, an appropriate objective function to be minimized is

$$d(\mathbf{x}) = \frac{1}{T_i} \int_0^{T_i} \times \left\{ (S_y - S_{ys})^2 + (R_y - R_{ys})^2 + S_x^2 + R_x^2 \right\}^{\frac{1}{2}} dt \tag{18}$$

where S_{ys} and R_{ys} denote the static equilibrium reactions, and $[0, T_i]$ is an operational time interval sufficiently long to allow d to take on a steady state value. Since the time-dependent behaviour of R_x, R_y, S_x and S_y are dependent on the design variables $\mathbf{x}, d$ is also a function of $\mathbf{x}$. This initial study is restricted to the design vector $\mathbf{x} = [a, b, h]^T$ (see Figure 2). The objective d is computed by the numerical solution of the initial value problem (13), whilst at the same time carrying out the numerical integration of (18). The time interval used here is $T_i = 39$ seconds.

The objective function d may now be minimized with respect to $\mathbf{x}$ using any standard numerical optimization algorithm. Initially it was decided to make use of the standard and 'off-the-shelf' IMSL optimization subroutines BCONF and UMCGF,[10] employing the BFGS quasi-Newton and conjugate gradient methods, respectively. Both methods failed to provide satisfactory convergence. The failures are ascribed to the existence of 'noise' in the objective function which arises as a result of small inaccuracies in the computation of the objective function. These inaccuracies are due to the fact that the objective function is itself the outcome of an approximate numerical integration procedure. This results in serious errors in the gradient evaluations and line searches required for carrying out the optimization.

The optimization was finally successfully performed by means of Snyman's [7] dynamical and heuristic LFOP1(b) optimization algorithm. This method, requiring no formal line searches, is a proven robust method with outstanding global convergence properties. To allow for the imposition of side constraints on the design variables and functional constraints on the stroke, use was made of a penalty function approach in which the objective function was modified by the addition of a corresponding penalty term whenever a constraint (see Appendix A) is violated. The convergence histories of the successful LFOP1(b) algorithm, together with that of the failed methods and for two different designs, are shown in Figure 5, (a) and (b). In both cases the dynamical method converged to the same local, and probably the global, minimum $[a; b; h]^T = [0.5993;\ 0.4294;\ 0.2571]^T$, with objective function value $d = 27.8929$ units.

Conclusion

The objective of the study, namely the optimal reduction of the transmission of dynamic forces through the adjustment of the spring support positions, was achieved.In particular reduction of more than 20% in the value of the transmission function, compared to that of two acceptable but otherwise arbitrarily chosen initial designs, were obtained.

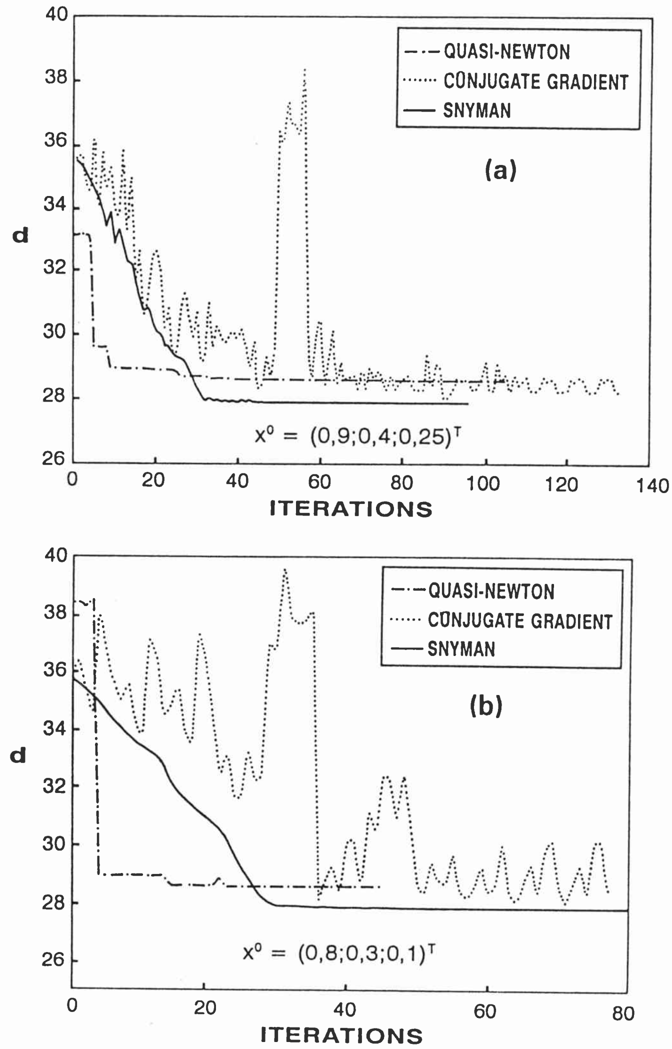

Figure 5 Convergence histories of the three optimization algorithms for two different initial designs

Since the mathematical model employed already takes into account other geometrical as well as material and operational parameters, future work will concentrate on achieving further reduction through optimization with respect to these additional design variables.

References

[1] Singh R. Analysis of a vibratory conveyor. *Indian Journal of Technology*, 1975, **13**, pp.500–504.

[2] Schofield RE & Yousuf M. The design of a linear 'out-of-phase' vibratory conveyor. *Journal of Engineering for Industry*, 1973, pp.42–47.

[3] Manohar L & Hoberock LL. Solids conveyance dynamics and shaker performance. *SPE Drilling Engineering*, 1988, **3**, pp.385–392.

[4] Maré P. Oscillating Systems (Pty) Ltd, Pretoria. Personal communication, 1993.

[5] Rademacher FJC. *Feeders and vibratory conveyors for bulk solids handling, TUNRA Ltd.* Department of Mechanical Engineering, University of Newcastle, NSW, Australia, 1974.

[6] Snyman JA & Vermeulen PJ. The mathematical modelling of the motion of a horizontal vibrating screen supported by ROSTA oscillating mountings. *R & D Journal*, 1992, **8**, pp.25–33.

[7] Snyman JA. An improved version of the original leap-frog dynamic method for unconstrained minimization: LFOP1(b). *Applied Mathematical Modelling*, 1983, **7**, pp.216–218.

[8] Fowles GR. *Analytic mechanics.* Holt Rinehards & Winston, 1977.

[9] Van Wyk AJ. *Die wiskundige modellering en optimering van 'n vibrasievoerder.* M Eng dissertation, University of Pretoria, 1993.

[10] IMSL Math/Library, Version 1.1, IMSL Inc, Houston TA, USA, 1989.

Appendix A

Geometrical, material and operational system parameters

Spring parameters:
- undeformed spring lengths ($q_5(0) = q_8(0) = 0.130$ m)
- feed end axial spring stiffness ($k_5 = 5300.0$ N/m)
- feed end torsional spring stiffnesses ($k_1 = k_2 = 46.68$ Nm/rad)
- discharge end axial spring stiffness ($k_6 = 7007.0$ N/m)
- discharge end torsional spring stiffnesses ($k_3 = k_4 = 60.86$ Nm/rad)

Mass parameters:
- mass of table ($m = 54.9$ kg)
- moment of inertia about axis through c.o.m. ($I_g = 11.70$ kg/m^2)
- gravitational acceleration ($g = 9.81$ m/s^2)

Excitation parameters:
- angular frequency ($\omega = 99.34$ rad/s)
- eccentric mass ($\mu = 5.249$ kg)
- eccentricity ($r = 0.0205$ m)
- excitation angle relative to table ($\sigma = \pi/4$ rad)
- radial position of excitor ($\rho = 0.188$ m)
- angular position of excitor ($\tau = 1.220$ rad)

Damping parameters:
- damping constant in x direction ($c_{xx} = 0.064$ s^{-1})
- damping constant in y direction ($c_{yy} = 0.011$ s^{-1})

Constraints imposed on design:

$0.3851 \leq a \leq 0.7851$
$0.2149 \leq b \leq 0.6149$
$0.1660 \leq h \leq 0.2560$
$0.003 \leq \text{stroke} \leq 0.005$

(bounds given in metres)