Kinematic analysis of a train of trailers

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Abstract

The paper deals with the development of a kinematic model suitable for the analysis of long trains of trailers. The model describes the kinematics of trailers with rear-wheel steering and is solved using a numerical method. Numerical results are discussed and compared with experimental results. A numerical algorithm to optimise the design of a train of trailers with rear-wheel steering is presented.

Nomenclature

n	Trailer element number, $n = 1, 2, 3 \ldots N$
θ_n	Angle between horizontal axis and trailer body
$\dot{\theta}_n$	Rate of rotation of the trailer body
A_n	Towing point
B_n	Axle pivot
D_n	Towing point of trailer $n+1$
V_{An}	Velocity of towing point at front of trailer
V_{Bn}	Velocity of axle centre point
ϕ_{An}	Angle between horizontal axis and direction of $\overline{V_{An}}$.
ϕ_{Bn}	Angle between horizontal axis and direction of $\overline{V_{Bn}}$.
L_n	Length, $A_n\, B_n$
L_o	Length, $B_n\, D_n$
C	Position of instantaneous centre of rotation
c/d	Steering constant for cross-over steering trailer

Introduction

In an industrial environment trains of trailers play an important role in the transporting of large amounts of goods by means of a single towing vehicle. The problem with long trains of trailers is that the train tends to cut into corners when being towed which means wide aisles are required for transport. It was decided to try and modify the trailer designs to reduce the amount of corner cutting experienced by existing trains of trailers. This can be done by introducing a form of rear-wheel steering to counteract the tendency to cut corners. These designs are optimised to obtain the best cornering performance. This reduces the aisle width required by trailer trains and allows more space for storage of goods or for longer trains to be operated.

In this field of research there are very few published articles dealing directly with the theory of trailers. The derivation of the equations of motion for a simple trailer using the method of instantaneous centres of rotation can be found in [1] and a discussion on the performance and optimisation of trailers is detailed in [2]. This deals with the optimisation of trailers trains with a rear-wheel steering system requiring the application of an actuator steering mechanism.

In this paper a method of optimisation is discussed which is suitable for long trains of trailers with a rear-wheel steering mechanism. The optimisation method of the trailer design has been based on a kinematic simulation of the system. The instantaneous centre of rotation technique has been applied to devise the kinematic equations. The equations are solved numerically and the optimisation procedure is detailed and applied.

Trailer models

Trailer models have been developed in this paper using generalised trailer elements. The generalised trailer element is shown in Figure 1 in which the angles used to describe the trailer configuration are shown. This generalised trailer element can be used to model various trailer designs. The first design is the simple trailer in which the rear axle is fixed to the trailer body at B_n. The second model, referred to as a cross-over trailer, is based on a steering mechanism which can be easily constructed and optimised as shown in Figure 2. It should be pointed out that the load-carrying bed is placed on turntable bearings at points E and H. The design is in use in certain warehouses but no attempt has yet been made to analyse and describe its kinematics. The kinematic analysis of the system enables one to investigate the trailer performance and to introduce suitable improvements.

Equations of kinematics

The kinematic equations consist of a velocity equation and constraint equations. The velocity equation has been obtained by applying the method of instantaneous centres of rotation. The constraint equations are based on the geometry of the system. The assumption is made that the tyres do not skid or track off a path that is perpendicular to their rotational axis.

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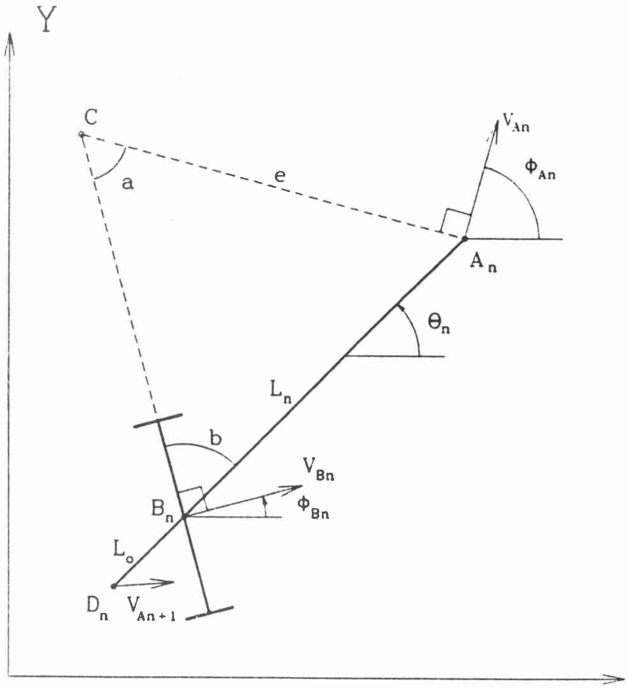

Figure 1 Model of generalised trailer element X

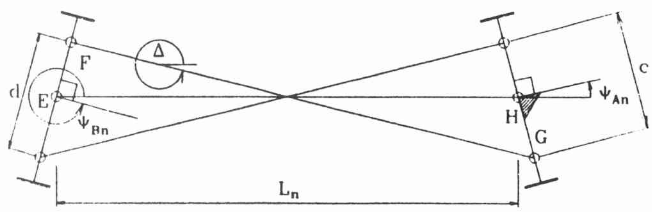

Figure 2 Cross-over steering trailer being towed by a simple trailer

To write the velocity equation the direction of the velocities at two points on the trailer body need to be known. The first velocity direction is obtained from the trailer towing path which must be defined for any time instant t. The second direction passes through the centre point of the axle B_n and is perpendicular to the axle, which is based on the assumption that the tyres do not skid. The trailer wheel can rotate about its point of contact on the ground but at any given time instant it can only translate horizontally in a direction perpendicular to the wheel's axis of rotation. Perpendicular lines are drawn to these directions and the instantaneous centre of rotation C is found at the intersection of these two lines. The velocity directions change with time and therefore the instantaneous centre of rotation continuously changes its position. The angular velocity of the trailer body is given by the following equation (details of the derivation can be found in Appendix A):

$$\dot{\theta}_n = \frac{|V_{An}| \sin(\theta_{An} - \theta_{Bn})}{L_n \cos(\theta_n - \theta_{Bn})} \tag{1}$$

The equation is to be solved for θ_n which allows the position of any point on the trailer body to be determined. The parameters V_{An} and ϕ_{An} are defined via a prescribed towing path. However additional constraints are required to determine the angle ϕ_{Bn}. From this generalised trailer element a simple trailer element can be obtained. For this simple trailer element the rear axle is fixed to the trailer body so that ϕ_{Bn} equals θ_n. For the trailer model shown in Figure 2 the closed loop EFGH can be used to generate the constraints. For the closed loop the following vector equation can be written:

$$\overline{EH} = \overline{EF} + \overline{FG} + \overline{GH} \tag{2}$$

From equation (2) the following constraints result:

$$-|\overline{EF}| \sin(\Psi_{Bn}) + |\overline{FG}| \cos(\Delta) - |\overline{GH}| \sin(\Psi_{An}) - L_n = 0 \tag{3}$$

$$|\overline{EF}| \cos(\Psi_{Bn}) + |\overline{FG}| \sin(\Delta) + |\overline{GH}| \cos(\Psi_{An}) = 0 \tag{4}$$

where $\Psi_{An} = \phi_{An} - \theta_n$ and $\Psi_{Bn} = \theta_n - \phi_{Bn}$.

Numerical method of solution and results

A numerical method has been applied to solve the non-linear differential equation (1) together with the constraint equations (3) and (4). The prescribed towing path has been used as a driving constraint. To solve the differential equation (1) a 4th and 5th order Runge-Kutta algorithm was used. The constraint equations (3) and (4) were solved for each time step within the Runge-Kutta algorithm by the Gauss-Newton method. The initial conditions for the simulation were the angles θ_n for each trailer element at time $t = 0$. MATLAB numeric computation software was used to implement the Runge-Kutta and Gauss-Newton procedures. The time taken to simulate the kinematics of the system ranged from 15 seconds for a short simple trailer train to approximately 5 minutes for longer cross-over trains on a 386 personal computer.

By solving equation (1) the orientation of the trailer element θ_n is found for discrete time instants for a particular path. Using the prescribed towing path and trigonometry the loci of the trailer extremities A_n and B_n are found and plotted. The simulation has been extended to analyse complete trains of trailers. In a train the only trailer that has a prescribed towing path is the leading trailer with all others following. In the solution procedure it is necessary to define the velocity at point D_n of each trailer element. This can be accomplished using the known prescribed velocity on the towing path and A_n as a reference point. Therefore the following equation has been used,

$$\overline{V_{Dn}} = \overline{V_{An}} + \overline{\omega_n} \times \overline{A_n D_n} \tag{5}$$

where $\omega_n = \dot{\theta}_n$.

In the solution procedure it is important to keep all angles used in equation (1) between 0 and 2π. This is in order to avoid singular points in the solution.

Some typical results of the computer simulation are shown in Figure 3. In this example a train of four trailer elements is towed around a quarter circle from the initial orientation. Details of each trailer element can be found in Table 1. The trailer elements 2 and 4 are typical designs with cross-over steering used to transport two standard pallets. The trailers 1 and 3 are simple elements attached to the cross-over trailers.

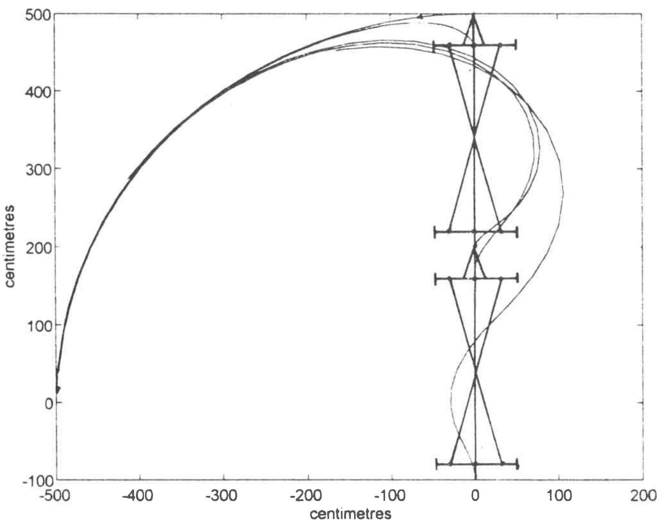

Figure 3 Sample results of computer simulation

The towing is performed in an anticlockwise direction as shown in Figure 3. Each other line is the path of D_n of each successive trailer element. The initial position of the train has been superimposed so that each element's path can be identified. The units on the axes are the same as those used to model the trailers. While it is best to keep to one set of units, say metres, it was found that for speed and accuracy of simulation it was best if units were kept in the region of 10 to 100.

Table 1 Trailer element details for sample numerical results

Trailer number	Length L_n [cm]	Length L_o [cm]	Trailer model	Width [cm]	Initial θ_n [radians]
1	40	0	simple	100	$\pi/2$
2	240	20	cross-over	100	$\pi/2$
3	40	0	simple	100	$\pi/2$
4	240	20	cross-over	100	$\pi/2$

Verification of numerical solution

Small scale trailer train models were built in order to verify results of the numerical simulation. The train models were towed along a prescribed path and the locus of point D_n for each element was plotted. The initial position of the trailer train was in a straight line and the towing path was a straight line perpendicular to the train's initial orientation. The numerical simulation was then performed for the train. The experimental results were compared with those obtained in the simulation. Two types of trailer trains were in the verification procedure. The first type, with dimensions specified in Table 2, consists of four simple trailer elements. The second type consists of two simple trailer elements and two cross-over trailer elements and the dimensions can be found in Table 3. The simulation results for the simple trailer train are shown in Figure 4 with the experimental results superimposed as a series of '+' signs.

Table 2 Details of simple experimental trailer

Trailer number	Length L_n [cm]	Length L_o [cm]	Trailer model	Initial θ_n [radians]
1	7.1	0	simple	$\pi/2$
2	16.5	4.2	simple	$\pi/2$
3	8	0	simple	$\pi/2$
4	16.5	0	simple	$\pi/2$

The simulation results for the cross-over trailer train are shown in Figure 5 with the experimental results superimposed. Comparison of experimental and simulated loci shows good correlation of results.

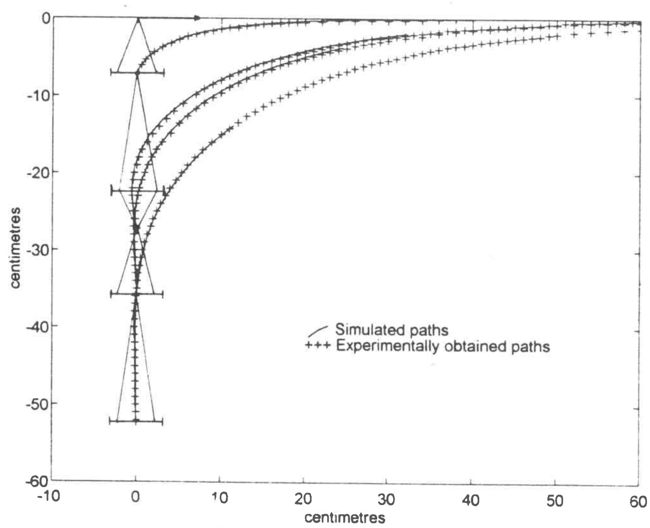

Figure 4 Simulated and experimental results of a simple trailer train

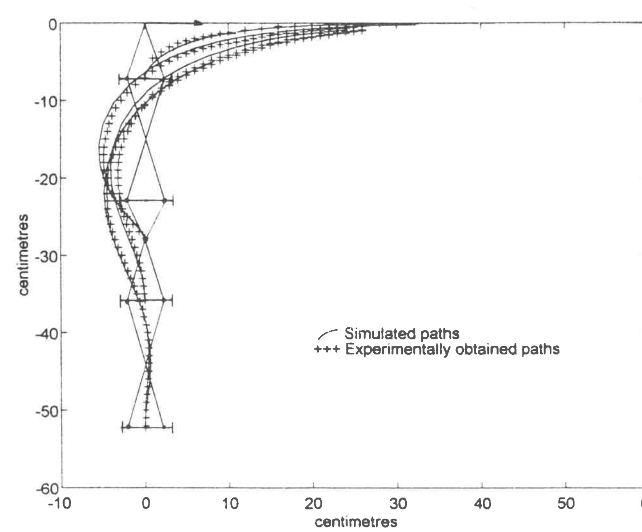

Figure 5 Simulated and experimental results of a cross-over steering trailer train

Differences occurred when the steering mechanism reached full lock and the tyres skidded as this violated the fundamental assumption of the simulation model.

Table 3 Details of cross-over steering experimental trailer

Trailer number	Length L_n [cm]	Length L_o [cm]	Trailer model	Initial θ_n [radians]
1	7.1	0	simple	$\pi/2$
2	16.5	4.2	cross-over	$\pi/2$
3	8	0	simple	$\pi/2$
4	16.5	0	cross-over	$\pi/2$

Optimisation of design

In designing a trailer train with rear wheel steering elements the road width required for the train to turn the corner should be taken into consideration. The design may be optimised and the optimal ratio of c/d (see Figure 2) can be determined in order to minimise the road width when cornering.

In the optimisation algorithm all trailer elements form a straight line and the leading trailer is towed in the direction perpendicular to this line, The ratio of c/d is changed from 0 (a simple trailer) to a prescribed value, usually 1.1 . For a particular c/d ratio the loci of points A_n and D_n as well as the loci of the four corners of each trailer body are found. For each simulated time instant the inner and outer road edges of the corner that could just accommodate the train are calculated. The most extreme edges found from the simulation define the road width required for the train to turn through the corner. Once this has been done for all c/d ratios, a graph of required road width vs c/d ratio is drawn.

Figure 6 shows an example of the optimisation curve for the train with dimensions specified in Table 1. The required road width becomes wider after the optimum design point due to the out-swing of the trailer becoming larger. The optimum c/d ratio is found to be 0.91 and the corresponding road width is 191 cm. Figure 7 shows the loci of trailer extremities with the required road width superimposed for the optimised trailer train.

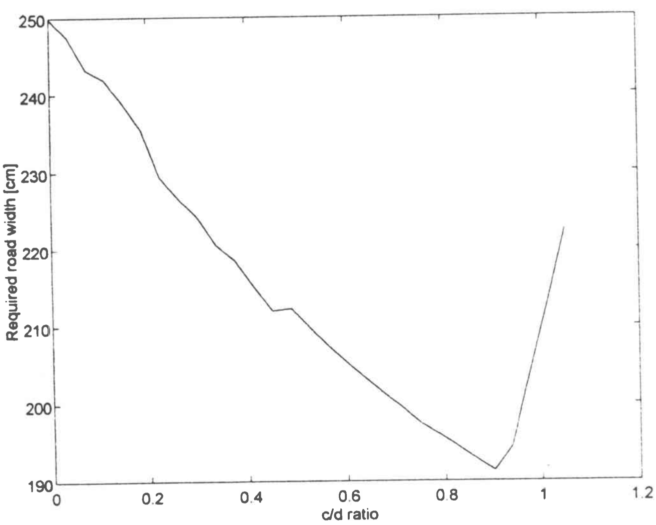

Figure 6 Graph showing effect of varying c/d ratio on the required road width

Discussion

The optimum c/d ratio varies in different trailer trains. This parameter depends on the length L_n and L_o of each trailer and on the number of trailer elements. Once the optimal design of the train has been found further simulations can be performed using different towing paths to see how the train reacts in different situations that may be encountered during operation. The described optimisation procedure produces the c/d ratio that reduces corner cutting and ensures the minimum out-swing in the worst cornering situation. In optimising the train it was assumed that all cross-over trailers in the train have the same c/d ratio. However, in a very long train this may not be the ideal case. Each trailer in the train follows a slightly different path which would suggest that a different c/d ratio for each cross-over trailer element would be better. This could only be applied to the situation in which the order of trailers in the train would not be changed. This type of optimisation can be accommodated in the existing optimisation algorithm. Having each trailer with the same c/d ratio allows for flexibility in trailer order within the train and still improves the cornering performance.

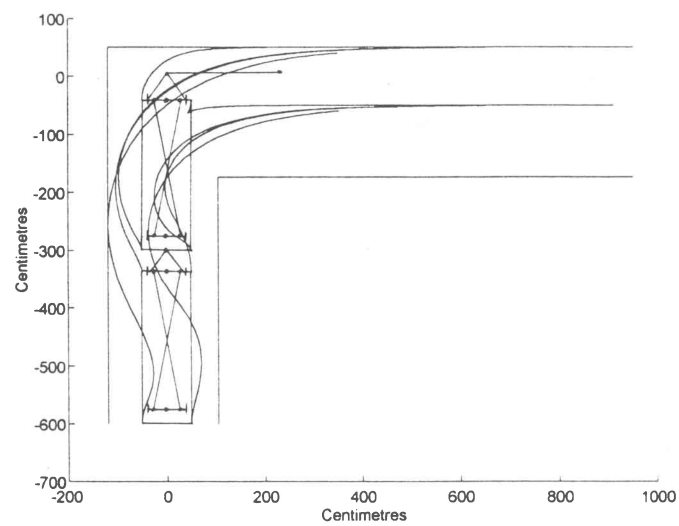

Figure 7 Locus of optimised trailer train turning through corner

In some areas the cross-over steering trailer is already in use but in most cases the ratio c/d is equal to 1. This is an improvement over simple trailers but it would not require much effort to change the ratio to the optimum value. In cases where simple trailer trains are used the added cost of applying trailers with a steering mechanism would have to be compared with the gains due to either floor area saved or due to larger trailers used on existing roads.

The cross-over steering mechanism is not the only one available for rear-wheel steering but the kinematic analysis and optimisation method described above can be used for any mechanism as long as the angle ϕ_{Bn} is defined. The analysis procedure can also be used in the case when a sophisticated rear-wheel control system is present.

Conclusion

Using the method of instantaneous centres of rotation it has been possible to derive equations describing the kinematics of a generalised trailer element. These equations have been applied to simulate the kinematics of a train of trailers. Good correlation of experimental and computer simulation results was achieved. Using this technique an effective optimisation procedure was devised and implemented in the design of trailer trains.

References

[1] Beggs JS. *Kinematics.* Hemisphere, Washington, 1983.

[2] Chen HF & Velinsky SA. Design of Articulated Vehicles for Low Speed Manoeuvrability. *Journal of Transportation Engineering ASCE*, 1992, **118(5)**, pp. 711–728.

Appendix A

The magnitude of the towing velocity $\overline{V_{An}}$ can be written in terms of rotation about point C:

$$\left|\overline{V_{An}}\right| = \dot{\theta}_n e, \tag{1}$$

where e is expressed in terms of trailer geometry and orientation as follows:

$$e = \frac{L_n \sin b}{\sin a} \tag{2}$$

The angles a and b are given by:

$$a = (\phi_{An} - \phi_{Bn})\,, \tag{3}$$

$$b = \left(\frac{\pi}{2} - (\theta_n - \phi_{Bn})\right) \tag{4}$$

Finally:

$$\dot{\theta}_n = \frac{|V_{An}| \sin(\phi_{An} - \phi_{Bn})}{L_n \cos(\theta_n - \phi_{Bn})} \tag{5}$$