Error budgeting for control system design

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Abstract

System accuracy is an important issue in control system design. The design goals for the different subsystems of a control system are often derived from 'rolling down' the total allowable error in the response of the control system to the different subsystems. Once the attainable subsystem errors are known (from analysis and/or tests), these values need to be 'rolled up' in order to determine the overall attainable system accuracy. The iterative process of rolling down the requirements, rolling up the attainable values, rolling down reviewed requirements, etc., is termed error budgeting. This paper defines different error budgeting approaches and illustrates some of the most important aspects by means of error budgeting for a satellite inertial measurement system.

Nomenclature

A/D	analogue to digital [converter]
hr	hour
IMS	inertial measurement system
mrad	milliradian
mV	millivolt
μrad	microradian
ppm	parts per million
RSS	root-sum-square
SF	scale factor
temp.comp.	temperature compensation
V/F	voltage-to-frequency [converter]

Introduction

Accuracy is a critical issue in the development of control systems. In many complex systems, zero error is not attainable and some error in the controlled variable has to be tolerated. System design can be done in a **top-down** manner (where the maximum allowable system error is defined, and then rolled down to the different subsystems); or it can be done in a **bottom-up** manner (where the attainable accuracies of subsystems are determined and then rolled up to determine the attainable overall system accuracy); or it can be done in an **iterative** manner (where requirements are first rolled down, and attainable values are then rolled up, until an optimum balance between technical performance, cost, and schedule is reached).This iterative process is termed error budgeting. The first part of this paper is aimed at providing a concise, but useful, description of the principles and procedures involved in error budgeting – aspects which are often neglected in existing literature on the subject (e.g. [2] and [4]). Thereafter, some of the most important aspects of error budgeting are illustrated for a satellite inertial measurement system – which is an example of a subsystem in a high-accuracy control system.

System design approaches

Table 1 defines the top-down, bottom-up, and iterative approach to system design in more detail. From the table it is clear that the iterative approach is the most flexible of the three, and that it is the most suitable when an optimum balance between technical performance, cost, and development schedule is pursued.

Error budgeting

Once a system's desired accuracy has been specified (typically based on the user requirement and the system's mission), the next step in the iterative error budgeting process is to determine the attainable accuracy, for comparison with the requirement. Then the iteration can start towards the point where an optimum balance between the goal and the attainable value is reached – within technical, financial, and schedule constraints. The major steps for determining a system's attainable accuracy by means of analysis during the design phase, are:

1. Define the system's major **functional blocks**.
2. Identify **potential contributors** to the system error in each of the system's functional blocks.
3. Perform a qualitative analysis of the contributors' **potential effects** on the system accuracy.
4. Identify the **dominant contributors** to the error, from the qualitative analysis.
5. **Quantify** the dominant error sources, and determine their combined contribution to the overall system error. This implies a *translation* from the values of errors in the functional blocks to values representing their effects on the overall error. For this translation process, three analysis routes can be utilised: static analysis, dynamic analysis, and system simulation.

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These three techniques are defined in more detail in Table 2.

Error budgeting example: Satellite inertial measurement system

Attitude control of earth observation satellites (e.g. for earth resource management) is an example of a control system which requires a high accuracy. Such a satellite's maximum allowable pointing error (or desired pointing accuracy) is therefore one of its major design parameters.

For a satellite making use of an inertial measurement system (IMS) as part of its attitude control loops, part of the satellite's desired pointing accuracy translates (*rolls down*) to a desired IMS measurement accuracy; which in turn translates to accuracy requirements at consecutive lower levels in the IMS hierarchy. Conversely, the attainable measurement accuracies of the different IMS subsystems combine to form the attainable IMS measurement accuracy; which in turn *rolls up* to form part of the attainable satellite pointing accuracy. Finding the optimum balance between the desired accuracy and the attainable accuracy clearly calls for *iterative error budgeting.* In this section, an overview is provided of the configuration and dominant errors of a typical satellite IMS, after which the principles of static and simulation-based error budgeting are illustrated for such an IMS.

Configuration of a typical satellite IMS

A typical satellite IMS is configured to measure satellite angular rates and incremental angles around three orthogonal measurement axes (X, Y, and Z).[3] For this purpose the IMS consists of a **sensor subsystem** and an **electronics subsystem**. The major components of a typical sensor subsystem are three dynamically tuned gyroscopes, a sensor block in which the gyros are mounted orthogonally, and shock mounts by means of which the sensor block and gyros are isolated from shocks and vibrations occurring on the satellite. For each gyro, the electronics subsystem contains components to perform electrical power conversion (*power supply*), to drive and control the gyro, to measure and process data, and to communicate with other satellite subsystems. Figure 1 shows a high level block diagram of one of the three measurement axes (i.e. around X, or Y, or Z) of a typical satellite IMS. Each gyro measurement axes uses a control loop to keep the gyro rotor's motion the same as the motion imposed around (and thus sensed by) that specific gyro axis. (With reference to Figure 1, the purpose of the gyro control loop is to maintain a zero rotor angle relative to the gyro casing.) The rotor motion, in reaction to the satellite's motion, is caused by torquer coils in the gyro, and the current in these coils is proportional to the angular rate experienced by the specific gyro measurement axis. The rotor motion

Table 1 Different approaches to system design

1. Top-down	2. Bottom-up	3. Iterative error budgeting
• The *system engineer* defines the overall desired system accuracy, and *rolls it down* to the first level of subsystems.	• Performance predictions (based on analysis and/or test results) are made from the lowest subsystem level, and *rolled up* in order to determine the best possible performance of the next higher system level.	• Initial requirements are set top-down — based on the system requirements and system mission.
• The system engineer for each of these subsystems then rolls down the specific subsystem's accuracy requirements to the next level of subsystems.	• This process is repeated up to the level of the complete system.	• These requirements are evaluated on sybsystem level (by means of analysis and/or tests), and the attainable values are fed back to the higher levels.
• This procedure is repeated down to the lowest level of subsystems.	• The performance of the overall system is then purely based on what is technically attainable within the constraints of cost and schedule.	• Based on the attainable values, the initial allocation can be reviewed and adapted, where necessary — and if possible.
• At each level, the requirements must be met without exception — i.e. all implemented subsystems must comply with the requirements.	• Each level thus accepts the performance which it gets from its lower level subsystems — without querying it.	• The total allowable error of the next higher level subsystem need not necessarily be increased, but a reallocation of allowable errors to different subsystems is often sufficient.
• This approach is only suitable for new developments, and where the desired system accuracy is of paramount importance — often without too much regard for development cost and schedule.	• This approach is typically used when existing designs are adapted for new applications; and when some technical performance has to (and can) be traded-off in favour of lower development cost and a shorter schedule.	• This process is repeated to reach a point of optimum balance between goals and attainable values — within technical, financial, and schedule constraints.
• This approach requires the person, allocating the allowable errors to lower level subsystems, to have a very good knowledge of the lower level's subsystems.	• In order to allow for some leeway during implementation, subsystem engineers roll up error values which are larger than what is really attainable — causing the predicted overall system performance to be worse than the true attainable performance. This can cause a loss of potential clients for the system.	• This approach is much less rigid than the other two approaches, since both the top-down requirements and the bottom-up attainable values are subject to revision, re-allocation and negotiation.
• This approach can lead to unrealistic design goals being imposed on lower level subsystems; and it is very rigid wince trade-offs between different subsystems' attainable accuracies are generally not done.		• This approach renders the opportunity to make trade-offs between technical performance, cost, and schedule, on the different system levels.

control current is measured as a voltage across a sense resistor, and is used to derive the incremental angle about the specific measurement axis, by counting pulses from a voltage-to-frequency (V/F) converter; and to derive the angular rate by means of an analogue-to-digital (A/D) converter and digital processor.

Importance of iterative system design

To illustrate the importance of the iterative approach to system design, consider the following example: For an IMS in a land-based vehicle, temperature control within narrow bounds, is often used to reduce the IMS temperature sensitive errors (thus leaving residual errors only). In satellites – where the available power is limited – it is more attractive, however, to use *temperature compensation* instead, whereby angles and angular rates measured by the IMS are adjusted according to the temperature of the IMS during the specific measurement. Since gyros are normally the most temperature-sensitive components in an IMS, restrictions on system complexity typically dictate that the temperature measurements for use in the temperature compensation, be restricted to the gyros only. It is clear that for an IMS with limited temperature compensation, decreased system accuracy is accepted in lieu of lower power consumption and lower system complexity.

The only way to determine how far this trade-off can be taken, is to calculate the effects of the temperature sensitive errors on the overall system accuracy; and this is done as part of the error budgeting process. It is obvious that if the desired measurement accuracy were allocated in a purely top-down fashion, the system complexity, power consumption, and cost could escalate beyond reasonable limits. The error budget can therefore enable the IMS designer to compare temperature control, with different levels of temperature compensation and this information can then be fed back to the higher level system engineer for decision making.

Static error budget

Static error budgeting is useful to determine the effects of different error sources on the overall measurement accuracy of a satellite IMS. Typically, such an error budget includes fixed, random, temperature-dependent, and angular acceleration dependent error sources. (Although the effects of fixed errors can largely be reduced by prelaunch calibration, some *residual error values* remain due to factors such as non-ideal calibration, fixed errors introduced during satellite launching, ageing of components, and mechanical creeping.) Not only can the contribution to the total IMS measurement error be determined for each error

Table 2 Analysis routes for determining attainable accuracy

	1. Static analysis	**2. Dynamic analysis**	**3. Simulation model**
Description	effects of parameters such as misalignments, biases, and drifts are analysed under specific system operating conditions.	Dynamic characteristics (gain and phase shift), and their effects on system error, are analysed for *analytically friendly* profiles of system inputs (e.g. sinusoidal or step inputs)	Effects of static parameters, dynamic characteristics, as well as actual input profiles, are analysed by means of a complete computer simulation model of the system
Inputs	• System operating conditions (limited) • Statistical values (e.g. 3σ values) of errors in each functional block	• System dynamic characteristics • Definition of system input profiles (typically restricted to sinusoidal and step inputs)	• Nominal system characteristics • Typical error values in each functional block • True system operating conditions and input profiles
Process	• Translation of each error source to its contribution to the final system error • Addition of different *translated errors* in root-sum-square (RSS) fashion [1]	• Determine gain and phase shift between input and output • Evaluate effects of the above on total system error	• Simulate system response for a specified input, and with specified error sources active in each functional block • Determine overall system error for specified errors and inputs
Outputs	• Effect of each error source on overall system accuracy • Statistical value (RSS) of total system srror — under specified (limited) operating conditions	System error due to gain and phase shift between input and output — for specific input profiles	• Effect of each error source (static or dynamic) on overall system accuracy • Expected total error — under specified operating conditions
Comments	This method is useful for: • preliminary error analysis • establishing orders of magnitude for errors • statistical analysis of individual and combined static characteristics	This method is mainly restricted to analysing the effects on system error, of gain and phase shifts due to the dynamic characteristics of the system	• This method is a combination and extension of the static and the dynamic approaches • Although the model can be complex, the method provides the most realistic results • The model can be used to determine worst-case errors, or it can also be used to determine statistical error values

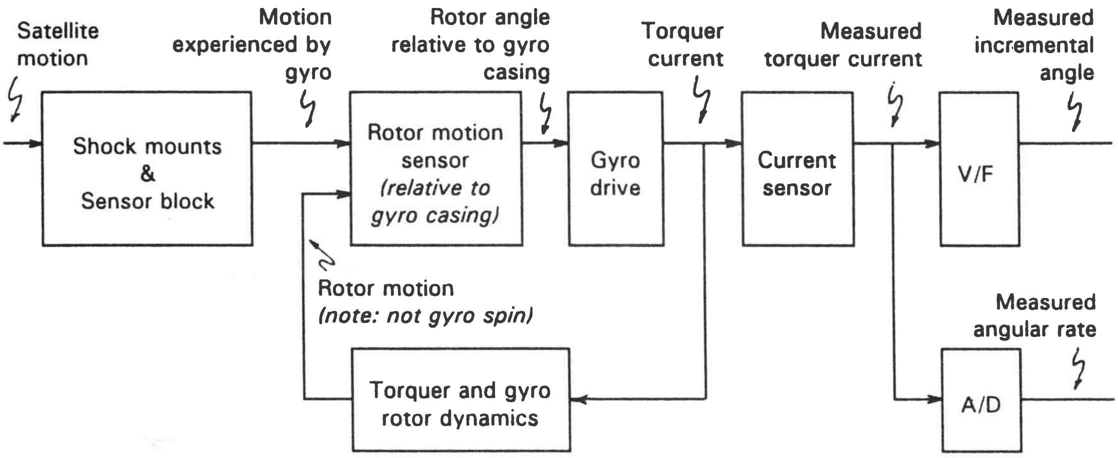

Figure 1 High level block diagram of one measurement axis of a typical IMS

source, the effects of combinations of different error sources can also be predicted. The static error budget can furthermore be used to evaluate specific circumstances, such as the effect on the measurement error, when neither temperature control nor temperature compensation is used, or when only gyro temperature compensation is used.

Two important steps in static error budgeting are identification of the dominant error sources and translation of its individual contributions to the overall system error. These steps are illustrated in Table 3, which contains:

- Typical dominant static error sources for an IMS measuring angular increments of a satellite. (These error sources are mainly due to one or more of the following: residual fixed errors after system calibration, launch-induced fixed errors, angular acceleration dependent effects, and temperature-dependent effects.)
- Typical values of the error sources. (These figures are typical for a high-accuracy IMS, but since they were rounded-off for use here, they do not represent any specific commercially available IMS.)
- Descriptions of the broad principles according to which the errors are *translated* (rolled up) to total system error.
- Contribution of each error source to the error in the measured satellite angular increment (i.e. the values of the error sources, translated to system level error). Two sets of results are shown: one set for the case where no temperature compensation or control is used on the IMS (i.e. temperature-sensitive errors have a large influence) and one set for the case where **gyro temperature compensation** is used (i.e. the influence of all gyro parameters which are temperature sensitive is reduced to only a *residual effect* – typically as if a gyro temperature variation of only 1°C occurs.)

The numbers obtained in the translation from error source values to system level measurement error were often rounded-off and are therefore not extremely accurate (some answers are in milliradians whilst others are in microradians). The intention was however not to reproduce the most accurate IMS error budget here, but rather to illustrate the principles and the trends that can be observed from such an exercise.

The individual error values, as used in Table 3, are statistical figures (typically 3σ values), with the total error being calculated as the root-sum-square (RSS) of the individual values. (RSS is used because the individual errors are considered to be independent.[1] The results in Table 3 indicate that, for the IMS considered here, gyro temperature compensation causes a twenty-fold increase in angular measurement accuracy. The same procedure can be used to verify what the effect on system accuracy would be if temperature compensation is also used, for example, for the most temperature sensitive electronic components. The error budgeting process can thus be used to investigate trade-offs between system complexity and system accuracy and, when used iteratively, it can largely contribute to optimization of system performance, cost, and development schedule.

Simulation model

Instead of the procedure used to compile Table 3, a complete simulation model can be used for determining the contribution of the IMS dynamics and the dominant static error sources to the total IMS measurement error. The simulation model creates the ability to evaluate the effects of individual and different combinations of error sources, whilst taking the IMS nominal and dynamic characteristics into account. The effects of different levels of temperature compensation can easily be verified by means of such a simulation model. Normal system modelling principles are used in this process, i.e.:

1. Identify the system's different functional blocks and compile a model which includes the nominal characteristics of each functional block.

2. Identify and quantify any error sources present in each of the functional blocks, and incorporate them into the model. Different approaches are possible for including the different error sources in the model:

(a) In order to determine the worst-case overall measurement error, the positive or the negative maximum values of all error sources can be used in the simulation model. However, this approach

Table 3 Typical dominant IMS static error sources and their effect on the angular error

Functional block	Dominant static error sources	Typical value	Translation method	Angular error	
				No temp. comp.	Gyro temp. comp.
Shock mounts and sensor block	Fixed misalignment — causing the orientation of the gyro measurement axes to differ from its intended orientation	170 µrad	• Make use of a directional cosine matrix to convert the maximum angular rate of the satellite from the misaligned axes to the true measurement axes • Make use of the satellite's maximum angular rate (e.g. 1°/s), and determine the resulting error in the angular rate measurements • Integrate over the time required for the satellite to traverse its full angular distance (e.g. 8 minutes)	0.5 µrad	0.5 µrad
	Temperature sensitive misalignment	7 µrad	• Multiply by full temperature range (e.g. 30°C) • Same procedure as above	0.7 µrad	0.7 µrad
Rotor motion sensor	Scale factor variations	250 ppm/°C	• The relationship between rotor motion sensor output voltage (V_{rs}), gyro rotor angle relative to the gyro casing (θ_r) and rotor motion sensor scale factor (SF_{rs}) is: $V_{rs} = SF_{rs}.\theta_r$ • If the rotor motion sensor scale factor increases, the same output voltage will result for a decreased rotor angle (and *vice versa*): $V_{rs} = (SF_{rs} + \delta SF_{rs}).(\theta_r - \delta\theta_r)$	17 mrad	0.9 mrad
	Bias	10 mV/°C	A bias added to (or subtracted from) the rotor motion sensor output voltage, is equivalent to an increase (or decrease) in rotor angle: $\delta V_{rs} = SR_{rs}.\delta\theta_r$	0.3 mrad	9 µrad
Torquer and gyro rotor dynamics	Scale factor asymmetry and variations	10 ppm/°C	• The relationship between torquer scale factor (SF_t), torquer current (I_t), and rate of change of gyro rotor angle ($d\theta_r/dt$) is: $d\theta_r/dt = SF_t.I_t$ • If the torquer scale factor (part of the control loop's feedback path) increases, the same torquer current will cause a larger rotor angle rate: $d\{\theta_r + \delta\theta r\}/dt = (SF_t + \delta SF_t).I_t$.	5 mrad	0.2 mrad
	Gyro drift	0.1 °/hr	Gyro drifts (in the feedback path of the rotor angle loop control loop) are integrated over the duration of a satellite scanning session	0.3 mrad	0.3 mrad
	Temperature sensitive gyro drift	2.10^{-3} (°/hr)/°C	Multiply with total temperature range (e.g. 30°C) in the case of no temperature compensation	8 mrad	0.2 mrad
Current sensor	Temperature sensitivity	5 ppm/°C	• Determine the change in resistance for the total change in temperature • An increase in the scale factor causes a proportional increase in the voltage across it • The resulting increase in the measured angle is: $\delta\theta = \delta V_r.SF_{v/f}$, where $SF_{v/f}$ is the scale factor of the total V/F channel	0.2 mrad	0.2 mrad
V/F channel	Scale factor variations	5 ppm/°C	The change in measured angle due to a change in total V/F scale factor, is $\delta\theta = V_r.\delta SF_{v/f}$, with V_r the voltage output from the current sensor	0.2 mrad	0.2 mrad
	Bias	1 pulse/ minute	Multiply the number of extra pulses with the part of the total V/F scale factor which converts number of pulses to equivalent angle	0.2 mrad	0.2 mrad
			Total angular measurement errors (rounded RSS values)	20 mrad	1 mrad

causes unrealistic simulation results, because in reality some errors cancel the effects of others.

(b) Random signs can be allocated to all error sources in the simulation model. This renders more practical simulation results than approach (a) above, since errors negating each other are thus introduced.

(c) In order to simplify the simulation program, errors occurring in the same functional block of the system can be added in root-sum-square fashion – provided they are independent.[1] Random signs are then allocated to the RSS errors of the different functional blocks. The effect of errors negating each other is still introduced, but the simulation model becomes less complex. In this case, the repeatability of the simulation results is better than for approach (b) above, since less randomness is incorporated into the model.

(Since the signs of errors are allocated randomly in both approaches (b) and (c), the simulation ought to be repeated a number of times – in Monte-Carlo fashion – in order to derive average and standard deviation values for the overall measurement error.)

3. Independently verify the model by means of careful reviews with the system's design experts.
4. Implement the simulation model by means of a digital computer program and verify the implementation by means of standard inputs (e.g. step functions) to, and expected outputs from, the simulation (first for individual functional blocks, then for different combinations of functional blocks, and then for the complete system).

Table 4 describes the major characteristics (nominal

Table 4 Major characteristics of IMS simulation model

Functional block	Nominal characteristics	Error sources	Inputs	Outputs
Shock mounts	Second order transfer function, representing the low-pass filtering characteristics of the shock mounts	Shock mount distortion — causing misalignment (root-sum-square) of residual fixed, temperature dependent, and angular acceleration dependent values)	Satellite motion (angular rate)	Satellite motion filtered by the shock mounts' transfer function, transformed by the shock mount misalignment angles, and imposed on the sensor block
Sensor block	Orthogonal mounting of three gyros	Block distortion — causing misalignment (root-sum-square) of residual fixed, temperature dependent, and angular acceleration dependent values)	Satellite motion filtered by the shock mounts' transfer function, and transformed by the shock mount misalignment angles	Satellite motion further transformed by the block misalignment angles
Rotor motion sensor	Scale factor for converting nett angular motion of the gyro rotor relative to the gyro reference plane, to a voltage	• Equivalent rotor motion sensor misalignment (root-sum-square) of residual fixed, temperature dependent, and angular acceleration dependent values) • Limited rotor angular motion (saturation due to rotor end stops) • Scale factor variations, asymmetry, and bias	Nett angular motion of the gyro rotor relative to the gyro reference plane (integral of the difference between satellite and rotor angular rate)	A voltage proportional to the rotor angle relative to the gyro reference plane
Gyro drive electronics	• Demodulator • Filters • Compensator • Transconductance amplifier	Temperature sensitive gain changes and biases in the filters, the compensator, and the amplifier	Voltage proportional to rotor angle	Current in torquer coils
Torquer and gyro rotor dynamics	Nominal torquer scale factor	• Scale factor variations and asymmetry • Gyro drift (root-sum-square of fixed, day-to-day, acceleration dependent and residual temperature dependent drifts)	Current in torquer coils	Rotor angular rate (note: not spin rate of the gyro motor)
Current sensor	Nominal scale factor	Temperature sensitive scale factor changes	Current in torquer coils	Voltage output from current sensor (proportional to satellite angular rates as sensed by the gyros)
V/F	Nominal V/F scale factor	• Scale factor variations (root-sum-square of temperature dependent, and residual fixed variations) • Residual V/F bias	Voltage output from current sensor	Measured satellite incremental angle (about a specific measurement axis)

and error sources) used for compiling a simulation model of a typical satellite IMS. With the simulation model, there is no need for a separate *translation* of errors to the overall IMS measurement error – the simulation takes care of that automatically. Because the individual error values are (as for the static error budget) statistical figures, the predicted overall error value is also a statistical value.

By making use of an IMS simulation model – including similar error sources to those used in Table 3, including gyro temperature compensation, and with the dominant dynamic characteristics (transfer functions) of the shock mounts, the gyro electronics, and the gyro included – the predicted measurement error was less than 1.1 milliradian. The question can arise: Why should the more complex simulation model be used if the results are similar to those obtained with the simpler static error budget? The answer is:

1. With the static error budget, in Table 3, only the maximum IMS measurement error (statistical value) can be calculated, whilst with the simulation model the designer can evaluate the measurement error as a function of time, or in the presence of noise signals, or for different frequencies of satellite motion, etc.

2. The simulation model can be used in different roles – such as for static error budgeting when the system's dynamic characteristics are removed from the model, or for dynamic error budgeting when the system's static errors are all made zero in the model, or for a combined static and dynamic analysis (such as in a Monte-Carlo type of statistical analysis).

3. With the simulation model, the translation of error sources to its system level effects are performed automatically when the simulation program is executed, whilst it requires complex calculations in the static error budget.

Although the simulation model is the most versatile of the three techniques described in Table 2, it is also the most complicated, because of the requirement for an accurate system model.

Conclusions

System accuracy is an important issue in control system design. In this paper, the iterative (*error budgeting*) approach to system design was defined alongside the pure top-down and pure bottom-up approaches. Whereas the pure top-down approach can lead to unrealistic design goals being imposed on lower level subsystems and the pure bottom-up approach can lead to the predicted overall system performance being poorer than the true attainable performance, the iterative approach renders an opportunity to make trade-offs between technical performance, cost, and schedule on the different system levels.

Three different analysis routes, static, dynamic, and simulation-based analysis, can be used for translating error sources in a system to their effect on overall system accuracy. The static and simulation-based routes were addressed in terms of the principles involved in error budgeting for the inertial measurement system of a high-accuracy satellite. Since a major portion of a satellite's required pointing accuracy is typically allocated to the IMS, measurement accuracy of the IMS is of paramount importance. Both analysis routes used here are very valuable for evaluating accuracy requirements, for establishing attainable accuracies, and for adapting requirements and/or designs to the extent that neither an over-design nor neglect of the system's accuracy requirements results. Static error budgeting provides less quantitative answers than simulation-based error budgeting, but the former is less complex to perform. Which of the two approaches should be followed is therefore dictated by a trade-off between error budget complexity on the one hand and accuracy of results on the other.

Although the iterative error budgeting approach was illustrated in this paper mainly by making use of a specific configuration of an inertial measurement system for satellite applications, the procedures and principles are widely applicable. Not only can accuracy requirements of other satellite subsystems be handled in similar fashion, but a wide variety of control systems can be designed as such.

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