MODELLING THE TURBULENT FLOW OF NON-NEWTONIAN SLURRIES

P.T. Slatter*

(First received August 1993; Final version October 1995)

The only reliable approach open to designers of pipeline systems conveying non-Newtonian slurries in the turbulent flow regime has been large scale pipe tests. This paper addresses this design problem, with particular emphasis on the theoretical modelling of the turbulent flow behaviour of these slurries in pipes. The literature and theory pertinent to the flow of slurries in pipes is examined. A new model for the prediction of the turbulent energy gradient is developed from widely accepted fundamentals. A particle roughness effect has been observed and turbulent flow is modelled using a new roughness Reynolds number to correlate the roughness function. Three test facilities were built for the establishment of a data base of non-Newtonian slurry behaviour — a tube viscometer and two pumped recirculating pipe test rigs. The experimental investigation covered wide ranges of diameter (5 mm to 200 mm nominal bore), mean pipe velocity (0.1 m/s to 10 m/s), slurry relative density (1.02 to 1.65), volumetric concentration (2% to 37%), solids relative density (2.4 to 2.8) and particle size range (d_{85} = 24 to 120 μm). Turbulent flow predictions using the new turbulent model are accurate and better than previous models, particularly in the rough wall region. The new analysis is based on physical behaviour and contributes to the understanding of the mechanisms involved.

Nomenclature

A	constant cross-sectional area	m^2
B	roughness function	
d	particle diameter	μm
D	internal pipe diameter	m
E	error function	
f	Fanning friction factor	
g	gravitational acceleration	m/s^2
H	manometer head difference	m
i	statistical identifier	
k	hydraulic roughness	μm
K	fluid consistency index	Pa s^n
L	pipe length	m
M	mass	kg
n	flow behaviour index	
N	number of items	
Q	volumetric flow rate of slurry	m^3/s
R	pipe radius	m
Re	Reynolds number	
t	time	s
u	point velocity	m/s
V	average slurry velocity	m/s
V_*	shear velocity	m/s
y	distance from the pipe wall	m
μ	dynamic viscosity	Pa s
μ'	apparent dynamic viscosity	Pa s
ρ	slurry or fluid density	kg/m^3
τ	shear stress	Pa
τ_y	yield stress	Pa
χ	von Karman constant	

*Associate Director, School of Civil Engineering, Cape Technikon, P.O. Box 652, Cape Town, 8000 South Africa

Subscripts

0	at the pipe wall
85	85th percentile of the particles passing
calc	calculated
obs	observed (experimental)
p	particle
r	roughness
s	solids
v	volumetric
w	water
x	representative

Introduction

This paper deals with the pipe flow of homogeneous non-Newtonian slurries. In particular, smooth wall, partially rough wall, and fully rough wall turbulent flows are examined, and a new analytical solution for determining head loss is offered which differs fundamentally from previous theoretical approaches.

Statement of the problem

The turbulent pipe flow of these slurries is not well understood, despite the large amount of research interest in this area.[1] Slurries having similar viscous properties are known to have dissimilar turbulent properties.[2] According to Shook & Roco[3] the predictions of the various theories in the literature differ significantly.

Recent attempts to predict the pipe flow energy requirements for these slurries in terms of their viscous characteristics have met with limited success[4,5] and errors of up to 50% have occurred. Any problems associated with the rheological characterisation of a slurry would increase the uncertainty of the results. The implications of errors of this magnitude could be that the wrong size pump or pipe diameter are specified and that the system will not operate at the required throughput. These errors are unacceptable. The errors are also general and other types of error may arise as each hydrotransport application is different. From a practical, engineering point of view, existing theoretical design methods for non-Newtonian slurries are inaccurate and the results lack confidence.

The only reliable approaches open to designers of pipeline systems conveying non-Newtonian slurries are either full scale pipeline tests, or scale up of tests over wide ranges of laminar and turbulent flow for the slurry under consideration.[6] Design procedures are therefore costly and inefficient.

Objective

The objective of this paper is to present a more reliable theoretical analysis for the flow of non-Newtonian slurries in pipelines. The analysis is based on widely accepted fundamentals and incorporates turbulent flow in smooth, partially rough and fully rough wall pipes. This analysis is shown to predict test data more reliably than previous models.

Analysis of turbulent flow of Newtonian fluids in pipes

Based on the Prandtl mixing length model, the velocity distribution for Newtonian turbulent flow in pipes is[7]

$$\frac{u}{V_*} = A \ln \frac{y}{k} + B \qquad (1)$$

B can be correlated using a roughness Reynolds number

$$Re_r = \frac{\rho V_* k}{\mu} \qquad (2)$$

The correlation is shown in Figure 1.

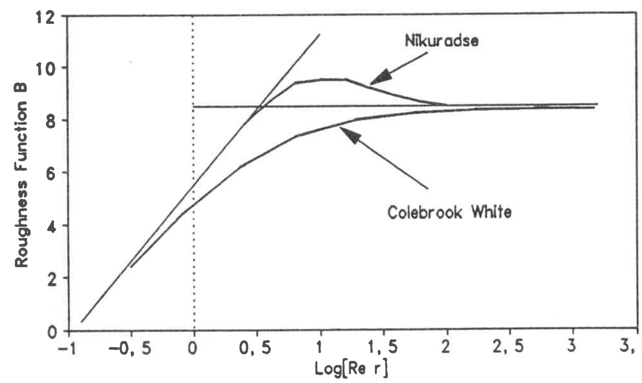

Figure 1 Roughness function correlation for Newtonian fluids

The oblique asymptote in Figure 1 is the line $B = 2.5 \ln(Re_r) + 5.5$ which represents smooth wall turbulent flow. The horizontal asymptote is the line $B = 8.5$ which represents fully developed or rough wall turbulent flow.

The top curve is the locus of data for pipes with uniform roughness from the experiments with sand roughened pipes by Nikuradse.[7]

The lower curve is the equation of Colebrook and White[8] and represents the locus of data for commercially available (randomly rough) pipes. This equation is widely used for the design of Newtonian pipelines. For fully developed rough turbulent flow, the roughness function is constant ($B = 8.5$), showing that the behaviour in this region is totally independent of the viscous characteristics of the fluid.

Non-Newtonian turbulent flow models

Non-Newtonian turbulent flow models can be divided into three categories. Firstly there are the models that have a strong analytical approach, such as those of Torrance[9] and Wilson & Thomas.[10] Secondly there are the models that are purely empirical, such as Bowen's[11] approach. The third category lies between these two extremes, such as the models of Dodge & Metzner[12] and Kemblowski & Kolodziejski.[13] Only the first are of interest in this paper.

The yield pseudoplastic model was found to best characterise the rheology of all the slurries tested for this investigation, and the method of Lazarus & Slatter[14] was used to extract the rheological constants τ_y, K and n.

Newtonian approximation

Standard Newtonian theory can be used, but a value for the viscosity of the fluid is required. Usually the term viscosity is meaningless once a non-Newtonian approach has been adopted. However, an apparent or secant viscosity μ' [15] can be defined. Note that μ' is not a constant for a given fluid and pipe diameter, but must be evaluated at a given value for τ_0.

The Torrance Model

Using the same mixing length model and method of derivation as for Newtonian turbulent flow, Torrance[9] derived a model for non-Newtonian turbulent flow in pipes using the yield pseudoplastic rheological model as the starting point. An important departure is that the von Karman constant is taken to be 0.36 n and is therefore dependent on the viscous characteristics of the fluid.

Similarly, Torrance derived a model for fully developed rough turbulent flow in which the von Karman constant is assigned the value 0.4 n. This model therefore predicts that the behaviour in this region is dependent on the viscous characteristics of the fluid – in contrast to Newtonian flow.

Torrance makes no comment on partially rough wall turbulent flow.

A characteristic of the Torrance model is that the rheological parameters are treated separately, in separate terms in the expression. This leads to the problem that the yield stress does not appear in the Reynolds number formulation.

The Wilson & Thomas Model

Wilson & Thomas[10] (also [15,16]) produced an analysis of the turbulent flow of non-Newtonian fluids based on enhanced micro-scale viscosity effects. This model predicts a thickening of the laminar sub-layer by a factor called the area ratio. This area ratio is defined as the ratio of the integrals of the non-Newtonian and assumed Newtonian rheograms (using the apparent or secant viscosity μ') under identical shear conditions. The thickened laminar sub-layer results in an increase in the mean velocity over that for an equivalent Newtonian fluid.

This model represents an improvement on the Newtonian approximation method. A method for scale up of turbulent data, based on the above model, has also been proposed.[10]

Rough wall and partially rough wall turbulent flow can be accommodated in the model by using the appropriate roughness when determining the equivalent Newtonian mean velocity. However, this can only be approximate, since the interaction between the pipe roughness and the laminar sub-layer will clearly be different when the thickened laminar sub-layer is present.

Discussion of literature

The literature on non-Newtonian turbulent flow is extensive and well-kown. This review concentrates on six areas which are of particular importance to the development of the present model.

Firstly, it is well established in the literature that there is a strong similarity between the turbulent behaviour of Newtonian fluids and non-Newtonian slurries, despite their significantly different behaviour in the laminar flow regime.[12,16,17,18,19,20,21,22,23]

Other researchers have shown that the velocity distribution in the turbulent core of turbulent non-Newtonian slurry pipe flow is similar to that of Newtonian fluids.[24] The logarithmic nature of these velocity distributions has been demonstrated experimentally[4,25,26] and can be accepted for the purpose of theoretical analysis.

Secondly, non-Newtonian slurries are widely regarded as homogeneous and their behaviour can apparently be successfully described by continuum models, especially in the laminar regime. It is customary to ignore the fact that solid particles are present. A key aspect of the new model is that this thesis in untenable. The presence of solid particles as an inherent component of the fluid becomes important when one considers the following:

1. One of the cornerstones of classical turbulence analyses is the existence of a laminar sub-layer. The non-Newtonian turbulent theory of Wilson & Thomas[10] predicts a thickened laminar sub-layer over and above that for an equivalent Newtonian fluid. However, the size of the solid particles which must be present in the laminar sub-layer are of a similar order of magnitude to the thickness of the laminar sub-layer. One possible conclusion is that the boundary layer is affected in some way by the presence of the solid particles.

2. The velocity gradients in the region of the pipe wall are known to be steep.[27] Calculations to determine the change in velocity which can be expected over the diameter of a particle under average test conditions show that it is of the order of 1 m/s in the region of the pipe wall. Such rapid changes in velocity will be impeded because the solid particles will resist shear. The velocity gradients in the region of the pipe wall are therefore so steep that the presence of solid particles must have a diminishing effect on these velocity gradients.

3. Clearly, the continuum approximation[28] must break down in the region of the pipe wall when the size of the solid particles becomes large compared with the scale of the modelling. The effect of the particles must be accounted for.

4. If the continuum approximation is untenable in the wall region, the implication is that the particles and the slurry pseudofluid will have to be considered as

separate phases at the crucial interface between the laminar sub-layer and the turbulent core. Particles of various sizes will be subjected to drag forces in an environment which is neither wholly laminar or turbulent, but somewhere in between. In the face of this extremely complex situation, it is logical to model the behaviour in terms of a dimensionless group which encompasses both the particle and fluid characteristics. A particle Reynolds number is one such dimensionless group.

The particles must therefore physically obstruct the theoretical steep velocity gradients in the region of the wall resulting in a decrease in the velocity gradient at the pipe wall. It is also known[7] that the effect of pipe roughness can cause a decrease in the velocity gradient at the pipe wall. However, in the case of slurries, the particles, although they are indeed sand particles, are not fixed or uniform, as in Nikuradse's experiments,[7] but they will have an effect similar to a surface roughness. It can therefore be postulated that the results will lie somewhere between the two data curves of Nikuradse and Colebrook & White,[8] as shown in Figure 1.

Thirdly, the partially rough wall turbulent flow region has been postulated to be relatively broad by researchers such as Dodge & Metzner[12] and Wilson & Thomas.[10] However, the above remarks on particle roughness, as well as the experiments by Park[26] indicate that the transition region is much narrower for non-Newtonian slurries than for Newtonian fluids.

Fourthly, in the derivation of the relationships in turbulent flow, it is standard practice to assume that the viscous stresses are negligible when compared with the turbulent stresses in the turbulent core region.[27] Because of the striking similarity between the turbulent behaviour of Newtonian fluids and non-Newtonian slurries, it would appear unlikely that the behaviour of the slurry in fully developed rough turbulent flow should depend on the viscous characteristics of the slurry. Wilson[6] has stated that turbulence is a process dominated by inertial forces. Indeed an absolute asymptote, independent of the viscous characteristics of the slurry, such as exists for Newtonian fluids, would be a useful engineering tool, in view of the highly sensitive nature of the yield pseudoplastic model.[5]

Fifthly, for the laminar flow of fluids with a yield stress, where $\tau < \tau_y$ the fluid cannot shear and plug flow occurs. Previous researchers[10,29] have assumed that the same phenomenon will occur in turbulent flow. However, the velocity profiles of Park[26] and Xu[4] show no such effect and indicate rather that the flow is turbulent over the entire core region. Plug flow has therefore not been admitted on the strength of this experimental evidence.

Sixthly, one of the problems that has arisen with previous models is that they apply only to the specific materials tested, or upon the test results used to generate correlation coefficients. These models then relate only to those specific circumstances and cannot be universally applied. One of the obvious ways of solving this problem is to ensure that the model reverts to the Newtonian model when the rheological parameters are relaxed to Newtonian conditions ($\tau_y = 0$, $K = \mu$ and $n = 1$). Note that this condition cannot guarantee universality – rather, it implies the opposite; any analysis which does not revert to the Newtonian form can never be universally applicable.

New analysis

The principle objective in the development of the new analysis is to accommodate the break down of continuum admissibility in the region of the pipe wall where the particles must have an effect because of their physical size. Furthermore, the new approach is an attempt to explain the behaviour of non-Newtonian slurries and to base the mathematical modelling on these quantitative descriptions.

The new analysis has therefore proceeded from the following initial assumptions based on the previous arguments:

- The velocity distribution is logarithmic and similar to the classical Newtonian turbulent velocity distribution over the entire core region.
- There is a roughness effect caused by the solid particles in the slurry.
- Fully developed rough wall turbulent flow does exist and the partially rough wall turbulent region is much narrower than for Newtonian fluids.
- Fully developed turbulent flow is independent of the viscous characteristics of the slurry.
- Plug flow does not occur.

Furthermore, the new model should revert to the Newtonian model.

Formulation of the velocity distribution

The basic velocity distribution for Newtonian turbulent flow in rough pipes is taken to be valid based on the above assumptions, i.e.:

$$\frac{u}{V_*} = A \ln \frac{y}{d_x} + B \quad (3)$$

The value of the coefficient A has usually been taken as the inverse of the von Karman universal constant. Now, turbulence is an inertial rather than a viscous process[6] and so viscous forces are taken to be negligible in the turbulent region.[27] Experimental evidence shows that the velocity distributions in Newtonian and non-Newtonian turbulent flow are similar. For these reasons the value of A has been chosen as the inverse of the von Karman universal constant, $1/\chi = 2.5$.

In order to correlate the roughness function B, it is necessary to formulate the roughness Reynolds number in terms of the yield pseudoplastic model. A problem with previous models has been the inadequate formulation of this important parameter, e.g. a formulation excluding the yield stress,[9,29] The new work on non-Newtonian Reynolds numbers (Re_2 in Slatter & Lazarus[30]) proves particularly valuable at this point.

By analogy with the Newtonian approach, the roughness Reynolds number for a yield pseudoplastic slurry can be formulated using the same basic form as Re_2 as follows:

$$Re_r = \frac{8\rho V_*{}^2}{\tau_y + K\left[\frac{8V_*}{d_x}\right]^n} \tag{4}$$

Higher turbulence intensities in the wall region reported by Park[26] provide strong experimental evidence in support of a particle roughness effect in the wall region.

An alternative derivation of the roughness Reynolds number can also be derived by combining the approach of Dedegil[31] and incorporating the fundamental Reynolds number definitions used by Slatter & Lazarus,[30] as follows:

$$Re_p = \frac{8\rho V_s{}^2}{\tau_y + K\left[\frac{8V_s}{d_p}\right]^n} \tag{5}$$

where d_p is the particle diameter and V_s is the velocity differential between the particle and the surrounding fluid.

As stated above, the velocity differential can only materialize once the continuum approximation breaks down in the wall region. Since this is a region of steep velocity gradient and indeterminate regime (neither fully laminar nor fully turbulent), an exact value of the velocity differential cannot be determined. In the face of this complex situation, the shear velocity could be used as a dimensionally representative velocity parameter.

From this formulation, equation (5), it can be seen that by introducing the shear velocity as a dimensionally representative velocity parameter in the wall region, and the representative particle size d_x, the roughness Reynolds number, equation (4), will result. This important dimensionless group, and the rationale behind its formulation, can be seen as part of the new analysis.

The solid particles must play a role and the roughness size d_x must be chosen accordingly. Since slurries typically contain a continuous range of particle sizes, a representative particle size must be found. The effect of roughness on turbulence can be thought of as an aggravation at the wall which stimulates turbulence. Clearly then the larger particles will have a more dominant effect on turbulence than the smaller particles. Also, the larger particles will shield the smaller ones, reducing their effectiveness in stimulating turbulence.[8] For the slurries tested, the d_{85} size was found to be a good representation of the turbulent roughness size effect of the solid particles in the slurry, i.e. $d_x = d_{85}$. This decision is supported by the sensitivity analysis in a later section. It is important to note that the Malvern instrument was used for particle size measurements – other methods may produce significantly different results.[32]

The mean velocity can be obtained by integrating over the cross section of the pipe yielding

$$\frac{V}{V_*} = \frac{1}{\chi}\ln\left(\frac{R}{d_{85}}\right) + B - 3.75 \tag{6}$$

Experimental data are now needed to correlate the roughness function B against the roughness Reynolds number.

Correlation of the roughness function

The roughness function B was correlated against the roughness Reynolds number in the same way as for Newtonian turbulent flow.

The correlation of data for the new model is shown in Figure 2, with the curves and asymptotes for the Nikuradse and Colebrook White loci. All turbulent data from the data base have been used in Figure 2 (some 500 data points).

The data lie above the Colebrook White curve.

Since the particles in the slurry are neither fixed nor uniform in size, as they were for Nikuradse's experiments, the roughness effect of the solid particles is not expected to be as great as in Nikuradse's experiments. This is reflected in Figure 2.

The figure also shows that the data lies close to the two asymptotes which describe the limits of behaviour of Newtonian turbulent flow. On the strength of these two points, the correlation chosen for this analysis is therefore the two asymptotes i.e.:

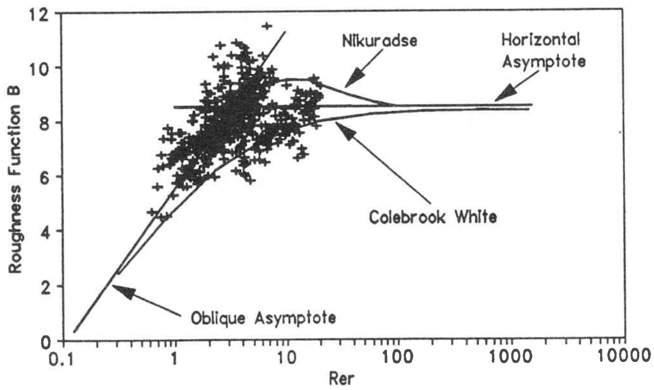

Figure 2 Roughness function correlation for non-Newtonian slurries

1. Smooth Wall Turbulent Flow

   If $Re_r \leq 3.32$ then $B = 2.5 \ln Re_r + 5.5$. This is analogous with smooth wall turbulent flow.

$$\Rightarrow \frac{V}{V_*} = 2.5 \ln\left(\frac{R}{d_{85}}\right) + 2.5 \ln Re_r + 1.75 \quad (7)$$

2. Fully Developed Rough Wall Turbulent Flow

   If $Re_r > 3.32$ then $B = 8.5$. This is analogous with fully developed or rough wall turbulent flow and will yield a constant value for the Fanning friction factor f.

$$\frac{V}{V_*} = 2.5 \ln\left(\frac{R}{d_{85}}\right) + 4.75, \quad (8)$$

which reduces to

$$\frac{1}{\sqrt{f}} = 4.07 \log\left(\frac{3.34\,D}{d_{85}}\right) \quad (9)$$

The average percentage error when calculating the roughness function, B, using this correlation is 9.2% with a standard deviation of 7.8%, and a log standard error of 0.0024.

This correlation produces a transition from the smooth to the rough flow condition which is abrupt.

The correlation further shows that the assumptions regarding the turbulent behaviour of non- Newtonian slurries are valid for the slurries tested.

Sensitivity analysis of the representative particle size

The above analysis relies on the assumption that the d_{85} size is the best representative particle size. In order to justify this assumption, a sensitivity analysis of the effect of various representative particle sizes would have on the accuracy of the analysis has been carried out.

A wall shear stress prediction error function can be defined as

$$E_\tau = \frac{\sum_{i=1}^{N} \frac{100\,|\tau_{0\,obs\,i} - \tau_{0\,calc\,i}|}{\tau_{0\,obs\,i}}}{N} \quad (10)$$

Table 1 Wall shear error sensitivity to representative particle size

d_x **percentile**	E_τ [%]	**Standard deviation**
d_{50}	12.88	9.88
d_{55}	12.24	9.54
d_{60}	11.59	9.19
d_{65}	10.91	8.89
d_{70}	10.18	8.63
d_{75}	9.42	8.49
d_{80}	8.75	8.46
d_{85}	8.58	8.99
d_{90}	9.79	10.35
d_{95}	13.57	12.95
d_{100}	42.67	26.98

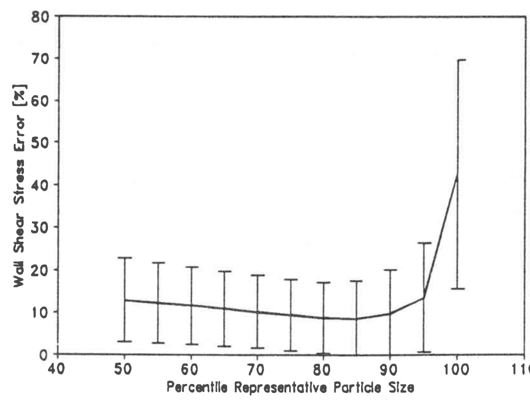

Figure 3 Wall shear stress error sensitivity to representative particle size

This error function gives an average percentage error. The error function for the new model and the standard deviation of this error is shown in Table 1 and Figure 3 for all tests in the data base (excluding the 200 mm diameter pipe results – see section on Pipe Roughness).

Table 1 and Figure 3 show that the d_{85} size provides the minimum error, and shows the sensitivity of the new model to a change in representative particle size. The asymmetry of Figure 3 is due to the shape of the particle size distributions which are much steeper for $d < d_{85}$ than for $d > d_{85}$.

Experimental work

Apparatus was built and test work performed specifically with the following objectives in mind:

- to measure the rheology accurately
- to test over as wide a range of flow velocities and diameters as possible
- to do test work using different particle size distributions
- to accumulate a database of test data for the evaluation of existing turbulent flow theory and the new model.

One of the most fundamental aspects of this investigation is that the turbulent pipe flow head loss can be predicted from the rheological data of the same fluid. Rheological data can only be obtained from tests under laminar flow conditions so it was vital that experiments were performed in order that reliable data over large ranges for both the laminar and turbulent regimes could be measured for the same non-Newtonian slurry.

Non-Newtonian mine tailings and kaolin clay slurries were tested in pipes of diameter ranging from 6 mm to 200 mm nominal bore with mean velocities ranging from 0.1 m/s to 10 m/s. An important aspect of the experiments is that the same slurry was used for each test set. A test set is a set of tests using different pipe diameters but the same slurry.

A database of 61 pipe tests was generated. This database was then used to evaluate theoretical models for the prediction of the behaviour of non-Newtonian slurries in pipes.

Apparatus

Three different sets of apparatus were used to gather experimental data for this investigation. These were the Balanced Beam Tube Viscometer, the Mini Rig, and the East Rig.

The Balanced Beam Tube Viscometer

The Balanced Beam Tube Viscometer (BBTV) developed by the author is a device for measuring the flow characteristics of a slurry.[33] The BBTV is, in fact, a miniature pipeline[34] and its use extends beyond viscometry.

This instrument consists of two pressure vessels which are located approximately 6 m apart at either end of a steel beam. This beam is centrally supported on a knife edge and a load cell is located under the left hand vessel. The vessels can be connected by transparent tubes of different diameter.

The prime mover is compressed air which forces the slurry through a selected tube at a controlled rate. The average slurry velocity is obtained from the mass transfer rate and the pressure drop across a known length of the tube is measured using a differential pressure transducer.

All the test section entry lengths can be changed to detect for undeveloped flow or time dependency.

The Mini Rig

The mini rig is a pumped pipeline test loop using small diameter PVC pipes. This rig consists of a Warman 1.5×1 solids handling centrifugal pump, a 3 m long test section with interchangeable clear PVC test sections of diameter 6 mm, 15 mm, and 25 mm nominal bore, and connecting pipes. The flow rate is measured with a 25 mm nominal bore Altometer magnetic flux flowmeter. Slurry is pumped from the 150 mm nominal bore pipe in the East Rig, through the Mini Rig to a weigh tank. This ensures that exactly the same slurry is tested in both the East and Mini rigs. The weigh tank is used to cheek the calibration of the flow meter during tests. The pump has a variable speed hydraulic drive.

All the test section entry lengths can be changed to detect for undeveloped flow or time dependency.

The East Rig

The east rig is a recirculating pumped pipeline test circuit with three test sections of diameter 80 mm, 150 mm, and 200 mm nominal bore.

Slurry is collected in a galvanised steel feed hopper which has a capacity of approximately 2 m^2. Slurry then passes directly from the hopper into the pump. The pump is a Mather and Platt 8×6 which is driven by a variable speed hydraulic drive.

Flow is measured using magnetic flux flow meters are located in the vertical pipe sections. Clear viewing and test sections are located in the return horizontal lines. The 200 mm line is steel, the others are PVC. The return lines are then fed back through an in-line heat exchanger and a pneumatic diverter valve into the hopper. The inline heat exchanger maintains the slurry at a constant temperature. The diverter valve feeds the weigh tank which is used for flow rate determination and flow meter calibration. For the slurries tested, no external agitation was necessary to maintain solids suspension in the hopper.

Pressure tappings

Differential pressure measurements are made from static pressure tappings located in the pipe walls of each of the horizontal test sections. The tappings have length to diameter ratios greater than four to ensure accurate readings.[35] The tappings are 3 mm in diameter and great care was taken to remove any burrs from the inside edge of each tapping.

Each tapping is fed through a valve to an isolation pod which collects any solids that may enter the pressure tapping. Each pod has a valve for flushing away collected solids with clear water. Clear water lines then connect the pod to the manometer and differential pressure transducer (DPT).

The test sections are preceded by unobstructed straight pipe of at least 50 pipe diameters.[24,35] The only exception is the 200 mm nominal bore pipe which, due to space constraints, has a straight entry length of 35 pipe diameters.

Determination of major parameters

The average slurry velocity and the wall shear stress are derived from the measured quantities as follows:

Average Slurry Velocity

The average or mean slurry velocity (V) is defined as the volumetric flow rate (Q) divided by the cross sectional area (A) of the pipe and is calculated

$$V = \frac{Q}{A} = \frac{4\,Q}{\pi\,D^2} = \frac{4\,M}{\pi\,\rho\,t\,D^2} \tag{11}$$

where M is the mass of slurry of density ρ collected in the weigh tank in time t seconds.

Wall Shear Stress

The wall shear stress (τ_0) is determined from the water manometer head difference (H) over a known length of pipe (L), i.e. the test section, as follows:

$$\tau_0 = \frac{\rho_w\,g\,H\,D}{4\,L} \tag{12}$$

Material

The following solids materials were used to make up the slurries for the tests.

Kaolin: Kaolin slurries were prepared from dry kaolin powder and pellets which were mixed with tap water to the required concentration and tested in the BBTV and the East and Mini rigs. Although all the material was obtained from the same deposit, the particle size distributions differed slightly.

Uranium Tailings: Uranium mining tailings slimes slurries were obtained wet from a mine. Various size fractions were obtained by mechanical sieving:

Slurry 1 $d < 100\ \mu m$
Slurry 2 $d < 250\ \mu m$

These slurries were then tested in the BBTV.

Gold Slimes Tailings: Various size fractions were obtained by mechanical sieving:

Tailings 1 $d < 500\ \mu m$
Tailings 2 $d < 106\ \mu m$
Tailings 3 $d < 62\ \mu m$
Tailings 4 $d < 42\ \mu m$

These slurries were then tested in the BBTV.[36]

Tap Water: The tap water had a pH of 9, was slightly super-saturated with respect to calcium carbonate (2 mg/1), total alkalinity was 35 mg/1 as $CaCO_3$, total calcium was 35 mg/l as $CaCO_3$, and ionic strength was less than 0.001 (molar scale).[37] Although the resulting mixtures were regarded as chemically stable, it should be noted that the rheology of these slurries can change with their ionic character. For this reason, tests on a given slurry were carried out on the same day. The time available for any chemically related rheology changes was therefore kept to a minimum.

Test procedure

The test procedure in all three sets of apparatus was similar and was as follows:

The apparatus was filled with slurry and the concentration adjusted as required by either adding solids or tap water. The slurry is then circulated at the same time to ensure thorough mixing.

A sample of the slurry is taken for relative density and particle size distribution tests and the transducers are calibrated.

The pods are connected to the tappings and the pressure measuring system flushed of air and solids.

The required slurry velocity is set and the transducer outputs are monitored via an automatic data logging routine. The data is them processed and the run repeated until sufficient points have been obtained.

Evaluation and discussion of the new analysis

The results of a typical test are shown in Figure 4 with the predictions of the theoretical models.

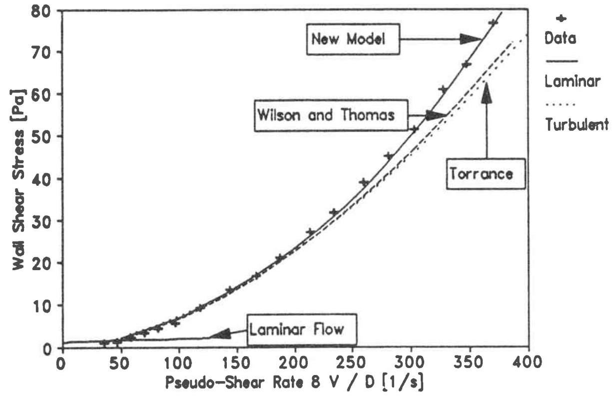

Figure 4 Typical test results showing turbulent flow predictions

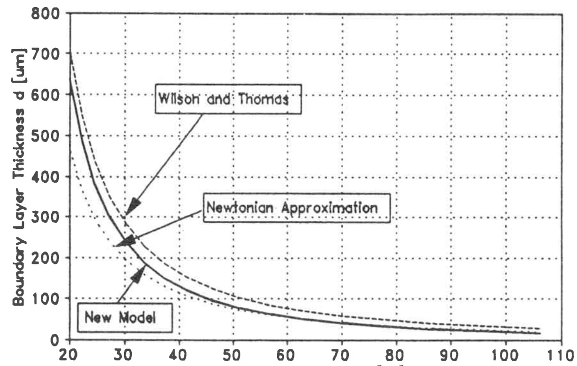

Figure 5 Laminar sub-layer thickness

Figure 4 shows that the new model provides a better prediction than the other models, particularly in the fully developed turbulent flow region.

The turbulent flow evaluations for the entire database are presented in Table 2.

Table 2 Turbulent model evaluation – whole database

	Torrance	Wilson & Thomas	New model
Av. % error	17.18	15.07	10.04
Log st. error	0.0050	0.0038	0.0024

Table 2 shows that the new model provides more accurate predictions than previous models for all the slurries tested.

Laminar sub-layer thickness

The thickness of the laminar sub-layer can be determined as the intersection of the velocity distributions in the laminar sub-layer and the turbulent core.[6] This thickness is plotted against wall shear stress in Figure 5.

[Base values used in this paper, unless otherwise stated: density = 1 130 kg/m^3; diameter = 100 mm; τ_y = 10 Pa; K = 0.03 Pa s^n; $n = 0.8$; d_x = 50 μm]

Figure 5 shows that the laminar sub-layer thickness predicted by the new model lies between the thicknesses predicted by the Newtonian approximation model and the model of Wilson & Thomas. The new model is closer to the predictions of Wilson & Thomas at low shear stresses and closer to the Newtonian approximation at higher shear stresses.

Smooth wall turbulent flow

The correlation of the roughness function B shows that smooth wall turbulent flow occurs if the roughness Reynolds number is less than 3.32. By analogy with Newtonian flow, smooth wall turbulent flow is characterised by an intact laminar sub-layer. In this region, the solid particles do not generate extra turbulence due to form drag.

Smooth wall turbulent flow is further characterised by a continuously decreasing friction factor. Unfortunately, due to the additive nature of both the constitutive rheological equation and the Reynolds number formulation, the equations do not resolve into a compact *Re-f* form. Also due to the additive nature of the fundamental relationships is the fact that the roughness size (representative particle size) does not vanish from the smooth wall equation as in the Newtonian case. However, the effect is small and has not affected the accuracy of the model for the slurries tested.

In this region, the new model predictions are similar to those of Torrance and Wilson & Thomas. As concluded earlier, these models perform well in this early turbulent region. The actual accuracy depends more on the accuracy of the rheological characterisation than the choice of model.

Fully developed or rough wall turbulent flow

Fully developed or rough wall turbulent flow is characterised by a constant friction factor and total obstruction of the laminar sub-layer by the particles. This constitutes a useful engineering tool, because in this region, the energy gradients depend only on the relative size of the particles (D/d_{85}), and provides an asymptote for the designer to work to in the absence of accurate rheological data. The correlation of Bowen[11] exploits this similarity of the fully developed turbulent flow region and it is used in his scale up law.

Partially rough wall turbulent flow

One of the characteristics of the new turbulent flow model is the abrupt change from smooth wall turbulent flow to fully developed or rough wall turbulent flow. However, as far as can be ascertained by observation of the test data graphs, this is the way in which the slurry behaves. An example of this is given in Figure 6. The abrupt change and therefore the absence of any significant transition region, is a true reflection of the real behaviour of these slurries. This is in sharp contrast to the Newtonian case, where the transition region spans several orders of magnitude of Reynolds number (see Moody diagram[38]). This abrupt change can also be seen in the results of Bowen.[11]

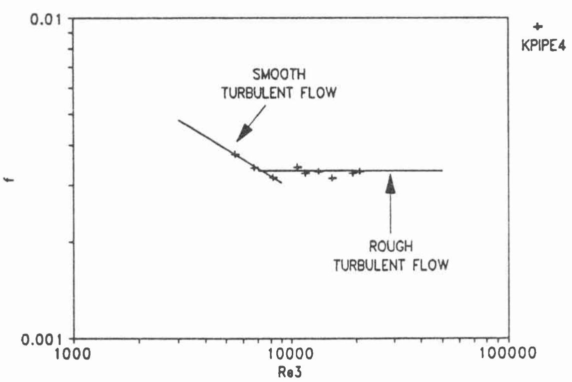

Figure 6 Data showing abrupt change from smooth to rough wall turbulence

Friction factor / Reynolds number diagrams

The new model can be plotted on a friction factor/Reynolds number diagram.

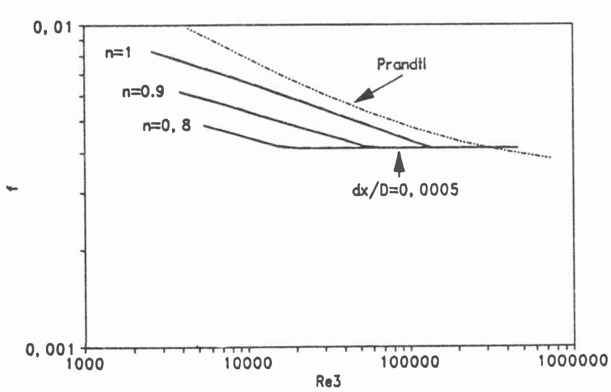

Figure 7 Friction factor vs Re_3 for the new model

Figure 7 emphasises the importance of the slurry rheology in the smooth wall turbulent region. The friction factor is constant in the rough wall turbulent region.

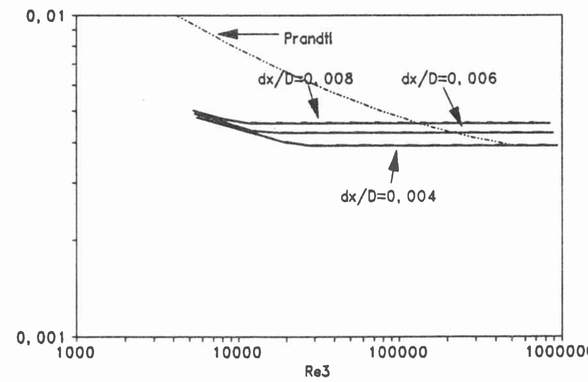

Figure 8 Friction factor vs Re_3; new model

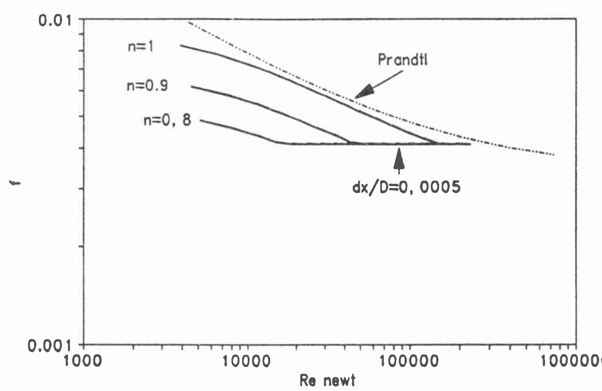

Figure 9 Friction factor vs Re_{Newt}; new model

Figure 8 shows the effect of different particle sizes for the same rheology. This is analogous to the relative roughness effect shown on the Moody diagram.

Figure 9 shows the convex nature of the smooth wall turbulent region when Re_{Newt} (using the secant viscosity – see section on Newtonian approximation) is used. A similar curved relationship is shown by Thomas & Wilson[16] for this range of n values.

All three diagrams show clearly the reduction in friction factor which is a characteristic of these slurries.[16]

Pipe Roughness

An interesting dilemma arises in the new turbulent flow model when the pipe roughness approaches or exceeds the representative particle roughness size. The 200 mm nominal bore steel pipe had a hydraulic roughness of 112 μm and the tests of Park[26] were in a steel pipe for which a reasonable hydraulic roughness is estimated at 45 μm. These values exceeded the representative particle roughness sizes. In these two cases the matter was resolved by using the larger value of either the representative particle size or the pipe hydraulic roughness when calculating the roughness Reynolds number.

Analysis of the Particle Roughness Effect

The effect of relative particle size on wall velocity gradient for the new model is shown in Figure 10.

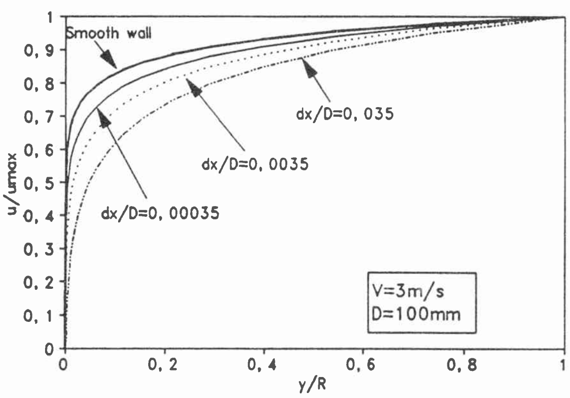

Figure 10 Effect of relative particle size on wall velocity gradient

This figure shows the turbulent velocity profile for the new model plotted for different representative particle sizes (d_x) for a constant average pipe velocity of 3 m/s.

Figure 10 shows that the effect of relative particle size in the new model is to reduce the velocity gradient in the wall region, in a similar way to conventional pipe roughness.

The new model does not take into account any other fluid/particle interaction which may occur.

Conclusions

Rheological characterisation

The correct procedure for obtaining the rheology of the slurries is critical in the smooth wall turbulent region and assumptions regarding rheological model should be kept as general as possible. The method employed here[14] has yielded consistently accurate rheological characterisations. Accurate rheology is particularly important for turbulent flow predictions in the smooth wall turbulent region.

The effect of solid particles

The presence of solid particles present in a non-Newtonian slurry affects both quantitatively and qualitatively the turbulent pipe flow behaviour of the slurry.

Particle roughness effect

The turbulent pipe flow behaviour of these slurries can be understood qualitatively in terms of a particle roughness effect.

Energy gradient prediction

Energy gradients for non-Newtonian turbulent slurry flow can be predicted using the rheology of the slurry and the particle size distribution. The predictive capability of the new model is more accurate than previous models for the test database.

For design purposes, the model can be used to accurately predict energy gradients for the turbulent flow of non-Newtonian slurries. This model is mathematically simple and easy to apply. This will facilitate more efficient design of pipe systems conveying non-Newtonian slurries.

Fully developed rough turbulent flow

The fully developed rough turbulent flow behaviour for non-Newtonian slurries in pipes is independent of the viscous characteristics of the slurry. This constitutes a useful engineering tool which can be used as an asymptote by designers.

When the pipe roughness exceeds the representative particle size, then the pipe roughness size should be used to model the flow.

The new model confirms the findings and method of Bowen[11] and provides a rationale for his correlation.

The particle roughness effect reduces the velocity gradient in the wall region, similar to the effect of conventional pipe roughness. Outside of the wall region, the continuum approximation has been accepted and no particle/fluid interactions have been considered.

Reversion to the Newtonian model

The new model reverts to the Newtonian model asymptotes under Newtonian conditions.

Final conclusions

Despite the fact that the new theoretical analysis is based on qualitative interpretations of physical behaviour, it remains, to some significant extent, an empirical description.

In the absence of any exact theoretical analysis, it is important, from an engineering perspective, to develop models which satisfactorily interpret experimental data and which provide a competent basis for design. The assumptions, approximations and simplifications present in the new analysis will doubtless prove controversial, but this is to be seen as healthy debate which, it is hoped, will further both the science and practice which comprises engineering technology.

The ultimate test of the new analysis is whether it is an improvement on existing analyses and models. The evidence presented in this paper shows that the new analysis does provide a significant improvement and can be used as a basis for practical design.

Acknowledgements

The author wishes to acknowledge the support of Prof. M.O. de Kock of the Department of Civil Engineering, University of Cape Town and the Council of the Cape Technikon, without whom this work would not have been possible.

References

[1] Mun R 1988. *Turbulent pipe flow of yield stress fluids.* MEngS thesis, University of Melbourne.

[2] Harris J & Quader AKMA 1971. Design procedures for pipelines transporting non-Newtonian fluids and solid liquid systems. *Brit. Chem. Engrg.*, **16**, 307–311.

[3] Shook CA & Roco MC 1991. *Slurry flow: principles and practice.* Butterworth Heinemann.

[4] Xu J, Gillies R, Small M & Shook CA 1993. Laminar and turbulent flow of kaolin slurries, *12th Int. Conf. on slurry handling and pipeline transport. Hydrotransport 12*, BHR Group, 595–613.

[5] Slatter PT 1994. Transitional and turbulent flow of non-Newtonian slurries in pipes. PhD thesis, University of Cape Town.

[6] Wilson KC, Addie GR & Clift R 1992. *Slurry transport using centrifugal pumps.* Elsevier Science Publishers Ltd, Essex, England.

[7] Schlichting H 1960. Chapter XX in *Boundary layer theory*, 4th edn, McGraw-Hill, New York.

[8] Colebrook CF 1939. Turbulent flow in pipes, with particular reference to the transition region between the smooth and rough pipe laws. *J. Instn. Civil Engrs.*, No. 4, Paper 5204, 133–156.

[9] Torrance BMcK 1963. Friction factors for turbulent non-Newtonian flow in circular pipes. SA Mechanical Engineer, **13**, 89–91.

[10] Wilson KC & Thomas AD 1985. A new analysis of the turbulent flow of non- Newtonian fluids. *Can. J. Chem. Eng.*, **63**, 539–546.

[11] Bowen RlB 1961. Designing turbulent flow systems. *Chem. Eng.*, **68,** 143–150.

[12] Dodge DW & Metzner AB 1959. Turbulent flow of non-Newtonian systems. *AICHE Journal*, **5**, 189–204.

[13] Kemblowski Z & Kolodziejski J 1973. Flow resistances of non-Newtonian fluids in transitional and turbulent flow. *Int. Chem. Eng.,* **13,** 265–279.

[14] Lazarus JH & Slatter PT 1988. A method for the rheological characterisation of tube viscometer data. *Joumal of pipelines*, **7**, 165–176.

[15] Wilson KC 1986. Modelling the effects of non-Newtonian and time-dependent slurry behaviour. *10th Int. Conf. on the hydraulic transport of solids in pipes, Hydrotransport 10,* Paper J1, 283–289.

[16] Thomas AD & Wilson KC 1987. New analysis of non-Newtonian flow- yield- power-law fluids. *Can J Chem Eng*, **65**, 335–338.

[17] Caldwell DH & Babbitt HE 1941. The flow of muds, sludges and suspensions in circular pipes. *Trans. Am. Inst. Chem. Eng.*, **37**, 237–266.

[18] Hedström BOA 1952. Flow of plastics materials in pipes. *Industrial and engineering chemistry*, **44**, 651–656.

[19] Metzner AB & Reed JC 1955. Flow of non-Newtonian fluids – correlation of the laminar, transition and turbulent flow regions. *AICHE Journal*, **1**, 434–440.

[20] Tomita Y 1959. A study on non-Newtonian flow in pipes. *Bull. JSME*, **2,** 10–16.

[21] Michiyoshi I, Matsumoto R, Mizuno K & Nakai M 1966. Flow of slurry through a circular tube. Friction factor for tubes. Part II, *Int Chem. Eng.*, **6**, 382–388.

[22] Edwards MF & Smith R 1980. The turbulent flow of non-Newtonian fluids in the absence of anomalous wall effects. *Journal of non-Newtonian fluid mechanics*, **7**, 77–90.

[23] Sive AW 1988. An analytical and experimental investigation of the hydraulic transport of high concentration mixed regime slurries. PhD thesis, University of Cape Town.

[24] Govier GW & Aziz K 1972. *The Flow of Complex Mixtures in Pipes.* van Nostrand Reinhold Co.

[25] Abbas MA & Crowe CT 1986. The effect of particle size and concentration on the flow properties of a homogeneous slurry. In *International symposium on slurry flows*, MC Roco & W

Wiedenroth (eds). New York: ASME Vol FED-38 109–112.

[26] Park JT, Mannheimer RJ, Grimley TA & Morrow TB 1989. Pipe flow measurements of a transparent non-Newtonian slurry. *Journal of fluids engineering*, **111**, 331–336.

[27] Janna WS 1983. *Introduction to fluid mechanics.*,Brooks/Cole Engineering Division.

[28] Lumley JL 1978. In *Topics in applied physics, Vol. 12; Turbulence*, (ed.) P Bradshaw, 2nd edn, Springer-Verlag, New York.

[29] Hanks RW & Dadia BH 1971. Theoretical analysis of the turbulent flow of non-Newtonian slurries in pipes. *AICHE Journal*, **17**, 554–557.

[30] Slatter PT & Lazarus JH 1993. Critical flow in slurry pipelines. *12th Int. Conf. on slurry handling and pipeline transport, Hydrotransport 12*, BHR Group, 639–654.

[31] Dedegil MY 1986. Drag coefficient and settling velocity of particles in non-Newtonian suspensions. In *International symposium on slurry flows.* MC Roco & W Wiedenroth, (eds). New York: ASME Vol FED-38, 9–15.

[32] Knight J 1993. Private communication, Physics Department, University of Cape Town.

[33] Slatter PT 1986. The rheological characterisation of non-Newtonian slurries using a novel balanced beam tube viscometer, MSc dissertation, University of Cape Town.

[34] Slatter PT & Lazarus JH 1988. The application of viscometry to the hydraulic transport of backfill material. *Conf. Backfill in South African mines.* (SAIMM).

[35] Hanks RW 1981. Course notes. *Hydraulic design for flow of complex fluids.* Richard W Hanks Associates, Inc. Orem, Utah. USA.

[36] Neill RIG 1988. The rheology and flow behaviour of high concentration mineral slurries. MSc dissertation, University of Cape Town.

[37] Loewenthal RE 1994. Private Communication, University of Cape Town.

[38] Moody LF 1944. Friction factors for pipe flow. *Trans ASME,* 671– 684.