Simplified thin-ship theory with application to the design of bulbous bows

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Nomenclature

C_w	wave resistance coefficient $(= R_w/\frac{1}{2}\rho V_\infty^2 S_w)$
Fr	Froude number $(= V_\infty/\sqrt{gL_{pp}})$
g	acceleration of gravity
i	square root of (-1)
L_{pp}	length between perpendiculars
N, N_x, N_z	number of panels: total, x-direction, z-direction
M	number of vertices for any one panel
R_f, R_{pv}, R_t, R_w	resistance: frictional, viscous pressure, total, wave
S_w	wetted surface area
u, v, w	x-, y- and z-direction components of the perturbation velocity vector
V_∞	free-stream velocity, ship speed
x, y, z	coordinates in the direction of the free stream, outward from the ship centreplane and vertically upwards from the fluid surface, respectively
y_s	local perpendicular distance of ship hull surface from ship centreplane
ϕ, ϕ_w	perturbation velocity potential, wavemaking velocity potential
κ	wave number $(= g/V_\infty^2)$
ν	wave angle
ρ	fluid density
σ	source strength

Abstract

A method for calculating the wave resistance of a ship by means of so-called Kelvin 'wavemaking' source panels on the centreplane of the ship is described. The method is a simplification of conventional thin-ship theory, which involves complicated Fourier expansions and which is very difficult to apply for hull forms other than those which can be described by simple analytical functions. A computer program to calculate wave resistance by means of this simplified theory was developed and used to calculate the wave resistance of a 24 m purse seine trawler. Results are presented for the trawler with and without a bulbous bow, at design speed and for two operating conditions. Fair correspondence between the prediction of the reduction in effective power requirement by means of the theory and by means of towing tank model tests was obtained.

Introduction

The total resistance of a ship can be subdivided into three parts,[1] i.e. the skin friction resistance R_f, the viscous pressure resistance R_{pv} (due to separation of the flow at the stern of the ship resulting in incomplete pressure recovery) and the wave resistance R_W (due to the continual supply of energy necessary to maintain the wave system that spreads out behind the ship):

$$R_t = R_f + R_{pv} + R_w \tag{1}$$

Arguably the simplest method for reducing the resistance, and hence the effective power requirement, of a given displacement ship with given operating conditions, is the addition of a bulbous bow. Kracht [2] divides the effect of the bulbous bow into two categories, i.e. (1) the interference effect and (2) the breaking effect. The interference effect refers to the interference between the free wave system of the ship and the waves generated by the bulbous bow itself, the geometry of which can be 'tuned' to cause partial cancellation of the ship wave system at normal operating conditions. The breaking effect refers to the attenuation of the bow wave system by improving the flow around the forebody and thus preventing the breaking of the bow waves. Thus the purpose of the bulbous bow is mainly to reduce the wave resistance, which can vary

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between about 15% of total resistance for a cargo vessel at low speed to about 60% for a destroyer at high speed.[3]

The purpose of this paper is to present a method for the calculation of wave resistance for displacement ships that is easy to apply and which yields reliable results for affordable work input. Such a method is essential for the initial design of a bulbous bow for a given ship.

Mathematical model

Various methods are available for calculating the flow around a ship hull, ranging from full Navier-Stokes solutions with turbulence modelling (see for example Larsson *et al.* [4]) through panel method potential flow solutions using panels distributed over the surface of the ship (see for example Jensen *et al.* [5]) to thin-ship theory which yields a potential flow solution for the flow around a ship in a similar fashion to that of thin airfoil theory for the flow around an airfoil. Both the Navier-Stokes and the full potential flow options require huge amounts of computational time, and are at present only used to compare two or three design options. For the purpose of initial design, thin-ship theory is more cost-effective: although most displacement ships cannot be considered as thin, the theory yields reasonable approximations of the *differences* between the wave resistances of various modified versions of the same basic ship.[6]

Michell [7] developed the theory of thin ships moving with a uniform velocity V_∞ into still water almost a century ago. The mathematical problem is to find the solution to Laplace's equation for the perturbation velocity potential,

$$\nabla^2 \phi = 0 \tag{2}$$

subject to the following boundary conditions:

1. The linearized no-throughflow boundary condition on the ship hull:

$$v = \frac{\partial \phi}{\partial y} = \frac{\partial y_s}{\partial x} \text{ on } y = 0 \tag{3}$$

2. The linearized free surface boundary condition:

$$\frac{\partial^2 \phi}{\partial x^2} + \kappa \frac{\partial \phi}{\partial z} = 0 \text{ on } z = 0 \tag{4}$$

3. The radiation condition, i.e. that the free surface far ahead of the ship must remain undisturbed:

$$\nabla \phi = 0 \text{ at } x \rightarrow \infty \tag{5}$$

4. The bottom condition, i.e. that the fluid far below the ship must remain undisturbed:

$$\nabla \phi = 0 \text{ at } z \rightarrow -\infty \tag{6}$$

Once the velocity potential has been found, the wave resistance of the ship can be obtained from the wake integral:[1]

$$R_w = \frac{1}{2}\rho \left[\begin{array}{c} \int_{-\infty}^{0} \int_{-\infty}^{\infty} \left(w_\infty^2 + v_\infty^2 - u_\infty^2\right) dy dz \\ + \frac{1}{\kappa} \int_{-\infty}^{\infty} u_\infty^2 \left(z = 0\right) dy \end{array} \right] \tag{7}$$

The subscript ∞ in Eq. (7) indicates that the perturbation velocities have to be calculated at an yz-plane at a distance very far astern of the ship.

Michell [7] used a Fourier expansion to arrive at the expression for the velocity potential. The resulting expression for the wave resistance of a ship is in the form of the sum of two triple integrals and one quadruple integral. Although the quadruple integral and one of the triple integrals vanish through symmetry, the mathematics involved in calculating the resistance by this means is very complicated, even for hull forms that can be described by simple mathematical functions.

A simpler solution method is that of representing the ship by means of Kelvin wave-making sources (see for example Gadd [8]) on the centreplane of the ship, i.e. on the xz-plane at $y = 0$. The velocity potential associated with a Kelvin source of strength σ located at $x = a$, $y = 0$, $z = -c$ (where c is a positive number) consists of three parts: the velocity potential of a Rankine source of strength σ located at $x = a$, $y = 0$, $z = -c$, the velocity potential of its image located at $x = a$, $y = 0$, $z = c$, and a wavemaking part given in complex notation by

$$\phi_w = \frac{\sigma}{2\pi} Re \int_{-\pi/2}^{\pi/2} \times \left[\begin{array}{l} -i\kappa_\nu \exp\left[\kappa_\nu \left\{ \begin{array}{c} (z-c) \\ +i(w-w_0) \end{array} \right\}\right] \\ + \frac{1}{\pi} \int_0^\infty \frac{\alpha}{\kappa_\nu - \alpha} \exp\left[\alpha \left\{ \begin{array}{c} (z-c) \\ +i(w-w_0) \end{array} \right\}\right] d\alpha \end{array} \right] d\nu \tag{8}$$

where $\kappa_\nu = \kappa \sec^2 \nu$, $w = x \cos\nu + y \sin\nu$, $w_0 = a \cos\nu$, ν is the angle between the direction of propagation of a wave and the free stream direction and α is a parameter of integration.

These Kelvin sources satisfy Laplace's equation, as well as the linearized free surface condition, the radiation condition and the bottom condition. A Kelvin source therefore represents a solution for a source-type flow in the presence of a free surface. The velocity potential for a ship represented by a source distribution $\sigma(x, 0, z)$ on its centreplane is obtained by replacing ϕ_w and σ in Eq. (8) with $d\phi_w$ and σdS, respectively, and integrating over the area of the centreplane. The source strength required at a certain point is determined by the no-throughflow boundary condition on the ship hull. Karamcheti [9] gives the corresponding magnitude of the required source distribution for a thin airfoil, which can be shown to hold also for a thin ship, as

$$\sigma = 2V_\infty \frac{\partial y_s}{\partial x} \tag{9}$$

Instead of using a continuously varying source distribution, the problem is simplified by dividing the centreplane area into a number of M-sided panels, on each of

which the source distribution is assumed to be constant (equal to its value at the geometrical centre of the panel). The value of M can be different for all panels, but usually triangular and quadrilateral panels are sufficient. It can be shown that the wavemaking velocity potential due to such a panel with constant source strength per unit area that is obtained by integrating Eq. (8) over the panel, is given by

$$\phi_w = -\frac{\sigma}{2\pi}\mathrm{Im}\int_{-\pi/2}^{\pi/2}\sum_{j=1}^{M} G_j(\nu)\, J_j(\kappa,\nu)\, d\nu \qquad (10)$$

where

$$G_j(\nu) = \frac{2A_j}{[(z_j - z_{j-1}) - i(w_j - w_{j-1})] \times [(z_{j+1} - z_j) - i(w_{j+1} - w_j)]}$$

and

$$J_j(\kappa,\nu) = \frac{1}{\kappa_\nu}\exp\left[\kappa_\nu \left\{ \begin{array}{l} (z+z_j) \\ +i(w-w_j) \end{array} \right\}\right] + \frac{i}{\pi}\int_0^\infty \frac{1}{\alpha(\kappa_\nu - \alpha)}\exp\left[\alpha \left\{ \begin{array}{l} (z-z_j) \\ +i(w-w_j) \end{array} \right\}\right] d\alpha$$

Here A_j is the area of the triangle formed by the vertices $(j-1)$, (j) and $(j+1)$, and $w_j = x_j \cos v$. Note that $j-1$ is defined as M for $j=1$, and that $j+1$ is defined as 1 for $j=M$.

The wavemaking velocity field which is induced by the submerged panel is obtained in the usual manner by differentiating Eq. (10). The resulting expressions for the perturbation velocity components very far astern of the ship due to N panels are given by

$$u_\infty = \frac{1}{\pi} Re \int_{-\pi/2}^{\pi/2} \sum_{k=1}^{N} \sigma_k \sum_{j=1}^{M} G_{j,k}(\nu) \times \exp\left[\kappa_\nu \{(z+z_{j,k}) + i(w-w_{j,k})\}\right] \cos\nu\, d\nu \qquad (11)$$

$$v_\infty = \frac{1}{\pi} Re \int_{-\pi/2}^{\pi/2} \sum_{k=1}^{N} \sigma_k \sum_{j=1}^{M} G_{j,k}(\nu) \times \exp\left[\kappa_\nu \{(z+z_{j,k}) + i(w-w_{j,k})\}\right] \sin\nu\, d\nu \qquad (12)$$

$$w_\infty = \frac{1}{\pi} Im \int_{-\pi/2}^{\pi/2} \sum_{k=1}^{N} \sigma_k \sum_{j=1}^{M} G_{j,k}(\nu) \times \exp\left[\kappa_\nu \{(z+z_{j,k}) + i(w-w_{j,k})\}\right] d\nu \qquad (13)$$

The expression for the wave resistance of the ship hull as represented by the panels of constant source strength is obtained by substituting Eqs. (11)–(13) into Eq. (7). The result is as follows:

$$R_w = \frac{\rho}{\pi\kappa^2}\int_0^{\pi/2} \cos^5\nu \sum_{k=1}^{N}\sum_{m=1}^{N} \left[\begin{array}{l} T_k(\nu)\, T_m(\nu) \\ +U_k(\nu)\, U_m(\nu) \end{array} \right] d\nu \qquad (14)$$

where

$$T_k(\nu) = 2\sigma_k \sum_{j=1}^{M} A_{j,k} \exp\left(\kappa z_{j,k} \sec^2\nu\right) \times \left[\begin{array}{l} Re(F_{j,k}) \cos(\kappa x_{j,k} \sec\nu) \\ +\, Im(F_{j,k}) \sin(\kappa x_{j,k} \sec\nu) \end{array} \right]$$

and

$$U_k(\nu) = 2\sigma_k \sum_{j=1}^{M} A_{j,k} \exp\left(\kappa z_{j,k} \sec^2\nu\right) \times \left[\begin{array}{l} Re(F_{j,k}) \sin(\kappa x_{j,k} \sec\nu) \\ -\, Im(F_{j,k}) \cos(\kappa x_{j,k} \sec\nu) \end{array} \right]$$

The function $F_{j,k}$ is the even part (with respect to $v=0$) of $G_{j,k}(\nu)/2A_{j,k}$ and is defined by the expression

$$1/F_{j,k} = \left[\begin{array}{l} (z_{j+1,k} - z_{j,k})(z_{j,k} - z_{j-1,k}) \\ -(x_{j+1,k} - x_{j,k})(x_{j,k} - x_{j-1,k})\cos^2\nu \\ -i(z_{j+1,k} - z_{j,k})(x_{j,k} - x_{j-1,k}) \\ +(x_{j+1,k} - x_{j,k})(z_{j,k} - z_{j-1,k})\cos\nu \end{array} \right]$$

Full details of the derivation of Eqs. (10)–(14) are given by Damant [10] for the specific case of quadrilateral panels.

Computer program

A FORTRAN computer program was developed for the calculation of wave resistance by means of Eq. (14) using triangular and quadrilateral panels. In the program, the integral in Eq. (14) is evaluated numerically by means of Gaussian quadrature, using 64 Gauss points (results generally differed by less than half a percent if 32 Gauss points were used). All the results reported here were calculated in double precision on the University of Stellenbosch VAX ALPHA 3000-800.

Several case studies were undertaken to verify that the program was working correctly. One of these was the calculation of the wave resistance of the hull form used by Michell [7] in his original paper. This hull form is defined by the expression

$$y_s(\mathrm{ft}) = \pm 4\,[1 + \cos\{\pi x(\mathrm{ft})/100\}] \times [1 + \cos\{\pi z(\mathrm{ft})/20\}]$$
$$-100 \le x \le 100;\ -20 \le z \le 0$$

where the origin of the coordinate axis is at the intersection of the vertical median plane, the midship plane and the plane of the undisturbed water surface. The projection of the hull surface on the centreplane is rectangular, and therefore the hull surface was divided into $N_x \times N_z$, rectangular panels for the computations described here. Panels were concentrated near the free surface, as shown in Figure 1 for a 32×8 subdivision.

Results of the wave resistance calculations are shown in Table 1 for a ship speed of 20 ft/s, the same as that used by Michell in the one and only calculation reported in his paper. The value of ρ was taken as that of seawater, i.e. 1023 kg/m^3, and the value of g as 9.807 m/s^2. It is evident from Table 1 that grid-independent results are obtained for $N_z = 32$, but not for N_x. The values in the last column were found by means of Richardson extrapolation for a second order scheme, i.e. by estimating the error of the solution on the finest grid as one third of the difference between the solutions on the finest and second finest grids. It is evident that the wave resistance calculated by means

of the proposed panel method is quite close to the 4181 N ('940 lbs. wt. about') calculated by Michell. The small difference that does exist can be ascribed to the difference between the two mathematical models. Slightly different values of water density and of g might also have been used by Michell in his calculation.

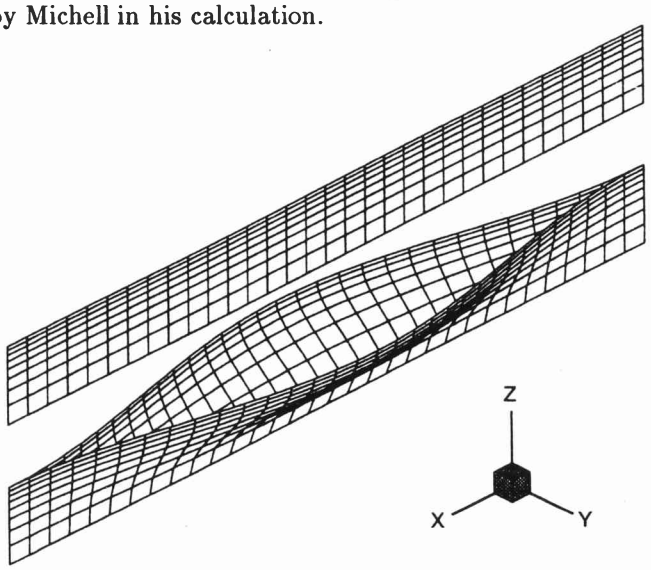


Figure 1 Panel distribution for Michell's hull form (actual panels on hull below, projection on centreplane above)

Table 1 Wave resistance (in Newton) calculated for the Michell hull form

$N_z \downarrow$ $N_x \rightarrow$	8	16	32	64	128	256	512	∞
8	2642	3255	3794	3951	3990	4000	4003	4004
16	2655	3268	3808	3966	4006	4016	–	4019
32	2656	3269	3810	3967	4007	–	–	4020
64	2656	3269	3810	3967	–	–	–	4019

The wave resistance coefficient of Michell's hull form as function of Froude number, calculated by means of the proposed panel method with a 128 × 32 subdivision and using 64 Gauss points for the numerical integration, is compared in Figure 2 with the Michell integral results, which were obtained by means of the method described by Wigley.[11] The difference between the two sets of results is only discernible at the higher Froude numbers, where the panel method values are too large by approximately half a percent compared to the analytical results.

Utilization for bulbous bow evaluation

Damant [10] conducted towing tests in the University of Stellenbosch towing tank on the 1/15 scale model of a 24 m purse seine trawler shown in Figure 3. The main characteristics of the trawler are summarized in Table 2. The tests were carried out at the two predominant load conditions which are detailed in Table 2, using the model with a normal bow as shown in Figure 3, and also with a bulbous bow fitted as shown in Figure 4. The bulb has circular cross sections, and was designed by exploring a large number of options by means of the proposed thin-ship theory presented in this paper, taking guidance from the recommendations of Kracht [2] and Couch & Moss.[12] Measured differences in total resistance (ΔR_t) are presented in Table 3 for Froude numbers in the range 0.34 to 0.44, which corresponds to the operating speed range of the trawler (10 to 13 knots).

Table 2 Trawler characteristics

Overall length	27.2 m
Length between perpendiculars	23.77 m
Maximum beam	8.0 m
Block coefficient	0.85
Design speed	11 knots
Departure condition draught	2.558 m
Departure condition displacement	260 tons
Trim by stern in departure condition	0.182 m
Full load draught	3.580 m
Full load displacement	430 tons
Trim by bow at full load	0.183 m

Table 3 Percentage differences in trawler resistance due to the bulbous bow

Departure load condition			Full load condition		
Fr	ΔR_t	ΔR_w	Fr	ΔR_t	ΔR_w
0.346	−3.7	−9.2	0.348	−6.5	−5.2
0.372	−5.8	−13.9	0.369	−5.6	−6.6
0.393	−6.2	−15.2	0.394	−3.3	−10.8
0.435	−11.8	−13.7	0.437	−4.7	−9.9

The differences in wave resistance due to the bulbous bow (ΔR_w) were also calculated by means of the proposed panel method. The results shown in Table 3 were obtained using 200 × 40 panel subdivisions similar to the 50 × 10 subdivisions shown in Figures 5 to 8. The results for the 200 × 40 subdivisions differed in general by less than half a percent from those using 100 × 20 subdivisions.

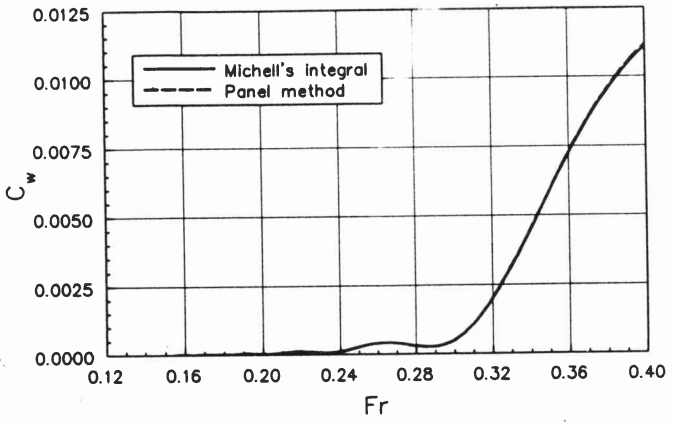


Figure 2 Theoretical wave resistance of Michell's hull form

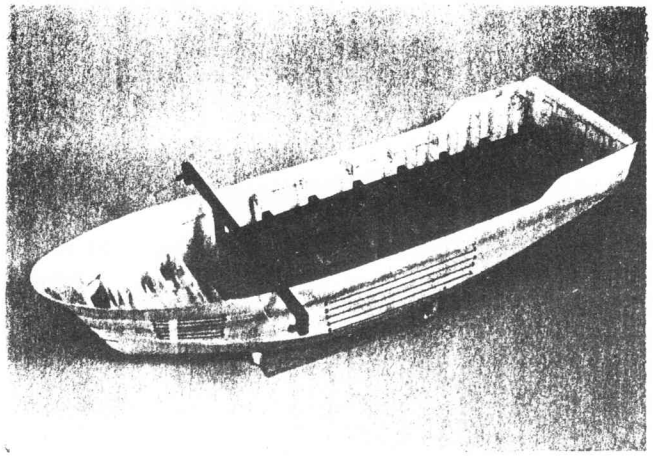


Figure 3 Purse seine trawler model with normal bow

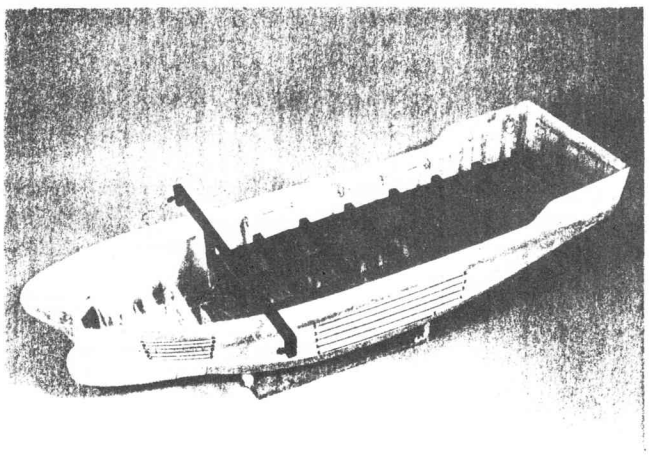


Figure 4 Purse seine trawler model with bulbous bow

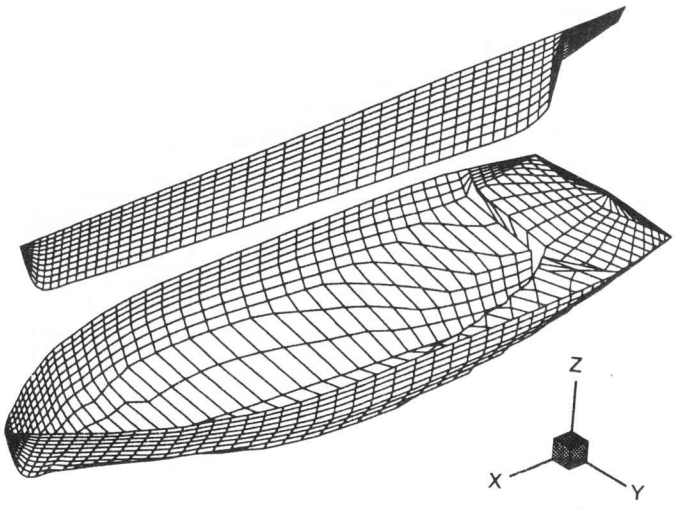


Figure 5 Panel distribution for trawler with normal bow in departure condition (actual panels on hull surface below, projection on centreplane above)

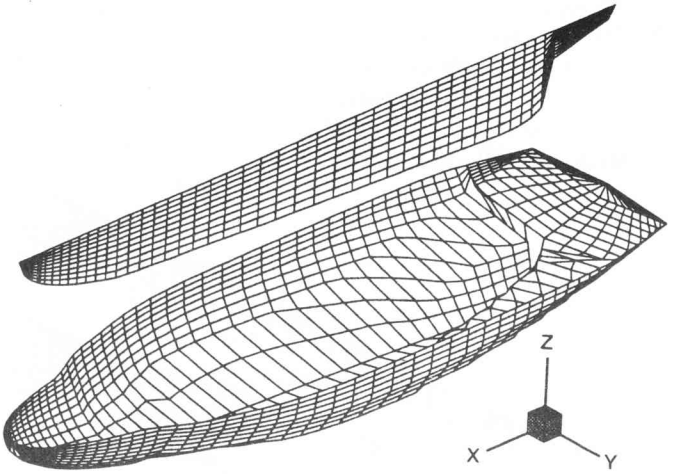


Figure 6 Panel distribution for trawler with bulbous bow in departure condition

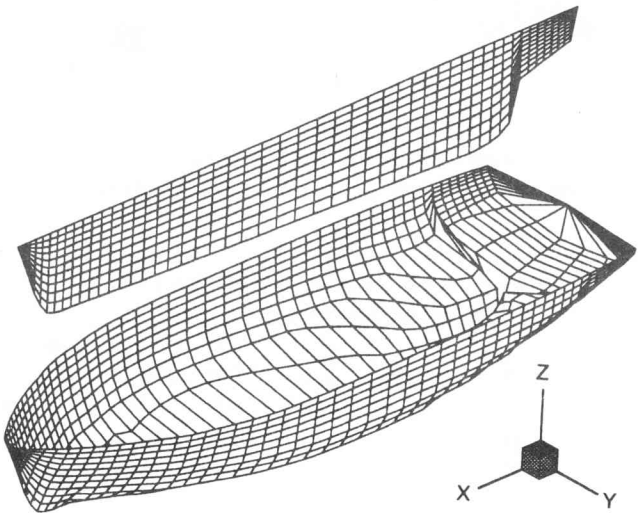


Figure 7 Panel distribution for trawler with normal bow in full load condition

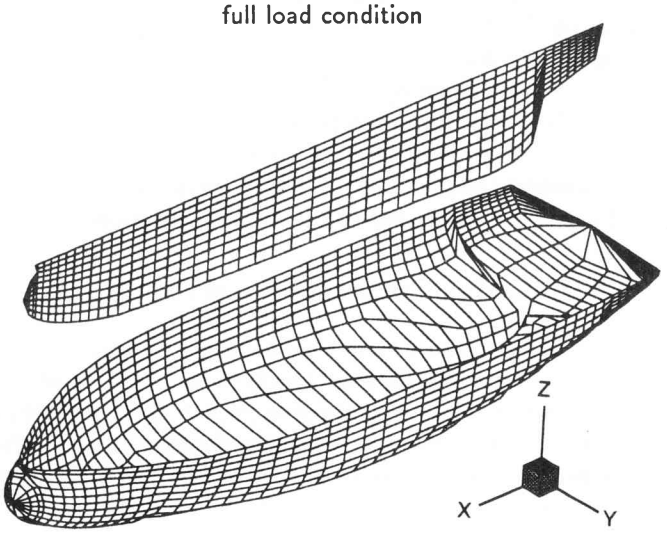


Figure 8 Panel distribution for trawler with bulbous bow in full load condition

It is evident that the decrease in wave resistance predicted by the proposed theory is generally of the same order of magnitude as the experimentally determined decrease in total resistance. Unfortunately it is not possible to compare directly the decreases in wave resistance, because wave resistance alone cannot be determined experimentally by means of the force measurements.

Concluding Remarks

It has been shown that the panel method proposed in this paper for implementing classical thin-ship theory yields very accurate results when compared with analytical solutions of the same theory. The method also proved to give reasonably satisfying predictions for the bulbous bow effect, even though the ship for which calculations were done cannot be considered as thin. Further work that is envisaged is to implement the method in a design optimization program.

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