Strain modal testing – a critical appraisal

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In performing modal tests it is conventional to measure either, acceleration, velocity or displacement as a response quantity. However, research indicates that modal testing based on strain measurement, appropriately termed strain modal testing (SMT), may provide the analyst with significant advantages such as modal testing efficiency improvement and condition monitoring benefits. Equipment requirements for SMT are however found to be less favourable than for conventional displacement modal testing. It may also be shown that the theory underlying SMT brings about fundamental changes in the nature of the data acquired and the ensuing analysis procedures. This paper critically assesses SMT and its potential applications and concludes that the analyst could benefit significantly from its judicious employment. SMT provides valuable additional information compared to displacement modal testing and should be employed complementary to conventional modal testing.

Nomenclature

A. Acronyms

DOF	degree of freedom
DFRF	displacement, velocity or acceleration frequency response function
DMT	displacement, velocity or acceleration modal testing
FEM	finite element method
FEML	finite element model
FRF	frequency response function
MT	modal testing, albeit strain or displacement oriented (consisting of both data acquisition and modal parameter extraction)
RDOF	rotational degree of freedom
SFRF	strain frequency response function
SMOD	structural modification
SMT	strain modal testing

B. Mathematical symbols

B.1 Roman

m	number of modes included in analysis
n	number of excitation and response measurement points
$\{E\}$	vector of strain response phasors (complex)
$\{F\}$	vector of excitation force phasors (complex)
$\{Q\}$	vector of response phasors in modal coordinates (complex)

B.2 Greek

$[\alpha]$	displacement transfer function (receptance) matrix
$[\beta]$	strain transfer function matrix
β_{ij}	strain transfer function: response at position i, excitation at position j
$[\Lambda]$	mass normalised spatial property matrix
$[\xi]$	mass normalised strain eigenvector (mode shape)
$\{_r\xi\}$	rth mass normalised strain eigenvector (mode shape) matrix
$[\phi]$	mass normalised displacement eigenvector (mode shape) matrix
$\{_r\phi\}$	rth mass normalised displacement eigenvector (mode shape)

B.3 Superscripts

-1	inverse of a square matrix
T	transpose of a matrix

Introduction

Until the early 1980s modal testing (MT) techniques were synonymous with the use of displacement responses and their derivatives with respect to time. Some interest has since been shown in the use of strain measurements in MT applications. However, this has been limited, which is quite surprising when one realises that displacement information from conventional displacement modal testing (DMT), while representing an important intermediate result in problem solution, is normally not the end product in structural integrity evaluation. In contrast, strains and stresses are the parameters that have been related directly to the strength of materials and can thus be used directly in integrity evaluation of structures that have to survive repeated dynamic loading (fatigue related loading). The SMT literature[1,2,3] shows that this method is able to produce modal information that is an important, if not essential, complement to the classic DMT techniques. It also indicates a need for research and investigation into the potential of SMT and its possible applications. With

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this paper it is endeavoured to critically appraise the advantages and disadvantages of SMT in order to encourage further research in this field.

Theoretical background of strain modal testing

A number of researchers have proposed methods of deriving analytical expressions for strain FRFs (SFRFs). These methods are based on:

- modal superposition applied to the strain mode shape matrix,[1]
- spatial differentiation of the translational DFRFs (which uses a mixture of continuous and discrete descriptions of the structure,[4]
- the fundamental derivation of the SFRFs for the particular structure under consideration,[5] and
- the solution of the continuous differential equation formulation (with boundary and initial conditions) of solid elastodynamics.[2,6]

All of these link the strain response, {E}, to the excitation force, {F}, through a relationship of the form:

$$\{E\}_{6n\times 1} = \{\beta\}_{6n\times 3n}\,\{F\}_{3n\times 1} \tag{1}$$

where

$$\{\beta\}_{6n\times 3n} = \{\xi\}_{6n\times m}\,[\Lambda]^{-1}_{m\times m}\,[\phi]^{\mathrm{T}}_{m\times 3n} \tag{2}$$

Here, $[\beta]_{6n\times 3n}$ is the SFRF matrix, $\{F\}_{3n\times 1}$ is the complex vector of excitation forces, $\{\xi\}_{6n\times m}$ is the mass normalised strain eigenvector matrix, $[\phi]_{3n\times m}$ is the mass normalised displacement eigenvector matrix and $[\Lambda]_{m\times m}$ is a symmetric, frequency-dependent matrix containing spatial system properties. Note the unsymmetric nature of $[\beta]_{6n\times 3n}$. This is caused by the mode shape matrices $[\xi]_{6n\times m}$ and $[\phi]_{3n\times m}$, whereas in the case of the DFRF matrix, $[\alpha]_{3n\times 3n}$, the matrix $[\Lambda]^{-1}{}_{m\times m}$ is pre-multiplied by $[\phi]_{3n\times m}$ instead of $[\xi]_{6n\times m}$, causing $[\alpha]$ to be symmetric. Also note that $[\beta]_{6n\times 3n}$ is rectangular in expression (2). This is due to the fact that only translational excitation coordinates have been used in the derivation. Inclusion of bending moments or couples in excitation will lead to a square SFRF matrix, but this would be equivalent to a DMT environment inclusion of translational and rotational excitation with translational response only. Thus, the main feature seen in equation (2) is the unsymmetric nature of the SFRF matrix, which implies that the SFRF corresponding to response at a general point j and excitation at a point k is different from the one corresponding to response at point k and excitation at point j. This is illustrated by means of SFRF measurements on a cantilevered beam specimen shown in Figure 1.

The nature of the SFRF matrix also causes some fundamental changes when mode shapes are to be extracted from measurements:

- extracting displacement mode shapes from SFRF measurements requires the strain response point to be fixed with excitation applied at every coordinate. This corresponds to the measurement of one row in the SFRF matrix. The measured displacement mode shape thus obtained is proportional to the displacement mode shape by a constant factor – this factor being the component of the associated strain mode shape (i.e. the entry (say ${}_r\xi_j$) in the strain mode shape corresponding to the fixed strain response point). To derive a mass normalised displacement mode shape, one needs to measure an additional point DFRF at the fixed point of strain measurement along with the SFRFS. The component of the displacement mode shape associated with the excitation point (say ${}_r\phi_j$) can then be calculated. Dividing the measured mode shape by ${}_r\phi_j$ will leave the value of ${}_r\xi_j$ as an entry in the resulting mode shape vector in the position in the vector associated with the excitation position. The measured mode shape vector can then be divided by ${}_r\xi_j$ to render the true displacement mode shape;
- extracting strain mode shapes from SFRF measurements requires an equivalent approach, with the only difference being that one column of the SFRF matrix must be measured, that is: the excitation point is kept fixed while response coordinates are varied. As in the previous case, the measured mode shape is proportional to the strain mode shape by a constant factor – this factor being the component of the associated displacement mode shape (i.e. the entry (say ${}_r\phi_j$) in the displacement mode shape corresponding to the fixed excitation point). The value of ${}_r\phi_j$ can be determined from the point DFRF measurement. The true strain mode shape can then be calculated by dividing the measured mode shape by the value found for ${}_r\phi_j$.

The unsymmetric nature of the SFRF matrix is thus seen to bring about fundamental changes to the measurement requirements of SMT as opposed to DMT. The role that point DFRF measurements could play in analysis of measurements illustrates that SMT should not replace DMT, but should rather be used as a complementary analysis tool.

Strain modal testing applications

In the appraisal of SMT and its possible applications various important benefits as well as drawbacks may be identified. In the following paragraphs these benefits and drawbacks are explored.

Benefits of SMT

(1) Correlation of a finite element model (FEML) with measurements is at best difficult to achieve. To improve the reliability of a dynamic FEML, the FEML will have to be correlated with dynamic measurement results such as mode shapes and natural frequencies

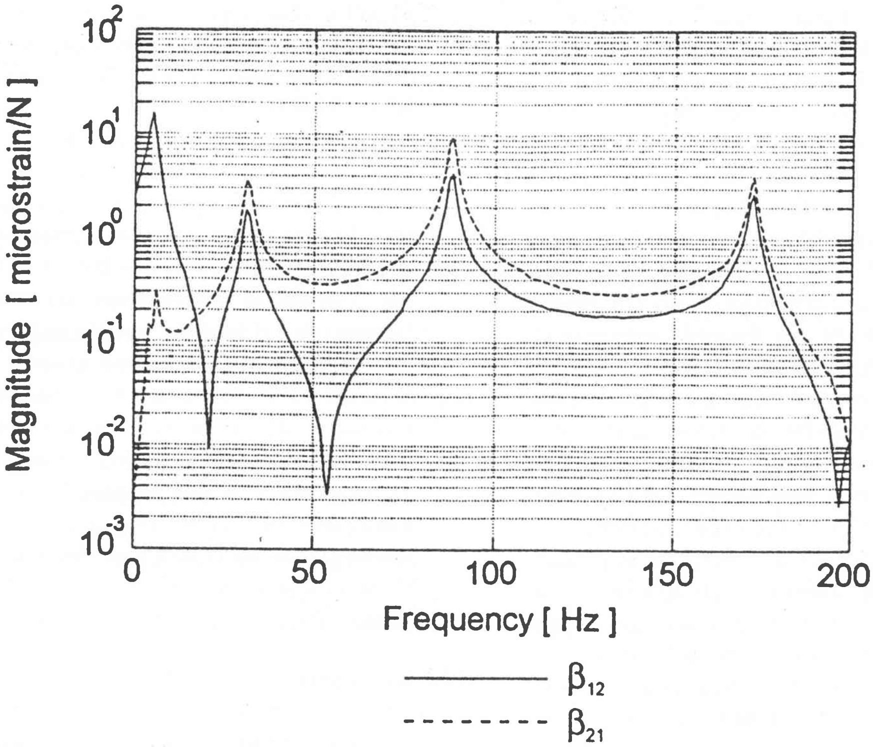

Figure 1 Measured transfer SFRFs with response and excitation positions swopped

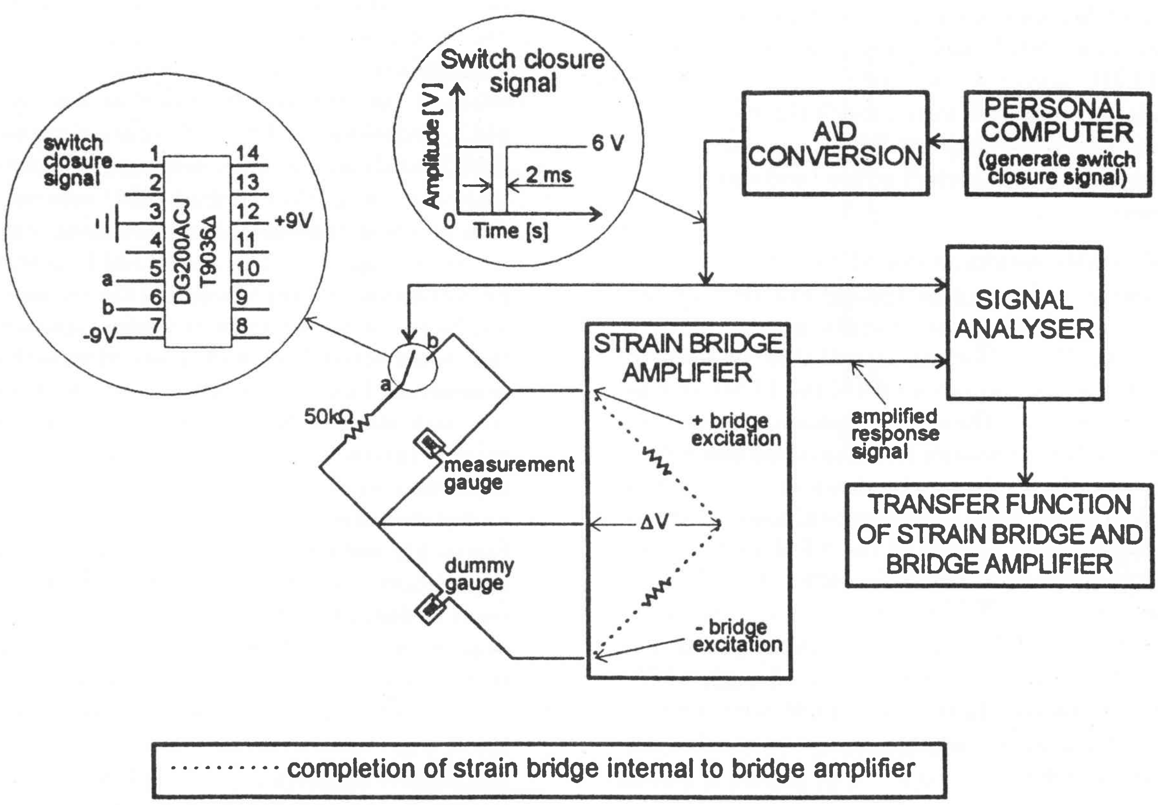

Figure 2 Schematic arrangement of strain measurement system dynamic calibration

from DMT. If the end results of a DMT on a structure are the natural frequencies (eigenvalues) and displacement mode shapes (displacement eigenvectors),[1,7] the problem exists that these results do not pertain directly to structural integrity, although they do provide a means of correlation with the FEML. The DMT results thus only indirectly alleviate the task of structural integrity evaluation. The displacement mode shapes must be manipulated further to obtain a position dependent vector of strain and/or stress response if detailed comparison or correlation with the stress or strain results of the FEM are desired. Seeing that this mathematical handling of the displacement vectors is based on the assumption of small deformations and that experimental errors (such as inaccurate measurements and modal truncation)[8] might already be present, it would be a great improvement if one were able to bypass manipulation of the experimentally obtained displacement eigenvectors. Such an approach is supported by research[9] showing that displacement data derived from accelerometer data are not of sufficient quality to predict dynamic stress or strain responses in a structure. This fact is aggravated by the frequent omission of RDOFs in DMT.[1] It would thus seem that DMT data are unfit for worthwhile prediction of dynamic strain or stress responses. However, SFRFs measured on the structure have undergone no mathematical manipulation and this diminishes the chance of propagation of measurement errors, that is to say, if FEML validation is to be done using stress or strain data. SMT assisted validation of the dynamic FEML above would enable one to complete the validation task in a structural integrity environment, whereas DMT assisted FEML validation does not necessarily ensure correct strain (and thus stress) prediction.

The first usable results obtained from the dynamic FEM analysis above will be the eigenvalues and displacement eigenvector components at the model's nodal points. This information will then be used to calculate the nodal displacements of the FEML due to the dynamic inputs. These displacements have to undergo interpolation through form functions and differentiation in order to derive the strain responses. It is preferred to apply these mathematical manipulations to the displacement results of the FEM rather than applying them to the experimentally obtained displacements from DMT. Thus, if the strain responses predicted by the FEM still result in the required correlation with those obtained directly through SMT, one can use the results from the FEM with greater confidence,[3] knowing that the assumptions within the FEML are sound.

SMT can thus serve as a FEML validation tool in a structural integrity environment and thus render more confident use of the FEML.

(2) Seeing that SMT allows more confident use of the FEM (as concluded in paragraph (1) above) SMT also enables one to make more confident fatigue life predictions by using a SMT validated FEML. In addition, when acceptable coherence of SFRFs is obtained it could be possible to use these curves directly for response predictions by means of superposition (assuming structural linearity) of strain response spectra. These spectra can be derived by multiplying the appropriate SFRFs by the known dynamic input spectra. The strain response spectra can then be used for fatigue life predictions. Assuming structural linearity, a fatigue life prediction can thus be made for any set of input force vectors without making use of a FEML. Excessive structural non-linearity of course rules out this possibility thus casting some doubt over life predictions made in this manner for structures failing in less than $\pm 10^4$ cycles of equivalent constant amplitude loading. The approach just described[10] should nevertheless enable one to quickly make a rough estimate of the fatigue life of an existing structure.

(3) By applying structural modification (SMOD) techniques to the measured SFRF data, the possibility exists of obtaining the strain response of the modified structure without having to modify or set up an FEML. With the strain response of the modified structure known, fatigue life predictions could be made for the modified structure as long as the properties of the original structure are known as input to the SMOD process, bypassing the use of an FEML. The result of such a modification would thus be the properties of the modified structure as described in the SMT environment. This combination of fatigue- and SMOD-analysis has been reported,[11,12] with the intention of using DMT in the SMOD process, although difficulty was experienced in converting displacement to strain response. The use of SMT in such SMOD predictions would thus facilitate the bypassing of this displacement to strain conversion. Another benefit that could derive from such a procedure is that the optimisation of a structure with respect to its fatigue life (through stepwise SMOD) will largely become a numerical instead of an expensive and time-consuming experimental process of durability or fatigue testing undertaken after each physically implemented SMOD. Durability testing cannot be discarded, but its area of application could be switched from product development to final product validation. This approach relies heavily on the assumption of structural linearity for strain response prediction but nevertheless could allow the prediction of quick estimates of the fatigue life of a modified structure. The combination of SMT and SMOD techniques could thus allow for improved fatigue design capability. A recent study of the application of SMOD in the SMT environment showed this pattern of thought to be a reachable target, although the application is currently limited to simple structures. SMOD using SFRFs as input to the unified frequency coupling approach[13] was applied to the coupling of two cantilevered beams (dimensions of 40mm×5mm×450mm and 20mm×15mm×450mm, respectively). The prediction is exact in an analytical environment. This is not the case for actual coupling of the substructures used, seeing that most practical coupling mechanisms introduce some properties to the coupled structure that are difficult if not impossible to incorporate in the prediction.[14]

(4) SMT also has advantages in determining the dynamic input forces to a structure. Research[5] has shown that the use of measured DFRFs in such force determination is an exceptionally sensitive operation. This is caused by the requirement that the DFRF matrix must be inverted. The referenced authors have shown that the SFRF matrix presents a less ill-conditioned inverse operation for the loading cases of bending couples and point applied forces than would be the case for the DFRF matrix. SMT thus provides one with a less ill-conditioned force prediction ability.

(5) Piezo-electric type of accelerometers are widely used in MT applications. These devices unfortunately decrease in sensitivity as the frequency of the motion decreases. However, strain gauges facilitate response measurement in the frequency range down to static loads. Use of this property has been reported[5] and with measured SFRFs found to be accurate to a lower frequency bound than measured DFRFs. Whereas the attempt to use DMT in the low frequency region usually leads to the use of larger and heavier accelerometers (influencing structural behaviour more severely) the use of strain gauges in this frequency region has no such detrimental effects. The small mass and size of strain gauges can thus be used in the SFRF measurement of structures that were previously considered too lightweight for accurate DFRF measurement[15] when using piezoelectric type of transducers. Transducers used in SMT thus have weight, size and low frequency sensitivity advantages over the piezo-electric type of transducers conventionally used in DMT.

(6) Rigid body motions are often of little importance in MT and can even become a hindrance.[9] The referenced authors experienced difficulty in determining dynamic stresses and strains from inertances, because their accelerometer data were 'polluted' by incomplete measurement of rigid body modes (theoretically occurring at 0 Hz but shifted up the frequency axis due to the finite stiffness of the method of 'free' suspension chosen). This difficulty could have been avoided by simply using SFRFs, seeing that rigid bodies undergo no deformation and strain gauges are therefore incapable of measuring rigid body modes. SFRFs thus contain no information pertaining to rigid body motion (in some instances this could be seen as a disadvantage, because inertial properties are lost together with the rigid body motion).

(7) DFRFs do not facilitate a direct measure of the problematic locations or excitation frequencies (problematic in the sense of possibly causing fatigue failure), as the frequencies or modes in the DFRFs with large displacements are not necessarily the modes that cause the highest strain values. However, inspection of SFRFs gives a direct indication of the problem frequencies (modes)[16] and locations,[17,10] on the structure where strain response reaches its highest values, because the SFRF containing the highest peak strain values indicates the modes that cause the largest strain field and also the physical response positions where these strains occur. This information can also be obtained by construction of time and position dependent strain and stress concentration fields when combining DMT and SMT.[1] This prospect becomes even brighter when one notes that the coherence of SFRFs are found to be 'good' in the vicinity of resonances.[1] The frequencies as well as the positions of most probable failure are thus indicated by a single set of SFRFs.[18,3] Using this information, the analyst can concentrate on problem areas for further analysis and possible prevention of fatigue failures, while information gathered from DFRFs will be useful in pinpointing the mechanism of failure. These areas can then be analysed in detail experimentally or analytically (for example: viewing a problem component as a substructure and assembling detailed FEMLs of these substructures only).[17] A further possibility is that of 'streamlining' the updating of the modal data base seeing that one can concentrate on the problem frequencies and derive a set of SFRFs with superior accuracy only in the vicinity of these frequencies. In short: augmenting DMT with SMT provides a means of increased productivity in MT, directly indicating the position and frequency where additional measurement effort is required.

It has been reported[2,4,15] that a set of modal parameters can be extracted by combining the measurement of point DFRFs at excitation locations with that of a row of SFRFs in the SFRF matrix, due to this matrix's unsymmetric nature. With the more accurate SFRFs in the vicinity of the problem frequencies, modal parameters of superior accuracy at these problem frequencies could be obtained with this set of parameters then possibly lending itself to a more accurate analytical simulation of the structure and of structural changes.

(8) SMT can be used as a tool in the pre-production stages of design where alpha or beta prototypes are available for testing. The information contained in a set of SFRFs makes them directly implementable (they pertain directly to structural integrity (paragraph **(1)** above) and aid in pinpointing problem areas and frequencies (paragraph (7) above)). Used as such, SFRFs can assist the engineer in curbing post-production costs as the SFRFs of the prototype provide immediate insight into problem area locations and frequencies before production starts. Final analysis could then be concentrated on these areas only.

(9) In using DMT as a condition-monitoring tool (continuous or regular set interval measurement of structure response and excitation), it is possible for a crack of a certain length at one location to cause a similar shift in natural frequencies and or DFRF amplitudes than would be caused by another crack of different length at a different location.[18] Using DMT as a condition monitoring tool thus points out future problem areas due for maintenance, but not necessarily uniquely so. For example: in practice it is thus possible to have an accelerometer positioned directly next to a growing crack, while the data from this accelerometer would not necessarily indicate that it is positioned in a high strain area. In contrast to this, a strain gauge situated at such a position and the SFRFs associated with its response will directly and uniquely show up the problem area and frequencies. The strain responses here also show the severity of failure.[19] Using SMT as a condition-monitoring tool therefore simultaneously provides information in four parts:

- identification of dynamic characteristics of the system under consideration,
- detection of possible failures in progress,
- unique detection of possible failure location, and
- indication of possible failure severity.

Having pointed out some major advantages of SMT employment, we now have a look at some of the facts that count to the detriment of this version of MT.

Drawbacks of SMT

(1) Requirements for successful dynamic strain response measurement are strict.[1,20] Some of the most noticeable of these are:

(a) bonding quality is of utmost importance (with bonding repeatability being a possible problem,[21] because each strain gauge bond is unique) seeing that phase delay and amplitude loss can be induced by inadequate bonding.

(b) the leads supplying the strain signal to the strain measurement bridge are a potential source of error as they are liable to introduce some resistive and capacitive effects in measurements. Calibration of an applied set of strain gauges should therefore be performed for a certain set of leads in use, seeing that each strain gauge is installed separately. In the case of DMT equipment, transducers are pre-calibrated and thus do not require the calibration of each individual measurement point.

(c) strain gauge measurement bridges or configurations act quite successfully as antennae in 'mains pickup' and therefore care should be taken in carefully grounding them. In comparison, DMT equipment is usually supplied as shielded units.

(d) deformation amplitudes tend lower as response frequency is increased and therefore the dynamic sensitivity of strain gauges at high frequency can become a problem.[1,5] Although successful SFRF measurements of up to 2 000 Hz using normal strain gauges have been made,[5] the success seems to depend on the dynamic properties of the structure so that some authors[1] have had to make use of semiconductor strain gauges to ensure success of SFRF measurements of up to 400 Hz. These strain gauges in turn carry inherent disadvantages, but it should be sufficient to keep in mind that one has to be careful of blindly using SMT in high frequency applications.

(e) amplitude and phase variations in SFRF measurements have been shown to occur. Typical values are 5 dB in amplitude drop with a linear phase drop of 115 degrees at 4 kHz[20] in one case and 8.3 dB in amplitude gain with a linear phase drop of 81.8 degrees at 512 Hz.[14] These effects thus have to be compensated for if testing takes place in frequency regions where meaningful variation from the true phase and/or amplitude values exists.

The use of SMT thus carries with it the disadvantage of requiring a more careful approach to response measurement than would traditionally be needed in DMT.

(2) Because of the phase delays and amplitude drops mentioned above, the calibration of strain gauge systems for use in SMT is much more difficult than that required by the DMT measurement system.[22] This situation in turn implies difficulty in detecting calibration errors.[23] A calibration set-up similar to that shown in Figure 2[14] is needed for proper dynamic strain gauge calibration. All information specified in the figure are the values used by the authors and are sufficient for calibration to 500Hz.

(3) Pending the discussion under points (1) and (2) directly above, it can be seen that the SMT measurement system's characteristics are less favourable than those used in DMT. This fact has led to the conclusion[1] that, although modal parameters can be extracted through combined SMT and DMT usage, SMT usage results in a set of less accurate modal parameters than when DMT is called upon to perform the same task. This is true as long as the analyst does not make use of all the advantages mentioned under

paragraph (7) of the benefits of SMT. The 'good' coherence of SFRFs around the resonance peaks[1] possibly leading to more accurate modal constants should be kept in mind. It would thus seem that DMT is a more valuable tool in mathematical model development than SMT.

(4) Only a prescribed two dimensional strain state can be measured on the surface of the test structure in SMT and transformation from the two-dimensional state to a global three-dimensional axis system is not possible. The orientation of the plane of two dimensional strain measurement is furthermore prescribed by the surface of the structure. The two dimensional effect seldom influences DFRF measurements to the same extent, seeing that displacement, velocity or acceleration in many practical applications can be assumed constant through the depth of, for instance, a beam or plate and the axis orientation can easily be modified in many instances by using mounting blocks for DMT response measurement equipment.

(5) When identifying displacement mode shapes through the use of SMT data, the use of excitation other than hammer impact in SMT can lead to a time-consuming exercise of moving the exciter from one excitation point to the next, seeing that extracting these shapes in SMT requires one to keep the response point fixed (see paragraph (2)). One should also take note of the fact that not all structures can be successfully excited by means of impact. It would thus seem that displacement mode shapes sometimes cannot be extracted from SMT data without experiencing difficulty through tedious measurement requirements.

(6) In some structures the stress and strain concentrations (like notches) occur locally, whereas strains in the largest part of the structure may be low (for example in a large steel truss structure) and would therefore be difficult to measure accurately. A possible problem might arise here: the local high strain occurrence could pass by undetected if no strain gauge is situated close to the stress concentration. To circumvent this, either prior knowledge of high stress concentration locations or close spacing of strain gauges is needed. However, the experience of the experimentalist in vibration problem solution is normally such that the locations of high strains are known. Strain gauges therefore need to be closely spaced only in locations where detailed information is sought. Such close spacing or repetitive MT could turn out to be more expensive than the use of DMT equipment due to the 'use once only' property of strain gauges.

(7) While experiencing the improved inverse force determination ability of SFRFs (see paragraph (4) of benefits of SMT), it is at the same time experienced that SFRFs are, in the loading cases of couples and point applied forces on beams in bending, less well-suited to response prediction than are DFRFs,[5] due to the quicker convergence of the modal series of the DFRF. For uniformly distributed loads (loads that are widely experienced in practice), the referenced authors expected the two MT methods' ability to predict response to be equivalent.

Conclusion

The SMT user could benefit significantly from its wise employment. It was seen that the use of SMT provides the opportunity to combine fatigue analysis with the extraction of dynamic characteristics of an existing component, to develop an optimised fatigue design procedure when combining SMT and SMOD, to directly indicate the physical locations and frequencies of problematic or peak dynamic strains in an existing structure and to 'streamline' the MT process. SMT results also need less handling before solutions are at hand. It is thus clear that SMT provides a method of making MT more solution oriented[3] and that it provides one with valuable information when used in conjunction with DMT.

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