A Mathematical Model of the Brake Shoe and the Brake Path System

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In this paper an improved mathematical model of the brake shoe – path system for post type drum brakes is presented. The model enables calculation of such brake properties as the brake factor, the pressure distribution between the brake shoe and path and the braking torque. The model comprises elastic properties of the brake elements and the preliminary geometry of contact between them. These properties are not taken into account in presently used methods of calculation.

Nomenclature

E	modulus of elasticity
I	moment of inertia
x	distance
P	applied force normal to shoe beam
p	pressure
H	component normal to P
L	length of shoe beam
M	bending moment
N	normal resultant of contact pressure
t	radial strain, thickness
k	rigidity of foundation sector
$k^{\prime}$	coefficient of elasticity
b	width of lining
w	shoe displacement
z	rim radial deformation
R	radius
T	friction force
S	shoe factor
μ	coefficient of friction
Δ	radial clearance
θ	angle
ψ	apparent radius of friction

Subscripts

s	shoe
r	rim
h, k	terminal sections of active part of shoe
ℓ	lining
w	wooden block

Introduction

The brake shoes and path systems are the only source of the controllable friction torque in mine hoists and as such determine the effectiveness and the dependability of the braking process as well as the safety of the whole winding installation. Observations on full-scale post type drum brakes have revealed several cases of deviations from the calculated value of the braking torque [1]. The divergence between actual and nominal (theoretically predicted) values of braking torque is caused by deviations between the actual and the nominal geometry of contact between the brake shoes and the rim, and the discrepancies between the actual and the used values of the coefficient of friction and applied forces. In particular the deviation from the theoretical sinusoidal pressure distribution on the friction lining is caused by considerable elastic strains of all elements in the braking system and errors in the dimensions of these elements as well as errors in their assembly. All these phenomena are ignored by presently used methods in friction brake calculations. The presently used methods are based on the controversial assumption that the brake shoes and drums are perfectly rigid. In this paper an alternative brake design method is suggested. As an example a mathematical model of the anchored type of post drum is presented. The model includes the elastic properties of the brake shoe-path system and the preliminary geometry of contact between them.

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Assumptions

It is assumed that each brake shoe-rim system is isolated from the other brake shoe interaction and any other static or dynamic influence emanating from the winding gear. The physical model of the system (figure 1) is based on the following assumptions:

1. The shoe beam is a weakly-curved bar with changing ridigity EI_s (x). The bottom end of the bar is privoted with one rotational degree of freedom. The bending moment in the bar is derived from a constant force P.
2. The other resolved component of the active force normal to P is H and can be omitted as it only induces 0 to 4% of the total

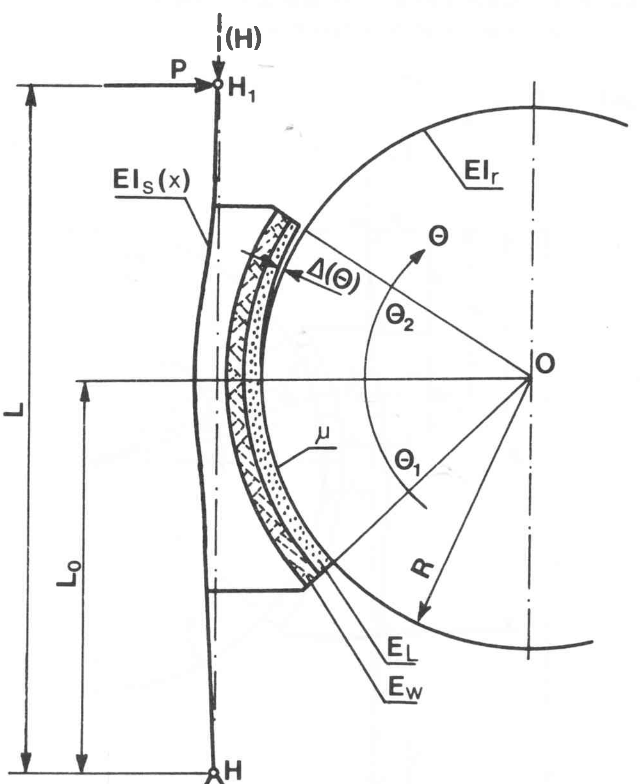


Figure 1 – Representation of the physical model of the brake shoe – path system

bending moment in the shoe beam depending on the position on the shoe beam.
3. The friction lining and the wooden lining make linear elastic foundations.
4. The brake rim is deformable in a radial direction and has a constant rigidity EI_r.
5. The radial clearance $\Delta(\theta)$ characterises the geometry of contact between the brake shoe and rim in the unloaded state, viz $P \rightarrow O$.
6. The pressure between the brake shoe and the rim is constant along the width of the lining.

Forces and strains in the brake system

Reactions in the elastic foundation

The problem of the geometry of contact between the brake shoes and the rim is solved by an approximate method. Therefore, the length of the shoe beam L is divided into n equal sections each of length $\Delta x = \frac{L}{n}$. The sections are designated by

$$x_i = i\Delta x$$

where:

$$i = 0, 1, 2, 3, \ldots n$$

In every section

$$x_h > x_i > x_k$$

where:

x_h, x_k – the terminal sections of the active part of the shoe, have corresponding sector i of the elastic foundation with central angle $\delta\theta_i$ and angular co-ordinate θ_i (figure 2).

An active force P causes a pressure on the surface of contact between the brake shoe and the rim. Consider that force N_i is the normal resultant of contact pressure on the surface of the sector i then $N_i = k_i t_i$

where: t_i – radial strain of the elastic foundation,
k_i – rigidity of foundation sector i.

and
$$k_i = \frac{b.R\delta\theta_i}{\frac{t_\ell}{E_\ell} + \frac{t_w}{E_w}} \qquad (1)$$

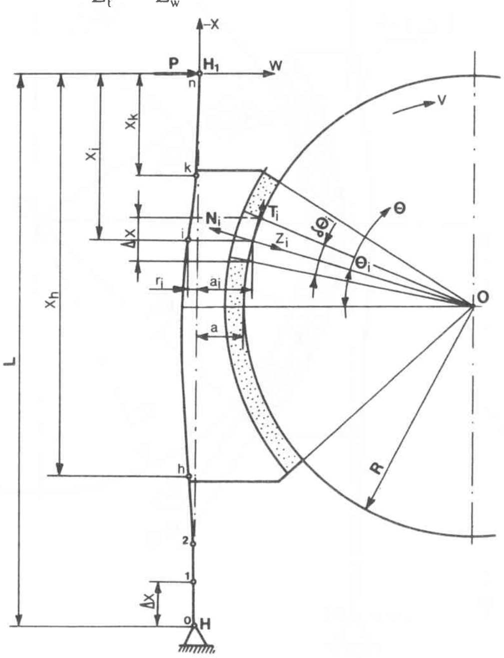


Figure 2 – Diagram of forces and displacements on the brake shoe sector i

where: b – the width of the lining
t_ℓ – thickness of the lining
t_w – thickness of the wooden block
E_ℓ – modulus of elasticity of the lining
E_w – modulus of elasticity of the wood

The radial strain of the elastic foundation t_i is a function of the shoe displacement w_i, the rim radial deformation z_i and the clearance Δ_i between the unloaded brake shoe and rim.

Hence: $$t_i = w_i \cos\theta_i - z_i - \Delta_i \qquad (2)$$

The resultant friction force T_i for the sector is

$$T_i = \vec{\mu}\, N_i \qquad (3)$$

where:

$$\vec{\mu} = \begin{cases} \mu \text{ – for the leading shoe} \\ -\mu \text{ – for the trailing shoe} \end{cases}$$

The brake shoe displacements

For a weakly-curved neutral axis of the shoe and for small displacements (as in practice the point H_1 never moves more than $\frac{1}{20}$ of the shoe length) the following differential equation (4) can be applied to shoe displacement calculations:

$$EI_{si} \frac{\Delta^2 w_i}{\Delta x^2} = M_i \qquad (4)$$

where: $\Delta^2 w_i = w_{i-1} - 2w_i + w_{i+1}$ (5)
EI_{si} – the rigidity of the shoe at point i
M_i – the bending moment at point i
w_i – the shoe displacement at point i

After substituting equation (5) the following equation may be obtained

$$\varepsilon_i = \frac{EI_{si}}{\Delta x^3} \qquad (6)$$

the differential equation takes the form:

$$\varepsilon_i (w_{i-1} - 2w_i + w_{i+1}) = \frac{M_i}{\Delta x} \qquad (7)$$

For the shoe point of support, viz. for $x_0 = 0$ displacement $w_0 = 0$. Bending moment in shoe point i (figure 2) can be calculated from equation (8):

$$M_i = (n-1)\Delta x P - \Delta x \sum_{\substack{j=i \\ j \geq h}}^{k} (j-i)(\cos\theta_j - \vec{\mu}\sin\theta_j)N_j$$

$$- \sum_{\substack{j=i \\ j \geq h}}^{k} (\sin\theta_j + \vec{\mu}\cos\theta_j)(r_i + a_j)N_j \qquad (8)$$

where: $a_j = a + R(1 - \cos\theta_j)$

After substituting (2) equation (8) takes the form:

$$M_i = (n-1)\Delta x P - \Delta x \sum_{\substack{j=i \\ j \geq h}}^{k} A_{ij} w_i - \Delta x \sum_{\substack{j=i \\ j \geq h}}^{k} B_{ij}(z_j + \Delta_j) \qquad (9)$$

where:

$$A_{ij} = B_{ij}\cos\theta_j \qquad (10)$$

$$B_{ij}=\left[\frac{r_i+a_j}{\Delta x}(\sin\theta_j+\vec{\mu}\cos\theta_j)+(j-i)(\cos\theta_j-\vec{\mu}\sin\theta_j)\right]k_j \qquad (11)$$

If (9) is substituted into (7) the following final differential equation for the brake shoe displacement is obtained:

$$\varepsilon_i(w_{i-1}-2w_i+w_{i+1})+\sum_{\substack{j=i\\ j\geq h}}^{k}A_{ij}w_j+\sum_{\substack{j=i\\ j\geq h}}^{k}B_{ij}z_j=(n-i)P-\sum_{\substack{j=i\\ j\geq h}}^{k}B_{ij}\Delta_j \qquad (12)$$

The brake path displacements

For modelling the brake rim deformation a circular ring with constant cross-section, compressed by radial concentrated forces N_0 distributed symmetrically (figure 3) is assumed. The radial displacement z of the rim in the section $\upsilon - \upsilon$ is expressed by equation (13) which is obtained from the one half-ring calculation using Castigliano principle.

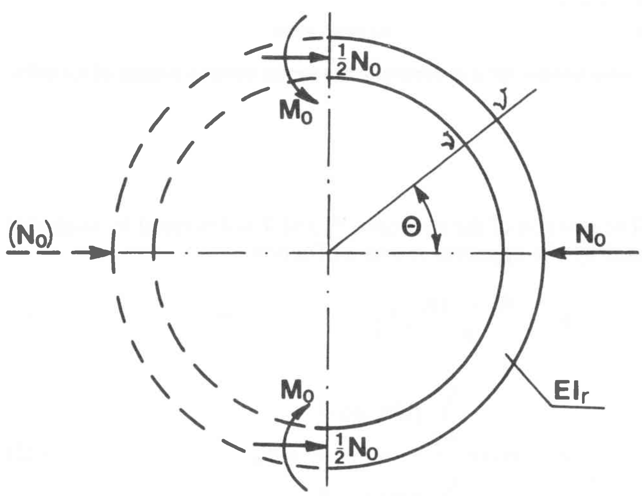


Figure 3 – Simplified model of the brake path

$$z = \frac{R^3}{EI_r}N_0\left[\left(\frac{\pi}{8}-\frac{1}{4}|\theta|\right)\cos\theta+\frac{1}{4}\sin|\theta|-\frac{1}{\pi}\right] \qquad (13)$$

Generalising let us assume that the rim displacement at the point i (described by angular co-ordinate θ_i) resulting from a concentrated force N_j applied to the point j is:

$$z_i = c_{ij}N_j \qquad (14)$$

where:

$$c_{ij}=\frac{R^3}{EI_r}\left[\left(\frac{\pi}{8}-\frac{1}{4}|\theta_i-\theta_j|\right)\cos(\theta_i-\theta_j)+\frac{1}{4}\sin|\theta_i-\theta_j|-\frac{1}{\pi}\right] \qquad (15)$$

is the rim displacement at the point i as a result of a unit force acting at the point j, if the condition $|\theta_i - \theta_j| \leq \frac{\pi}{2}$ is satisfied.

Using superposition principles the rim displacement at point i from the contact pressure resultants N_j on the separate sectors of the lining can be expressed as

$$z_i = \sum_{j=h}^{k}c_{ij}N_j \qquad (16)$$

After substitution of (2)

$$z_i = \sum_{j=h}^{k}F_{ij}w_j - \sum_{j=h}^{k}G_{ij}z_j - \sum_{j=h}^{k}G_{ij}\Delta_j \qquad (17)$$

where:

$$F_{ij} = c_{ij}k_j\cos\theta_j \qquad (18)$$

$$G_{ij} = c_{ij}k_j$$

The formation and solution of the matrix equation

The matrix equation for the brake shoes and the brake path system were formed utilising n − 1 equation type (12) and k − h + 1 equations type (17) as well as the equation of moment equilibrium for the shoe about its point of rotation:

$$M_0 = n\Delta xP - \Delta x\sum_{j=h}^{k}A_{0j}w_j - \Delta x\sum_{j=h}^{k}B_{0j}(z_j+\Delta_j) = 0 \qquad (19)$$

The values of the shoe and the path deformations at the respective sector borders are the solution of the matrix.

As the continuity of the contact between the shoe and the path can be disturbed the solution of the matrix equation is put on revision. The radial strains of the foundation are calculated from equation (2) using obtained values of the shoe and the rim deformation. If for any point $x_h \geq x_i \geq x_k$ the value of the foundation strain $t_i < 0$, repeated computation of the system is done with the coefficient of elasticity $k_i^* = 0$ for these points. Since the solution must also satisfy the inverse condition, the computation is completed when the following relations are met:

$$ti\begin{bmatrix}\geqq 0 \Leftrightarrow k_i^* = k_i\\ < 0 \Leftrightarrow k_i^* = 0\end{bmatrix} \qquad (20)$$

The above conditions exclude a solution with negative elasticity, which does not appear in this case.

Results and Conclusions

Computation of the matrix equations has been carried out in FORTRAN. The obtained values of the brake shoes and the brake path displacements were used for the calculation of the following properties describing a brake system as a kinematic pair of friction.

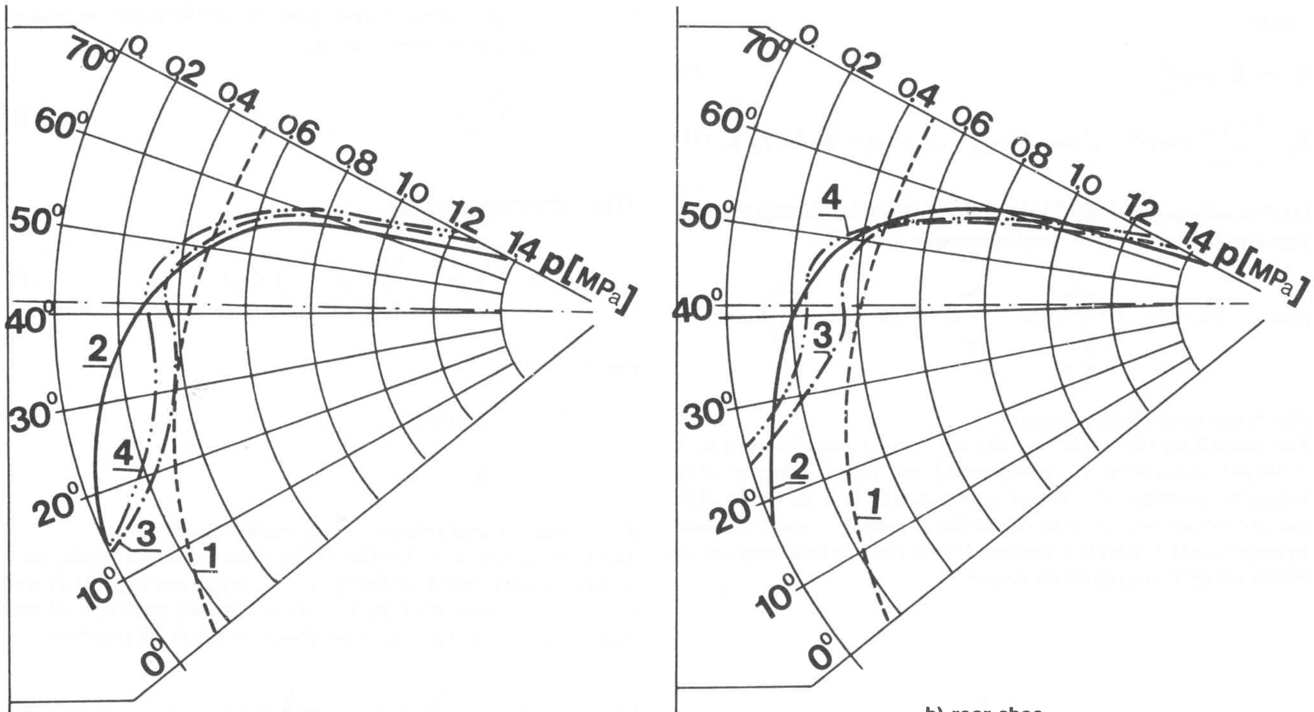
Pressure distribution

For the respective sector i of the lining with the angle co-ordinate θi when the radial strain $t_i > 0$ the pressure may be calculated from

$$p_i = \frac{k_i}{bR\delta\theta_i}(w_i\cos\theta_i - z_i - \Delta_i) \qquad (21)$$

if $t_i \leqq 0$ then $p_i = 0$

Position of the resultant of the normal forces N and the friction forces T.



a) front shoe b) rear shoe

Figure 4 – Pressure distribution between shoes and path in the 4L-4000 mine winder for a pressure in the brake service engine of 0,4 MPa
1 – theoretical sinusoidal distribution
2 – results obtained by measurements
3 – calculated values for the leading shoe
4 – calculated values for the trailing shoe

The position of the resultants N and T is described by angle θ[1] and by the apparent radius of friction

$$\psi = \frac{R + \Delta R}{R}\left[1\right].$$

$$\theta = \operatorname{arctg}\frac{\sum_{i=h}^{k} p_i\delta\theta_i \sin\theta_i}{\sum_{i=h}^{k} p_i\delta\theta_i \cos\theta_i} + \theta_0 \tag{22}$$

$$N = \frac{bR}{\cos(\theta - \theta_0)}\sum_{i=h}^{k} p_i\delta\theta_i \cos\theta_i \tag{23}$$

The apparent radius of friction ψ may be calculated from the braking torques equilibrium:

$$\mu N\psi R = \sum_{i=h}^{k} \mu(bRp_i\delta\theta_i)\, R \tag{24}$$

then

$$\psi = \frac{bR\sum_{i=h}^{k} p_i\delta\theta_i}{N} \tag{25}$$

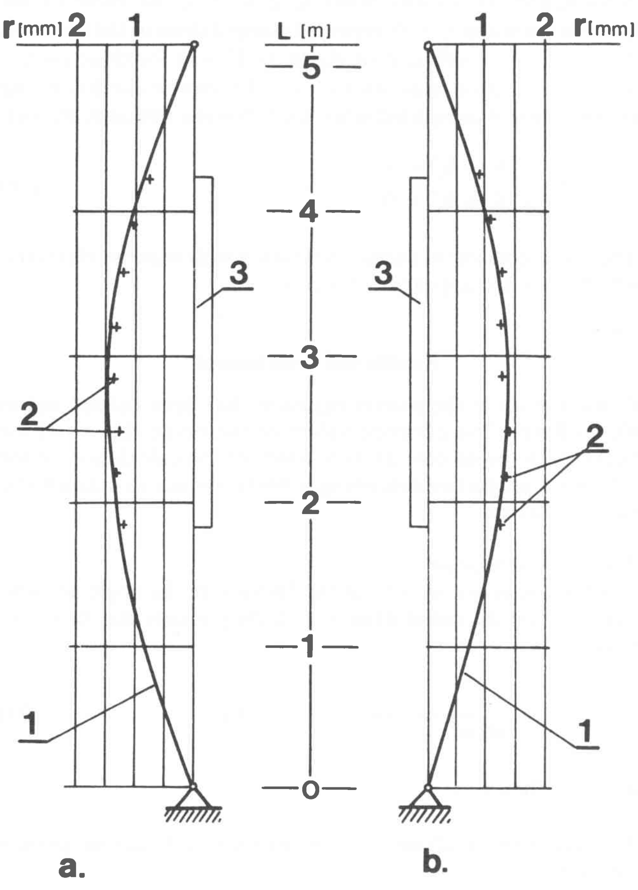
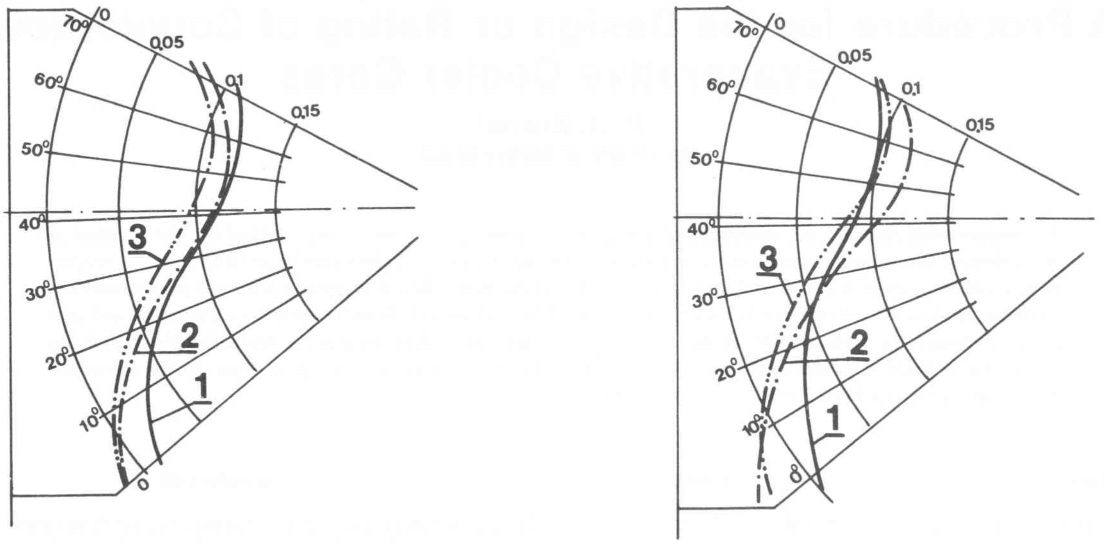


Figure 5 – Relative deformation r of the brake shoe beams in the 4L-4000 mine winder for a pressure in the brake service engine of 0,4 MPa
1 – calculated values, mean values for leading and trailing shoes
2 – results obtained from measurements
3 – lining, a) front shoe b) rear shoe



a) front shoe

b) rear shoe

Figure 6 – Brake path displacement z in the 4L-4 000 mine winder for a pressure in brake service engine of 0,4 MPa
1 – results obtained from measurements
2 – calculated values for the leading shoe
3 – calculated values for a trailing shoe

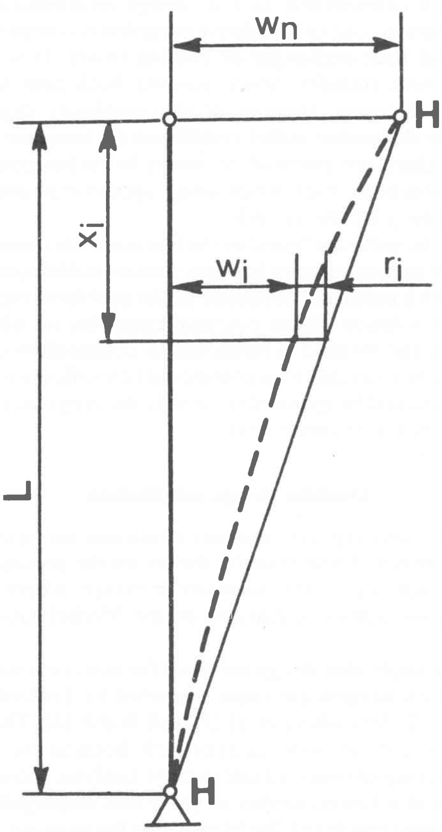


Figure 7 – Diagram for the shoe relative deformation calculation

Shoe factor

The shoe factors for both leading and trailing shoes can be calculated from:

$$S = \frac{\gamma \psi}{\delta\left(\frac{1}{\mu} \sin \theta \pm \cos \theta\right) \mp \psi} \tag{26}$$

where

$$\gamma = \frac{L}{R}$$

$$\delta = \frac{OH}{R}$$

For the selected mine hoist with the post type drum brakes the calculations and full-scale measurements of the pressure distribution and the shoe and path displacements have been conducted. The comparison between the calculated and the measured values are presented in figures 4, 5 and 6. The results show that the proposed method of calculation gives a more accurate prediction of the friction brake performance.

Therefore, the presented mathematical model of the brake shoe and path system may be used for brake design after establishing optimisation criteria.

Bibliography

1. Scieszka, S.-F. and Barecki, Z., "Geometry of Contact between Brake Shoes and Drums", *The South African Mechanical Engineer*, September 1984, pp. 324-329.