COMMENT

An Improvement to the Fluidity Approach to Non-Newtonian Flow

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Many substances exhibit non-linear rheological behaviour and some undoubtedly show a yield point, as witnessed by their not achieving a plane surface even when left for prolonged periods. Nevertheless under non-rigid body motion all the substances of interest exhibit no reference configuration of permanent significance and are therefore true fluids. In practice the measurement of yield stress for plastoviscous fluids is difficult and its value not easy to assign with any certainty, therefore a considerable latitude exists on this matter. The reported experimental value is often no more than an extrapolated intercept on the shear stress axis on a flow curve, and is therefore a fictitious yield stress.

In laminar flow through a tube, there will be a core of rigid body motion for plastoviscous fluids that exhibits a yield. Because of the uncertainty about the actual value of the yield stress there will be a corresponding uncertainty about the radius of this solid core. Hence, in performing an integration over the tube radius the expression found for the volumetric flow rate is not exact.

Thus, in a recent paper [1] a rheological fluidity model of the form

$$\dot{\gamma} = J\,\tau^{m} - \alpha \tag{1}$$

is taken and integrated over the whole tube radius to find laminar regime volumetric flow. Strictly speaking this is not an exact method because for shear stresses less than the yield stress, the rheological model is undefined.

Although the error introduced by this method is not large, it is possible to improve the treatment in the following way. This may be more conveniently done using dimensionless forms. Thus the type of plasto-fluidity model originally considered was,

$$\dot{\gamma}^{+} = \tau^{+m} - 1 \tag{2}$$

where

$$\dot{\gamma}^{+} = \dot{\gamma}/\alpha;$$
$$\tau^{+} = \tau/\tau_y \text{ and}$$
$$\tau_y = (\alpha/J)^{1/m}$$

is an apparent yield stress.

In dimensionless form, the volumetric flow rate is then given by,

$$Q^{+} = \frac{1}{\tau_w^{+3}} \int_o^{\tau_w^{+}} \tau^{+2} f(\tau^{+})\, d\tau^{+} \tag{3}$$

where

$$Q^{+} = \frac{8Q}{\pi D^3 \alpha}$$

The above two relations correspond with equations (10) and (20) in the original work.

Now consider a rheological model of the form,

$$\dot{\gamma} = \tau^{+m} - A\,\tau^{+a} \tag{4}$$

Where $a << 1$ but is otherwise undefined at the moment. The coefficient A may be chosen such as to make the integral in equation (3) vanish when the upper limit is unity. This is the case of incipient flow where the wall shear stress equals the yield shear stress. Thus,

$$\int_o^1 \tau^{+2} f(\tau^{+})\, d\tau^{+} = 0 \tag{5}$$

Substituting equation (4) into equation (5) gives,

$$A = \frac{3 + a}{3 + m} \tag{6}$$

Hence a more accurate representation of laminar pipe flow for a plastoviscous fluid is obtained with the following form that is extended to values of $\tau \geq 0$

$$\dot{\gamma}^{+} = \tau^{+m} - \left(\frac{3 + a}{3 + m}\right) \tau^{+a} \tag{7}$$

Then the dimensionless volumetric flow rate is given by,

$$Q^{+} = \frac{1}{(m + 3)} (\tau_w^{+m} - \tau_w^{+a}) \tag{8}$$

obviously, $Q^{+} \to 0$ as $\tau_w^{+} \to 1$, which is a logical condition. Also if $a << 1$, then for $\tau_w^{+} >> 1$

$$Q^{+} \to \frac{\tau_w^{+m}}{m + 3} \tag{9}$$

From equation (20) of the original paper $Q^{+} \to 0$ as

$$\tau_w^{+} \to \left(\frac{m + 3}{3}\right)^{\frac{1}{m}} \tag{10}$$

which is not exact. But the upper limit is the same as equation (9) above, which is satisfactory. If in equation (8), $a << 1$ then,

$$\tau_w^{+} = [(m + 3)\,Q^{+} + 1]^{\frac{1}{m}} \tag{11}$$

and it is possible to change the subject, as in the original paper.

Equation (20) of the original paper will tend to underestimate the volumetric flow rate as $\tau_w^{+} \to 1$.

In the turbulent flow analysis section there are typographical errors in equations (42), (49) and (55), which should read as follows,

$$\Phi\,(N_{RF}^{+m}\, N_F^{+}) - \chi\,(N_{RF}^{+m}\, \xi,\, N_F^{+}) = \psi\,(\xi) \tag{42}$$

$$\psi = -A^{+} \ln \xi + B_2^{+} \tag{49}$$

$$\frac{1}{\sqrt{C_f}} = A \ln \left[\frac{N_{RF} \cdot C_f^{1-\frac{1}{2m}}}{1 + N_F/6\sqrt{C_F}} \right] + C \tag{55}$$

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Reference

1. Harris, J., "A Fluidity Approach to Non-Newtonian Laminar and Turbulent Flow", *R and D Journal*, Vol. 2, No. 1, March 1986, p. 6.