On the dynamic excitation of large Industrial Centrifugal Fan Impellers

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When an impeller of a large centrifugal fan is rotating through a pressure field, dynamic excitation of certain impeller modes can take place in a resonance-like fashion. This paper discusses the conditions required for such an excitation to take place and the dynamic amplification factors thereof. Excitation due to aerodynamic and instability effects were not considered.

Nomenclature

(C)	Damping matrix
F	Forcing function
f	Frequency
(K)	Stiffness matrix
K	Fourier component number
(M)	Mass matrix
M	Number of modes considered
m_r	Modal mass
n	Number of blades
N	Number of complete revolutions after which response is sought
r	Mode number
t	Time
P	Number of impulses
Q	Response factor
u_i	Deflection response at point i
α	Portion of time τ
ξ	Percentage of critical damping
θ	Angular
ψ_{ir}	Modal deflection at point i (mode r)
τ	Time between impulses
ω	Circular frequency

Introduction

There have been numerous cases of centrifugal fans, operated by utilities, developing cracks in their impellers during operation. In some instances, the cracks have resulted from high cycle fatigue conditions although the corresponding dynamic stress levels were below 30 MPa peak to peak. This situation implies that the static stress levels in the fans are so high that only relatively small dynamic excitation is required to generate and/or propagate a crack.

In view of these observations it is clear that the dynamic characteristic of the fan impeller must be carefully considered so as to minimise any possibility of dynamic excitation. Unfortunately a detailed mathematical model to determine dynamic stresses is too complex to be applied practically, due to the lack of knowledge of the dynamic pressure distribution. Therefore, a practical approach will be to identify potentially dangerous modes and to attempt to assess their magnification factors whenever necessary.

A designer using a finite element model analysis may find that several natural frequencies exist near blade passing frequency as well as in the region of running frequency higher harmonics. Practically, it will be almost impossible to eliminate all natural frequencies around the possible energy source frequency regions. Consequently it is a matter of deciding which natural frequencies and modes are likely to be excited and therefore considered. Essentially excitability can be established by experimental techniques should a variable speed facility be available. This is not possible with large utility fans and therefore analytical investigations were undertaken to determine when an impeller is due to be excited and the amplification factors involved.

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Excitation due to harmonic distribution pressure field

As much as it is desirable to design a fan to work under symmetrically distributed pressure fields, there will usually be some net assymmetric pressure acting perpendicular to the shroud and distributed unevenly around the impeller periphery.

Since our objective is to determine conditions for excitability, rather than developing detailed response formulation, we shall assume a pressure field acting close to the impeller periphery (where the outlets have their major impact on the shroud) and in a direction perpendicular to the shroud. Consequently the pressure forces can be defined by $F(\theta)$ where the radial distribution is being ignored.

Providing that these forces have a cyclic distribution, they can always be expressed in terms of a Fourier series:

$$F(\theta) = F_o + F_i \operatorname{Sin} \theta + \ldots + F_k \operatorname{Sin} k\,\theta \tag{1}$$

The cos terms were omitted for simplicity as they merely change the initial positioning of the pressure field in relation to the impeller modes, without affecting the generality of the discussion.

Consequently we shall consider excitation due to a general harmonic forcing term $F_k \sin k\theta$. F_o will have no consequence on dynamic excitation.

If the impeller rotational frequency is ω_o, the pressure field can be considered to rotate relative to a stationary impeller at $-\omega_o$.

Therefore a discription of the pressure forces acting on the impeller taking into consideration the rotation is

$$F_k \sin k\,(\theta + \omega_o t).$$

This expression represents a wave function whereby the pressure field is rotating in the opposite direction to that of the fan.

Knowing the forcing function, the dynamic equation of the fan can be written as:

$$(M)\ddot{u}(\theta,t) + (C)\dot{u}(\theta,t) + (K)u(\theta,t) = F^{(K)}(\theta,t) \tag{2}$$

and using the modal superposition method, a solution can be obtained by applying the modal generalised force to each associated mode and then combining the results to arrive at a solution for a pressure component F_k [1].

$$u^{(K)}(\theta,t) = \sum_{r=1}^{n} \frac{P_r^{(K)}(t)\ \psi_r(\theta)}{m_r[\omega_r^2 + j2\omega\omega_r\zeta_r - \omega^2]} \tag{3}$$

Where:

u – deflection function
(M) (K) (C) – mass, stifness and damping matrices of the impeller
ψ_r – modal function at natural frequency ω_r
m_r – modal mass
ζ_r – modal damping
P_r – modal generalised force

The modal parameters m_r, ζ_r, ω_r and ψ_r can be available from a finite element (FEM) analysis of the impeller. ζ_r has to be assumed.

It is noticed that the solution contains terms pertinent to both continous and discrete systems. Strictly speaking this is correct only when (M) (K) (C), are based on an infinite number of modes. However, the investigation in this paper is based on FEM information being available, therefore it was convenient to mix the two representations in one formulation. Since the impeller is a continuous system, the formulation and the discussion will be valid taking into consideration the accuracy of the modal mass m_r.

In order to examine the generalised forces acting on the impeller, the impeller mode shapes should be considered.

From our experience it was observed that cracks usually occur on the fan shrouds and central plate. Therefore we shall mainly consider modes involving deflection of shrouds. Being a circular plate the shroud outer diameter will have mode shape described by A sin $r\theta$ where r is sometimes referred to as the diametrical modal number.

Typical modes of double inlet forces draft fans (12 blades, 3,68 m diameter, rotation 740 (rpm) are shown in figures 1 to 5 in the form of unfolded shroud and central plate lines with the blades in between.

The generalised forces associated with these modes are:

$$P_r^{(K)} = \int_o^{2\pi} \sin r\theta \, F_K \sin K(\theta + \omega_o t) \, d\theta$$

$$= F_K[\cos k\omega_o t \int_o^{2\pi} \sin r\theta \sin K\theta \, d\theta + \sin K\omega_o t \int_o^{2\pi} \sin r\theta \cos K\theta \, d\theta]$$

$$P_r^{(K)} = \begin{cases} O & \text{for } K \neq r \\ F_K\pi \cos K\omega_o t = F_r\pi \cos r\omega_o t & \text{for } K = r \end{cases} \quad (4)$$

Evidently, a necessary condition for such an excitation to occur is that the excited mode shape should match the force field shape.

Since the generalised force exhibits harmonic behaviour, the harmonic excitation solution can easily be applied for the deflection of any point u_1:

$$u_i^{(K)} = \frac{F_r\pi\psi_{ir} \cos r\omega_o t}{m_r[\omega_r^2 + j2r\omega_o\omega_r\zeta_r - (r\omega_o)^2]} \text{ for } K = r \quad (5)$$

The maximum response will be at resonance (for small ξ) where $\omega_r = r\omega_o$.

The corresponding resonance amplification factor will be $\frac{1}{2\xi}$

Since the above discussion applies to any pressure distribution, it can equally be applied to a local pressure field acting on a small portion of the impeller circumference. In this case the fol-

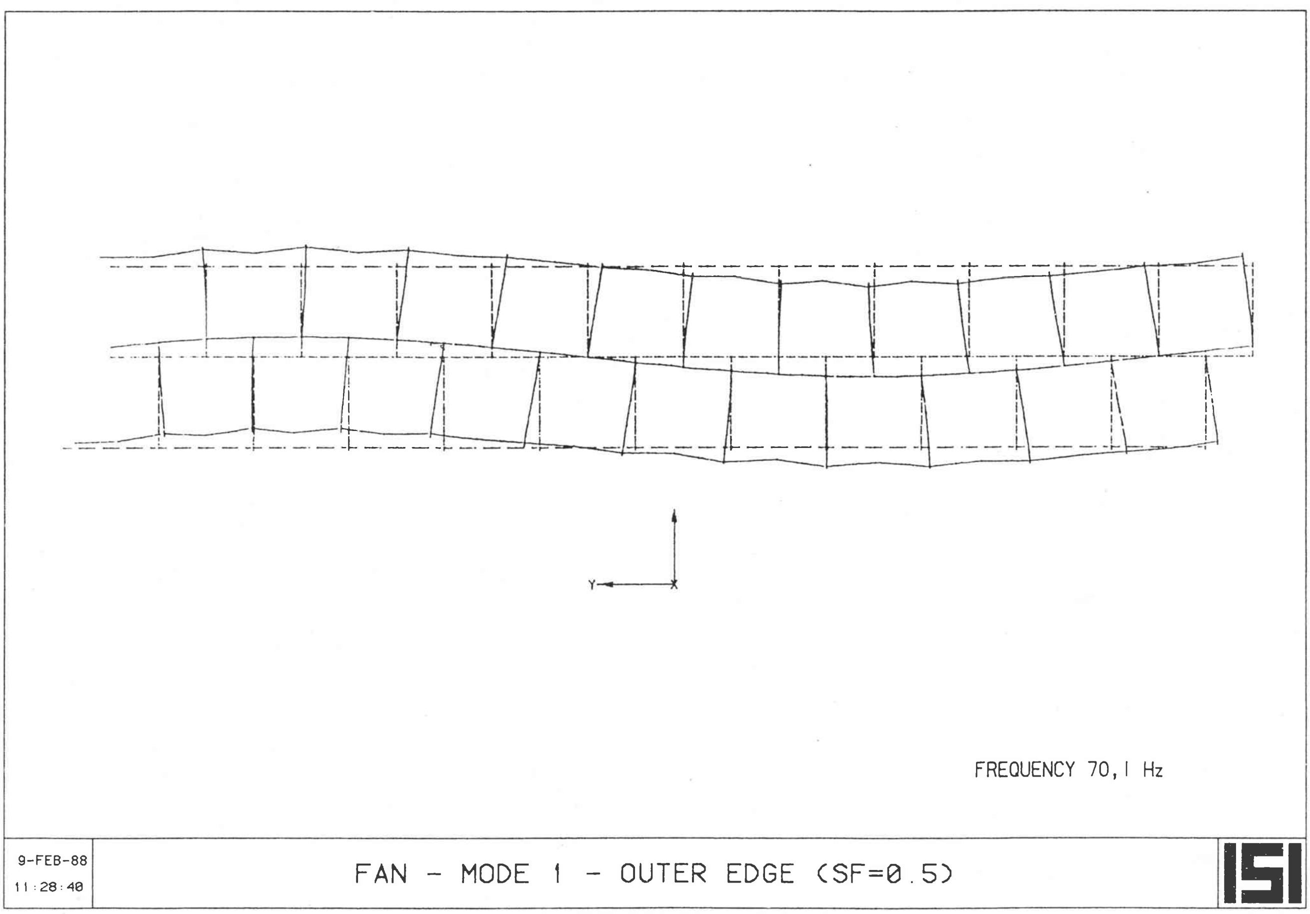

Figure 1 – Fan (Mode 1), outer edge

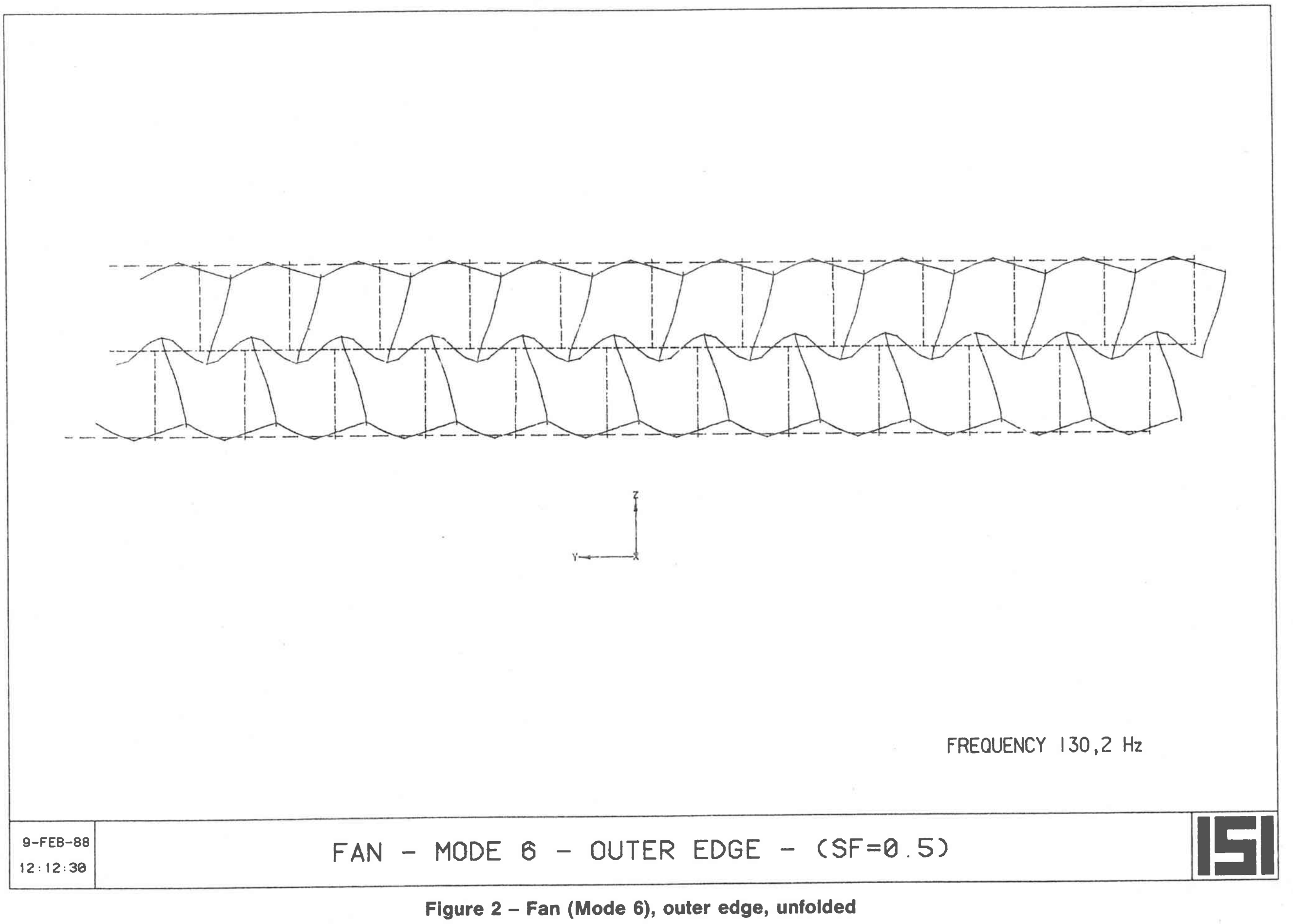

Figure 2 – Fan (Mode 6), outer edge, unfolded

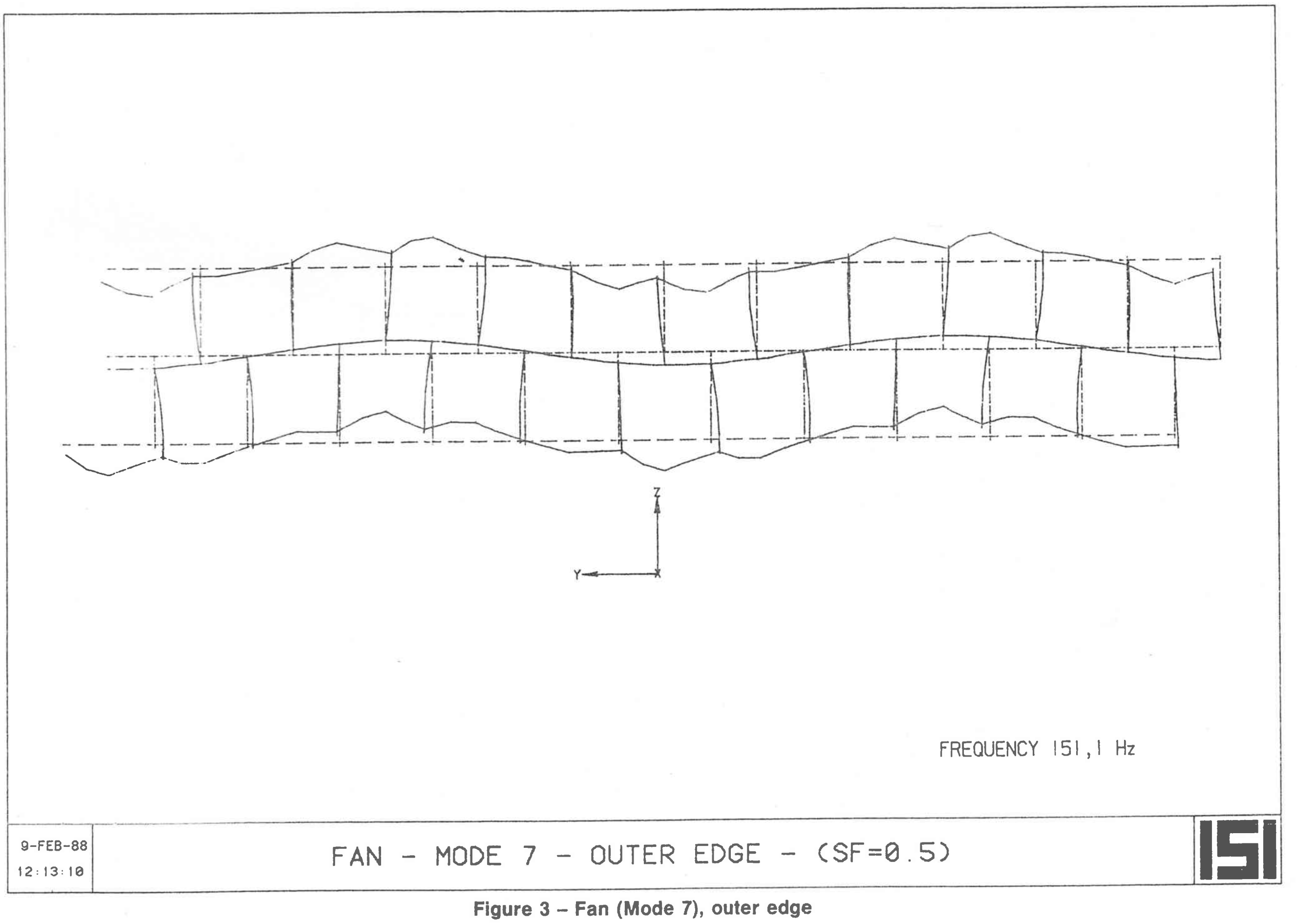

Figure 3 – Fan (Mode 7), outer edge

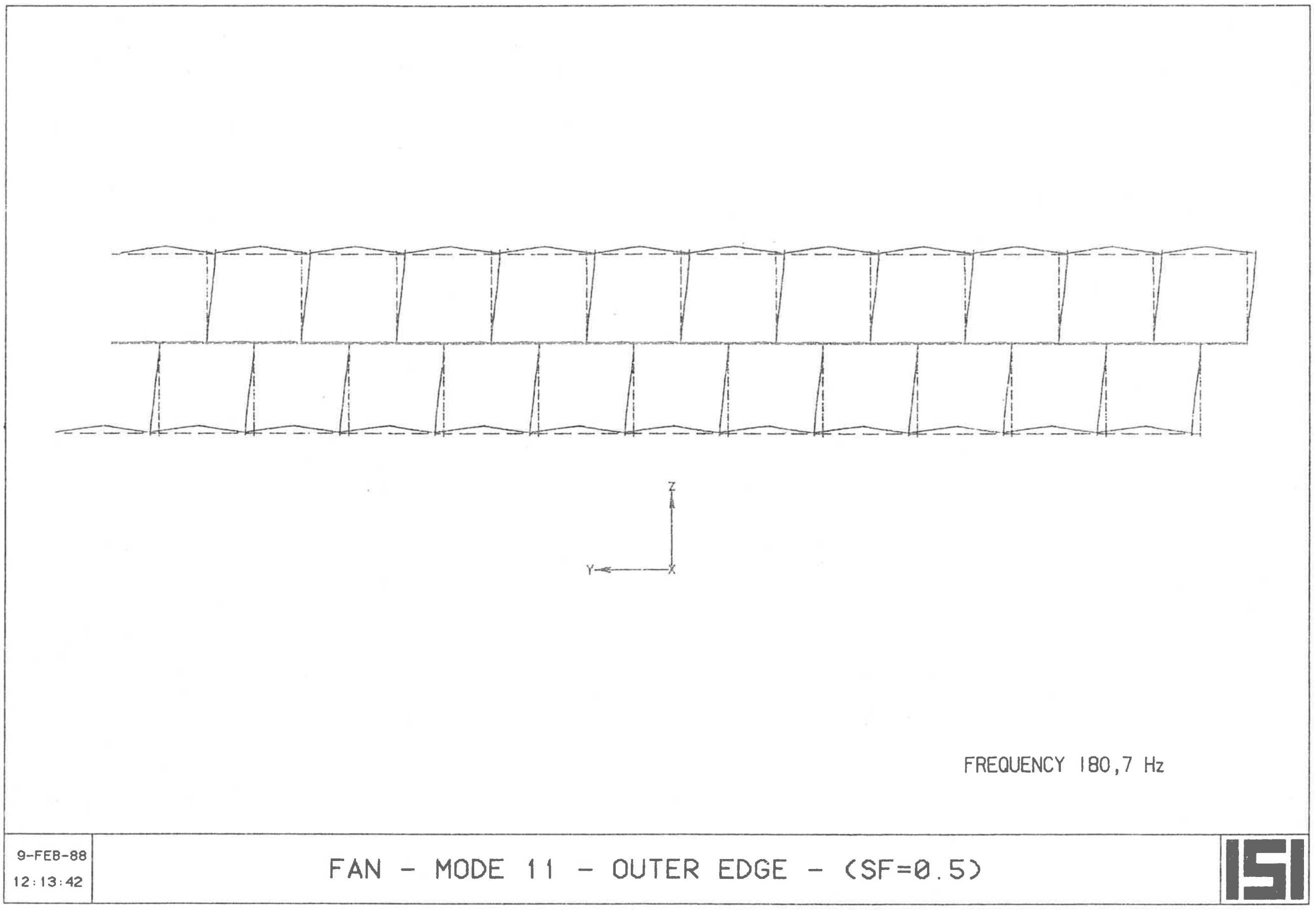

Figure 4 – Fan (Mode 11), outer edge

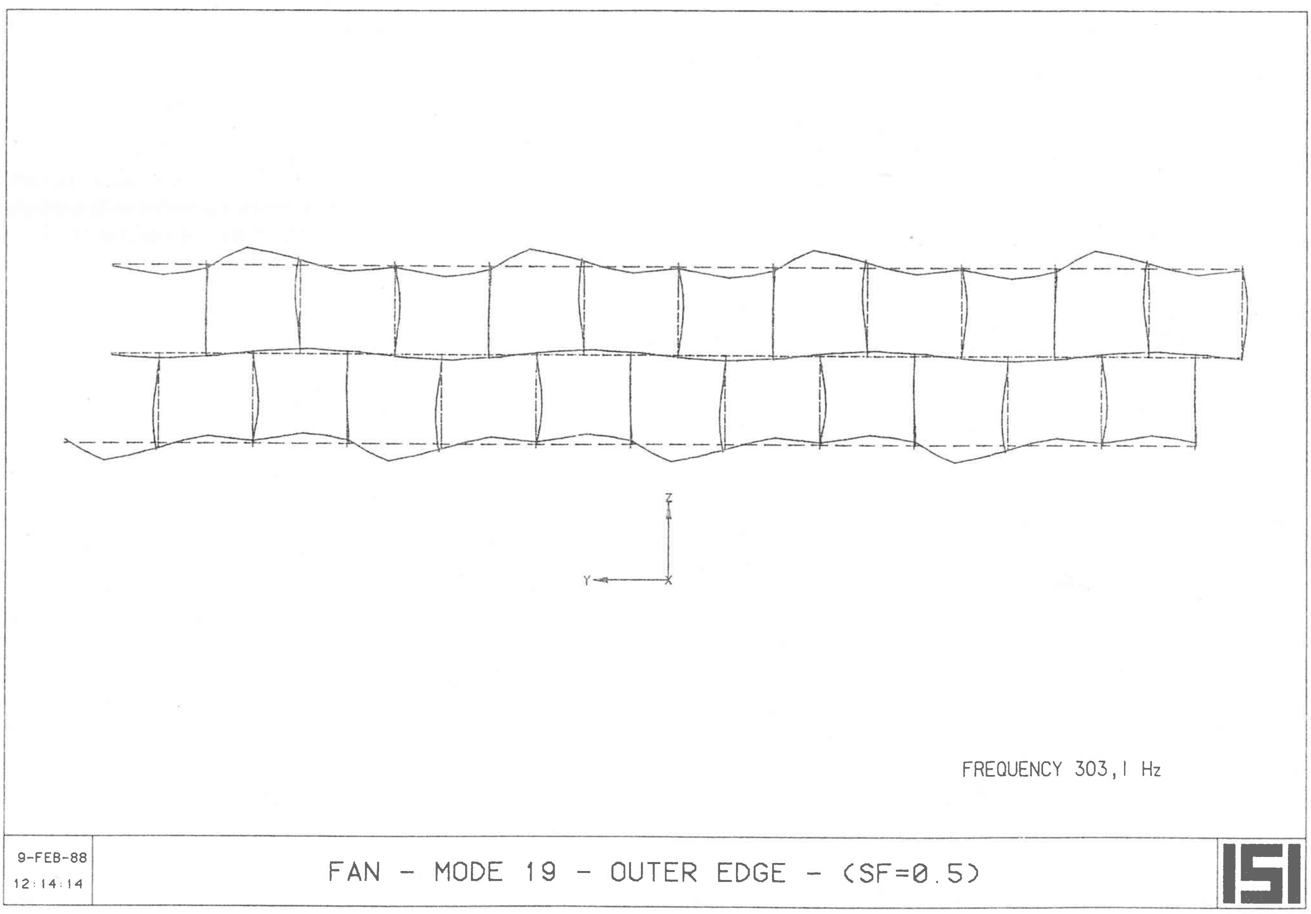

Figure 5 – Fan (Mode 19), outer edge

lowing discussion could assist in the visualisation of the excitation mechanism [2].
Consider a shroud having r diametrical modes passing through a localised force field P, figure 6-1.

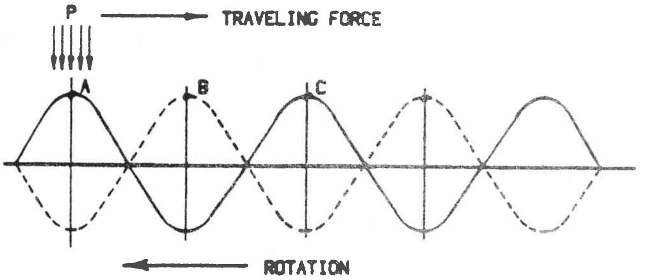

Figure 6-1 – Deflected shroud at r diametrical mode

If the rotational frequency of the fan is f_o, the time required to pass a distance AB, which represents half the distance between two modal peaks of the shroud, is $\frac{1}{2f_o r}$. While the impeller is rotating, the shroud is deflected. Therefore if force P is applied at point B when the dotted deflection has been reached, then P is ready to contribute further positive work into the system. This will increase the response in a resonance fashion.
For such a situation to occur, it is necessary that:

$$\frac{1}{2f_o r} = \frac{1}{2f_r} \qquad (6)$$

or

$$\omega_r = r\omega_o$$

as obtained earlier.

Excitation due to blade passing

Every time a blade passes over the discharge nozzle, it experiences a forcing action acting on the blade surface. In the case of an impeller having n blades, the impeller is excited by n forces with a time delay $\tau = \frac{2\pi}{n\omega_o}$ between them. For simplicity an impulse loading will be considered. However, once a detailed loading is known the discussion can always be expanded by using different excitations.

The response of an arbitrary point u_i will be summation of the impact response acting on each blade. Therefore for one revolution of the impeller we have:

$$u_i = \sum_{K=1}^{n} \sum_{r} \frac{\psi_{ir}\psi_{Kr}}{\omega_{\xi r} m_r} e^{-\xi_r \omega_r [t-(K-1)\tau]} \quad \mathrm{Sin}\ \omega_{\xi r}[t - (K - 1)\tau] \qquad (7)$$

Where

ψ_{ir} – modal deflection of the response point i
ψ_{kr} – modal deflection of the blade excitation point k
n – number of blades
$\omega_{\xi r}$ – damped neutral frequency

From the discussion that led to formula (6) it is evident that maximum response will occur when the time between impulses is that which allows a build-up of energy. Let us then consider two cases of possible mode shapes associated with the blade's deflection. These are presented in figure 6 as cases (a) and (b).

In case (a) maximum response requires:

$$\tau = \frac{1}{f_o n} = \frac{1}{f_r} \Rightarrow f_r = nf_o. \qquad (8)$$

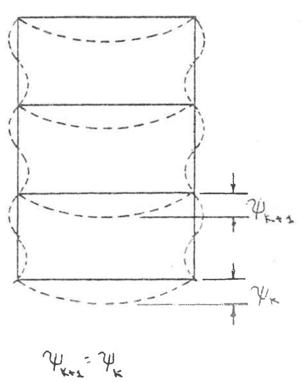

Figure 6-2 – Case (a)

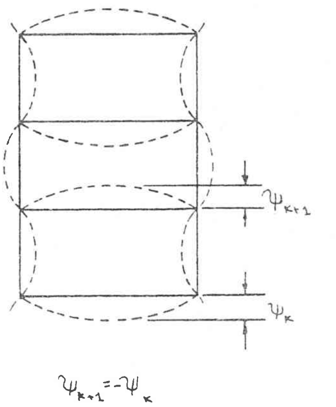

Figure 6-3 – Case (b)

In case (b) maximum response requires:

$$\tau = \frac{1}{f_o n} = \frac{1}{2f_r} \Rightarrow f_r = \tfrac{1}{2}nf_o \qquad (9)$$

Practically it implies that only a limited number of modes which exhibit similar behaviour should be considered in formula (7).

Moreover, substituting (8) and (9) into (7) for the two cases respectively assuming $\omega_r = \omega_d$ we get:

For case (a) $\psi_k = \psi_{k+1}$; $\tau = \frac{2\pi}{\omega_r}$ we get:

$$u_{ir} = \sum_{k=1}^{n} \frac{\psi_{ir}\psi_{kr}}{\omega_r m_r} e^{-\xi_r \omega_r t}.e^{\xi_r 2\pi(k-1)} \sin[\omega_r t - 2\pi(k - 1)]$$

$$= \frac{\psi_{ir}\psi_{kr}}{\omega_r m_r} \sum_{k=1}^{n} e^{\xi_r 2\pi(k-1)}.e^{-\xi_r \omega_r t}\ \mathrm{Sin}\ \omega_r t \qquad (10)$$

For case (b) $\psi_k = -\psi_{k+1}$; $\tau = \frac{\pi}{\omega_r}$ we get:

$$u_{ir} = \sum_{k=1}^{n} \frac{\psi_{ir}\psi_{kr}}{\omega_r m_r} e^{-\xi_r \omega_r t}.e^{\xi_r \pi(k-1)}\ \mathrm{Sin}[\omega_r t - \pi(k - 1)]$$

$$= \frac{\psi_{ir}\psi_{kr}}{\omega_r m_r} \sum_{k=1}^{n} e^{\xi_r \pi(k-1)}.e^{-\xi_r \omega_r t}\ \mathrm{Sin}\ \omega_r t \qquad (11)$$

It is evident that in both cases the solution is a summation of sinusoidals having the same frequency. This summation can repeat itself with the increase in the number of revolutions and must eventually reach a maximum value due to the existence of damping. Performing the summation over an unlimited number of impulses p we get for case (a):

$$u_{ir} = \frac{\psi_{ir}\psi_{kr}}{\omega_r m_r} \mathrm{Sin}\ \omega_r t \sum_{k=1}^{p} e^{2\xi_r \pi(k-1) - \xi_r \omega_r t}$$

If we write

$$t = t^* + (p - 1)\tau = t^* + (p - 1)\frac{2\pi}{\omega_r}$$

where t* is the time measured immediately after p impulses and further denote the summation as a response factor Q

then:

$$Q = \sum_{k=1}^{p} e^{2\xi_r \pi(k-1)} e^{-\xi_r \omega_r [t^* + (p-1)\frac{2\pi}{\omega_r}]}$$

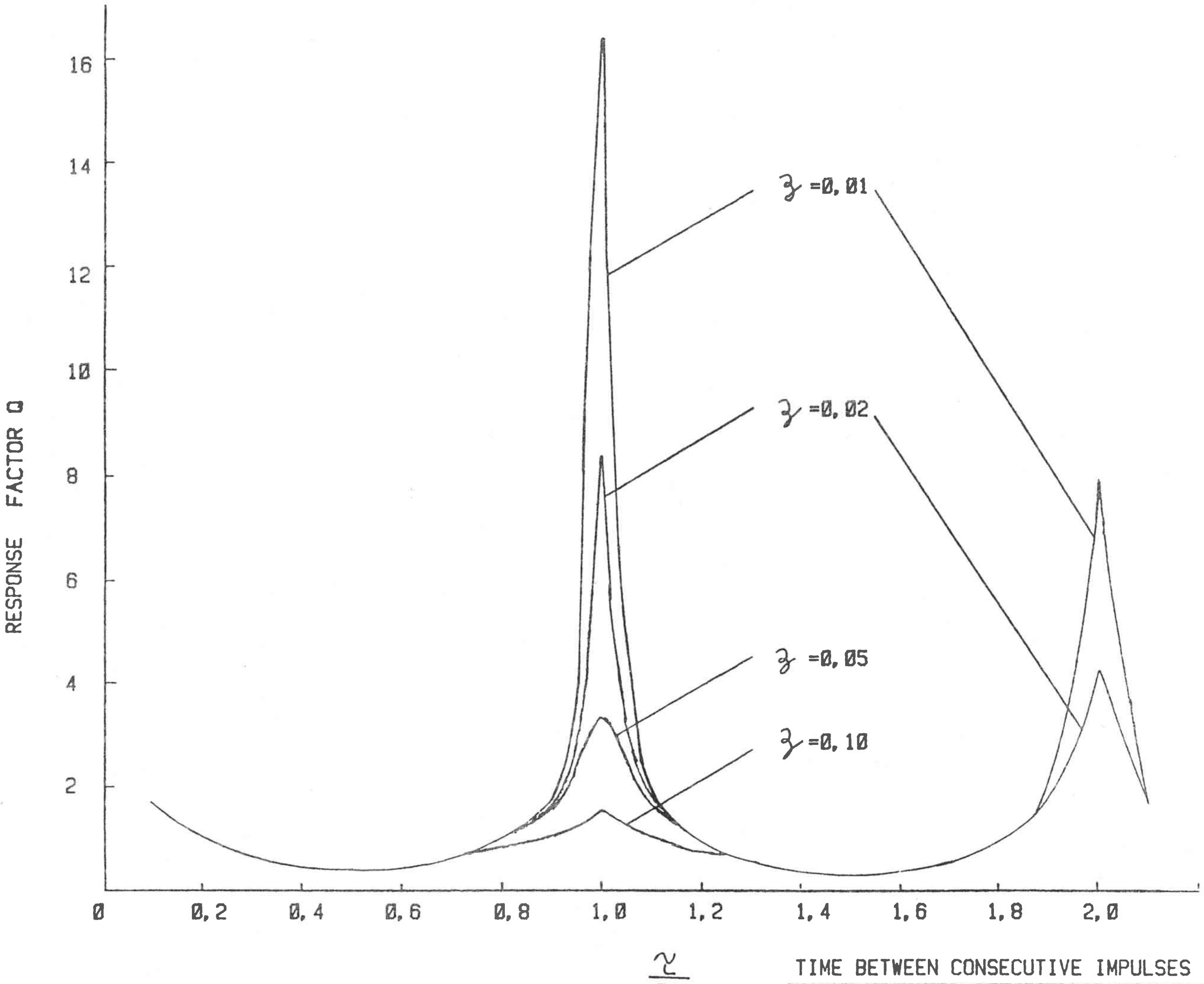

Figure 7 – Case (a)

After performing some mathematical manipulation and considering a large number of impulses, Q will reach a limiting value:

$$Q = \frac{e^{-\xi_r \omega_r t^*}}{1 - e^{-2\xi_r \pi}}$$

for $t^* = \frac{\tau}{4} = \frac{2\pi}{4\omega_r} = \frac{\pi}{2\omega_r}$ the limiting value of Q becomes:

$$Q\ max = \frac{e^{-\xi_r \frac{\pi}{2}}}{1 - e^{-2\xi_r \pi}}$$

In a similar way for case (b) we get for the limiting value of Q:

$$Q = \frac{e^{-\xi_r \omega_r t^*}}{1 - e^{-\xi_r \pi}}$$

for $t^* = \frac{\tau}{2} = \frac{\pi}{2\omega_r}$ the limiting value of Q becomes:

$$Q\ max = \frac{e^{-\xi_r \frac{\pi}{2}}}{1 - e^{-\xi_r \pi}}$$

Table 1 gives maximum values of Q for different values of damping.

TABLE 1

	RESPONSE FACTOR Q_{max}			
ξ	0,01	0,02	0,05	0,10
Case (a)	16,17	8,21	3,43	1,83
Case (b)	31,85	15,92	6,36	3,17

For values of τ different from case (a) and (b) a numerical solution can be obtained by using the response expression:

$$u_i = \sum_{r=1}^{m} \sum_{v=0}^{N} \sum_{k=1}^{n} \frac{\psi_{ir}\psi_{kr}}{m_r \omega_r} e^{-\xi_r \omega_r [t-(v.n+k-1)\tau]} \quad (12)$$

$$.\sin \omega_r[t - (V.n + k - 1)\tau]$$

provided that $t > [N.n + (n - 1)]\tau$

where:

n number of blades
N number of complete revolutions after which response is sought
M number of modes to be considered.

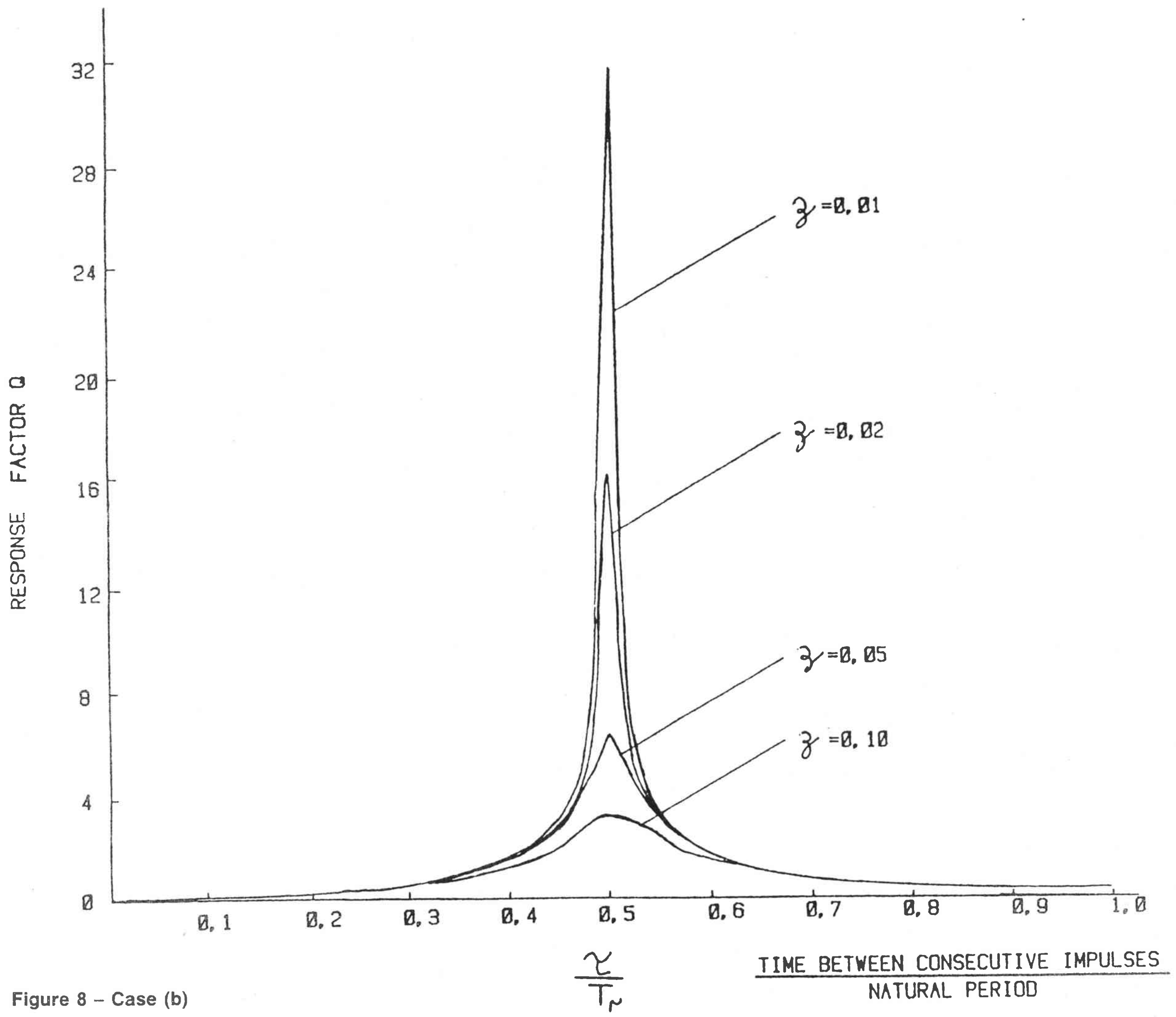

Figure 8 – Case (b)

In the previous examples we have established the maximum response in two resonance-like situations. In order to determine the resonance amplification factor for other mode cases, the maximum response has to be calculated for different relations of τ and ω_r, so that an on and off resonance situation can be compared.

To facilitate the numerical investigation the following points were taken into account:

1. The maximum response was calculated after a large number of pulses $p = N.n + n\text{-}1$
2. After a steady state has been reached, the response between the impulses must be repetitive and therefore only a portion of $t^* = \alpha\tau$ had to be considered, $0 \leq \alpha \leq 1$.

Consequently the following expression for t should be substituted in the response.

$$t = \alpha\tau + (N.n + n - 1)\tau = (N.n + n - 1 + \alpha)\tau$$

By changing τ in relation to ω_r (or the modal period T_r) and searching for maximum response by changing α between 0 to 1, the response factor Q was calculated and plotted in figure 7 and 8 for the previously described cases (a) and (b).

The results confirmed the maximum response values obtained beforehand, (table 1).

Equation (12) is therefore suggested as a tool aimed at establishing the excitability of fan impeller modes to blade passing frequency.

Conclusion

This paper has discussed the conditions required for the excitation of a fan impeller structure by a uniform pressure field and blade passing impulses. Aerodynamics and instability were not considered. Viscous damping was assumed.

In the case of an axial pressure field excitation, it was shown that for a resonance situation to exist, the impeller natural frequencies *and* their associated modes have to be considered. In this case the expected resonance amplification factor is the usual $1/2\xi$

In the case of blade impulse excitation, a computer program was developed to calculate the response of different modes at different running speeds. The calculation was demonstrated on two likely to be excited modes and the results were presented in the form of frequency response plots, where the resonance amplification factors were evident.

When combining the above results with experimental stress data taken during normal off-resonance operating conditions, one can assess the severity of a particular resonance should it occur as a result of a change in operational conditions. This sort of comparative investigation is of particular interest when the excitation energy cannot be quantified and dynamic stresses can therefore not be directly determined.

In our particular example corresponding to a fan which had to be assessed during the design stage and represented in figures 1 to 5, the necessary conditions for resonance excitation are not fulfilled; mode number 1 natural frequency of 70,1 Hz is much higher than 1 x 12,33 Hz (one diameter mode and 12,33 Hz rotating frequency).

Modes number 6 and number 11 exhibit shapes which could be excited by blade passing. However, in the case of mode shape number 6 considering the two outlets, the natural frequency of 130,2 Hz is far from 24 x 12,33 = 295,9 Hz. (Excitation at 12 x 12,33 Hz will result in cancellation of energy). In the case of mode shape number 11, the natural frequency of 180,7 Hz is far from the blade passing frequency 12 x 12,33 = 148,0 Hz.

Modes number 7 and 9 cannot be excited by blade passing due to the inconsistency of the blade deformation shape resulting in energy cancellation.

Evidently, the assessment revealed no close resonance situation and therefore the calcultion of amplification factors were unnecessary. However, the above procedure can be used to assess amplification factors whenever required.

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