Large Separation Factors in Short Bowl Centrifuges

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The possibility of obtaining large separation factors in short bowl centrifuges is discussed in this report. A suitably modified internal configuration, e.g. employment of an internal single start spiral, could enforce a desired counter current flow pattern over a long active separating length, thus creating conditions for large separation factors in a short centrifuge. Desired flow patterns are assumed and a simplified model used to indicate the influence and sensitivity of various geometrical and flow parameters on centrifuge performance in a parametric study. A comparison is made between separating characteristics of a conventional and a spiral centrifuge operated under similar conditions.

Nomenclature

A peripheral speed parameter, $\frac{\Delta M\, v_w^2}{RT}$

A′ peripheral speed parameter, $\frac{M\, v_w^2}{2RT}$

B diffusion parameter, $\frac{\rho_w v_w r_w}{\rho D}$

C_1
C_2 constants defined in text
D diffusion coefficient
F mass flow function
I_p internal mass flow in the direction of product extraction point
I_s internal mass flow in the direction of the waste extraction point
K_0,
K_1 dimensionless mass velocity constants defined in text
M mole mass
ΔM mole mass difference
P product flow
P′ normalised product flow
$= \frac{P}{2\pi\rho_w v_w r_w^2}$ for the conventional and
$= \frac{P}{\rho_w v_w b r_w}$ for the spiral centrifuge
R universal gas constant
T absolute temperature
X mole fraction of desired component
X* mole fraction of recirculation stream at feed point
V feed mass flow
V(X) value function
W waste mass flow
W′ normalised waste flow
$= \frac{W}{2\pi\rho_w v_w r_w^2}$ for the conventional and
$= \frac{W}{\rho_w v_w b r_w}$ for the spiral centrifuge
Z length of centrifuge
b width of spiral groove
d diameter
n number of spiral turns
p pressure
r radius
v speed
w speed component in direction of main flow
x dimensionless radius, $\frac{r}{r_w}$
z′ distance measured in direction of main flow
z dimensionless distance, $\frac{z'}{r_w}$
α separation ratio
ϵ separation factor
θ cut
ρ density
δU separative power

Subscripts

o feed
p product
s waste
w wall
c flow inversion point
i inner
t total

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Introduction

It has been shown [4] that it is possible to efficiently enrich uranium for commercial purposes e.g. from a natural concentration to 3% and deplete it to 0,3% in one step with long high speed centrifuge rotors.

The purpose of this paper is to investigate if short bowl centrifuges are also capable of enriching to commercial grade material without cascading.

It is demonstrated that reasonably efficient one step enrichment is theoretically possible even at relatively low wall speeds. A critical assumption is that a favourable internal counter current spiral flow could be induced by an internal single start spiral, fixed on the inside of the cylindrical wall of a short bowl centrifuge. See figure 1. The influence of various geometrical and flow parameters of the spiral centrifuge was investigated with a simplified analytical model based on the Cohen [1] equation. Top hat mass velocity profiles were assumed.

Construction of this kind of centrifuge will present a very severe and complex practical engineering problem. The possibility of the creation of the desired flow pattern is another matter of concern and is speculatively discussed. Furthermore it is shown that the maximum separative power is equivalent to that of a conventional centrifuge of the same size and operated at the same wall speed. Therefore the only advantage of the spiral type centrifuge would be that it may be designed to do one step enrichment within a relatively short bowl.

Feed, product and waste streams per machine are very small and may present control problems.

Approximate solutions

Starting with the well known partial differential equation of Cohen [1] it can be shown [3] that with the following assumptions, namely:

(i) Solid body rotation of the gas mass at the angular velocity of the centrifuge
(ii) No radial flow of gas
(iii) $\frac{\partial X}{\partial z}$ is independent of x
(iv) $\frac{\partial^2 X}{\partial z^2} = 0$
(v) ρw is independent of z
(vi) $X(1 - X)$ is treated as a constant

the equation for axial concentration distribution can be written in the following non-dimensional format:

$$\frac{\partial^2 X}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{XP'}{C_2} + X(1 - X)\frac{C_1}{C_2}\right) \quad (1)$$

The general solution of this equation is:
(a) for the enriching section

$$\frac{X_p}{X^*} = \frac{\left(1 + \frac{P'}{C_1}\right) \exp\left(1 + \frac{P'}{C_1}\right)\frac{C_1}{C_2} z_{t_p}}{1 + \frac{P'}{C_1} \exp\left(1 + \frac{P'}{C_1}\right)\frac{C_1}{C_2} z_{t_p}} \quad (2)$$

(b) for the stripping section

$$\frac{X^*}{X_s} = \frac{\exp\left(1 - \frac{W'}{C_1}\right)\frac{C_1}{C_2} z_{t_s} \quad \frac{W'}{C_1}}{1 - \frac{W'}{C_1}} \quad (3)$$

In the case of the conventional centrifuge:

$$P' = \frac{P}{2\pi \rho_w v_w r_w^2} \quad (4)$$

$$W' = \frac{W}{2\pi \rho_w v_w r_w^2} \quad (5)$$

$$C_1 = A \int_0^1 Fx \, dx \quad (6)$$

$$C_2 = \frac{1}{2B} + B \int_0^1 \frac{F^2}{x} dx \quad (7)$$

$$F = \int_0^1 \frac{\rho w}{\rho_w v_w} x dx \quad (8)$$

$$z_{t_p} = \frac{z_p'}{r_w} \quad (9)$$

$$z_{t_s} = \frac{z_s'}{r_w} \quad (10)$$

$$z_t = z_{t_p} + z_{t_s} \quad (11)$$

$$\therefore \; Z = z_p' + z_s' \quad (12)$$

The corresponding equations for the spiral centrifuge follow below. In this case z' and z are measured along the length of the spiral groove. It is assumed that the thickness of the spiral can be neglected in relationship to the channel width b. Furthermore it is assumed that an average width can be used because most of the gas mass is concentrated near the circumference of the bowl.

$$P' = \frac{P}{\rho_w v_w r_w b} \quad (13)$$

$$W' = \frac{W}{\rho_w v_w r_w b} \quad (14)$$

$$C_1 = A \int_0^1 F \times dx \quad (15)$$

$$C_2 = \frac{1}{B} + B \int_0^1 F^2 \, dx \quad (16)$$

$$F = \int_0^x \frac{\rho w}{\rho_w v_w} dx \quad (17)$$

$$z_{t_p} = \frac{2\pi r_w n}{r_w} = 2\pi n_p \quad (18)$$

$$z_{t_s} = 2\pi n_s \quad (19)$$

$$z_t = 2\pi n = z_{t_p} + z_{t_s} \quad (20)$$

$$n = n_p + n_s \quad (21)$$

Separation factors and separative power

Consider the relevant flows and corresponding concentrations in the following diagram of a counter current centrifuge

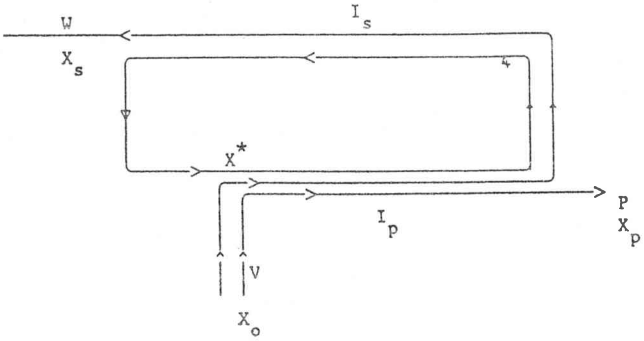

Figure 1 – Counter current centrifuge

If the cut for the centrifuge is defined by

$$\theta = \frac{P}{V} \quad (22)$$

the mass balance equations give

$$\theta = \frac{X_o = X_s}{X_p - X_s} \quad (23)$$

It can be shown that

$$X_p = X_o \left(\frac{\frac{X_p X^*}{X^* X_s}}{(1 - \theta) + \frac{X_p X^*}{X^* X_s}\theta} \right) \quad (24)$$

$$\text{and } X_s = X_o \left(\frac{1}{(1 - \theta) + \frac{X_p X^*}{X^* X_s}\theta} \right) \quad (25)$$

to provide a relationship between X_p, X_s, X_o if θ is known and also X_p/X^* and X_s/X^* from equations (2) and (3).

The mole fraction X_o of the desired component in the feed stream is normally known and the product and waste concentrations (X_p and X_w respectively) could therefore be calculated.

Separation ratios, separation factors and separative power given by the following equations can then be determined.

Separation ratios

$$\alpha_{po} = \frac{X_p}{1 - X_p} \Big/ \frac{X_o}{1 - X_o} \quad (26)$$

$$\alpha_{os} = \frac{X_o}{1 - X_o} / \frac{X_s}{1 - X_s} \quad (27)$$

$$\alpha_{ps} = \frac{X_p}{1 - X_p} / \frac{X_s}{1 - X_s} = \alpha_{po}\alpha_{os} \quad (28)$$

Separation factors

$$\epsilon = \alpha - 1 \text{ (in general)} \quad (29)$$

$$\text{e.g. } \epsilon_{ps} = \alpha_{ps} - 1 \quad (30)$$

Separative power of the centrifuge

$$\delta U = P\left(V(X_p) + \left(\frac{1 - \theta}{\theta}\right) V(X_s) - \frac{V(X_o)}{\theta} \right) \quad (31)$$

$$\text{where } V(X) = (2X - 1) \ln\left(\frac{X}{1 - X}\right) \quad (32)$$

the simple value function.

Normally the separative power of a centrifuge is proportional to its length if all the other parameters are constant.

Therefore a parameter of interest for comparative purposes will be:

$$\frac{\delta U}{Z}$$

where Z = Z for the conventional
and Z = nb for the spiral centrifuge.

Calculation of C_1, C_2 and P'

Two shell top hat (uniform) mass flow profiles, shown on the diagram, are assumed to simplify calculations. These profiles are integrated when determining flow functions and the constants C_1, C_2 and P' and for this reason they will provide good first order values for α and $\delta U/Z$. When comparing the two types of centrifuges on a relative basis these profile assumptions will even be more applicable.

It is thus assumed that

$$\frac{\rho w}{\rho_w v_w} = K_o \quad \text{for } x_i \le x \le x_c$$

$$= -K_1 \quad \text{for } x_c \le x \le 1 \quad (33)$$

$$= 0 \quad \text{for } 0 \le x \le x_i$$

For the conventional centrifuge

$$P' = \frac{K_1}{2}\left(x_c^2\left(\frac{K_o}{K_1} + 1\right) - \frac{K_o}{K_1} x_i^2 - 1 \right) \quad (34)$$

$$\text{or } \frac{K_o}{K_1} = \frac{2P'/K_1 + (1 - x_c^2)}{(x_c^2 - x_i^2)} \quad (35)$$

$$C_1 = \frac{AK_1}{4}\left[\frac{K_o}{K_1}\left\{ x_c^2\left(1 - \frac{x_c^2}{2}\right) - x_i^2\left(1 - \frac{x_i^2}{2}\right) \right\} - \frac{(1 - x_c^2)^2}{2} \right] \quad (36)$$

$$C_2 = \frac{1}{2B} + \frac{BK_1^2}{4}$$

$$\left[\left(\frac{K_o}{K_1}\right)^2 \left(\frac{(x_c^2 - x_i^2)(x_c^2 - 3x_i^2)}{4} + x_i^4 \ln \frac{x_c}{x_i} \right) \right.$$

$$+ \left(\frac{K_o}{K_1}(x_c^2 - x_i^2) + x_c^2 \right)^2 \ln \frac{1}{x_c} - \left(\frac{1 - x_c^4}{4} \right)$$

$$\left. - \left(\frac{K_o}{K_1}(x_c^2 - x_i^2) + x_c^2 \right)\left(1 - x_c^2\right) \right] \quad (37)$$

The internal mass flows are given by:

$$I_p = \pi \rho_w v_w r_w^2 (x_c^2 - x_i^2) K_o \quad (38)$$

$$I_s = \pi \rho_w v_w r_w^2 (1 - x_c^2) K_1 \quad (39)$$

$$\text{Furthermore } \frac{P}{I_p} = \frac{2P'}{(x_c^2 - x_i^2)K_o} \quad (40)$$

$$\text{and } \frac{P}{I_s} = \frac{2P'}{(1 - x_c^2)K_1} \quad (41)$$

For the spiral centrifuge

$$P' = K_1\left[x_c\left(\frac{K_o}{K_1} + 1\right) - \frac{K_o}{K_1} x_i - 1 \right] \quad (42)$$

$$\text{or } \frac{K_o}{k_1} = \frac{\frac{P'}{K_1} + (1 - x_c)}{(x_c - x_i)} \quad (43)$$

$$C_1 = \frac{AK_1}{2}\left[\frac{K_o}{K_1}\left(\frac{x_i^3}{3} - \frac{x_c^3}{3} + x_c - x_i \right) - \left(\frac{2 - 3x_c + x_c^3}{3} \right) \right]$$

$$C_2 = \frac{1}{B} + BK_i^2\left[\left(\frac{K_o}{K_1}\right)^2 \right.$$

$$((x_c - x_i)^2 + x_i x_c^2 - \tfrac{2}{3}x_c^3 - \tfrac{1}{3}x_i^3) \quad (44)$$

$$\left. - \frac{K_o}{K_1}(x_c - x_i)(1 - x_c)^2 + \tfrac{1}{3}(1 - x_c)^3 \right] \quad (45)$$

$$I_p = \rho_w v_w r_w b(x_c - x_i) K_o \quad (46)$$

$$I_s = \rho_w v_w r_w b(1 - x_c) K_1 \quad (47)$$

$$\frac{P}{I_p} = \frac{P'}{(x_c - x_i)K_o} \quad (48)$$

$$\frac{P}{I_s} = \frac{P'}{(1 - x_c)K_1} \quad (49)$$

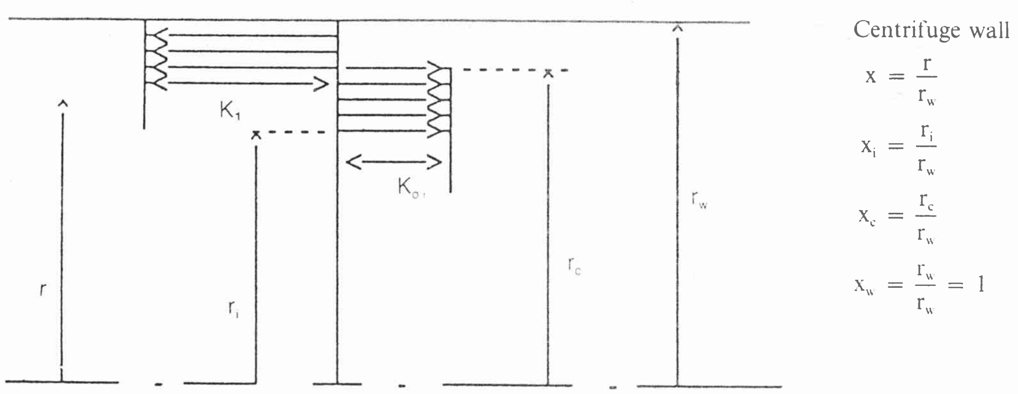

Figure 2 – Mass flow profiles

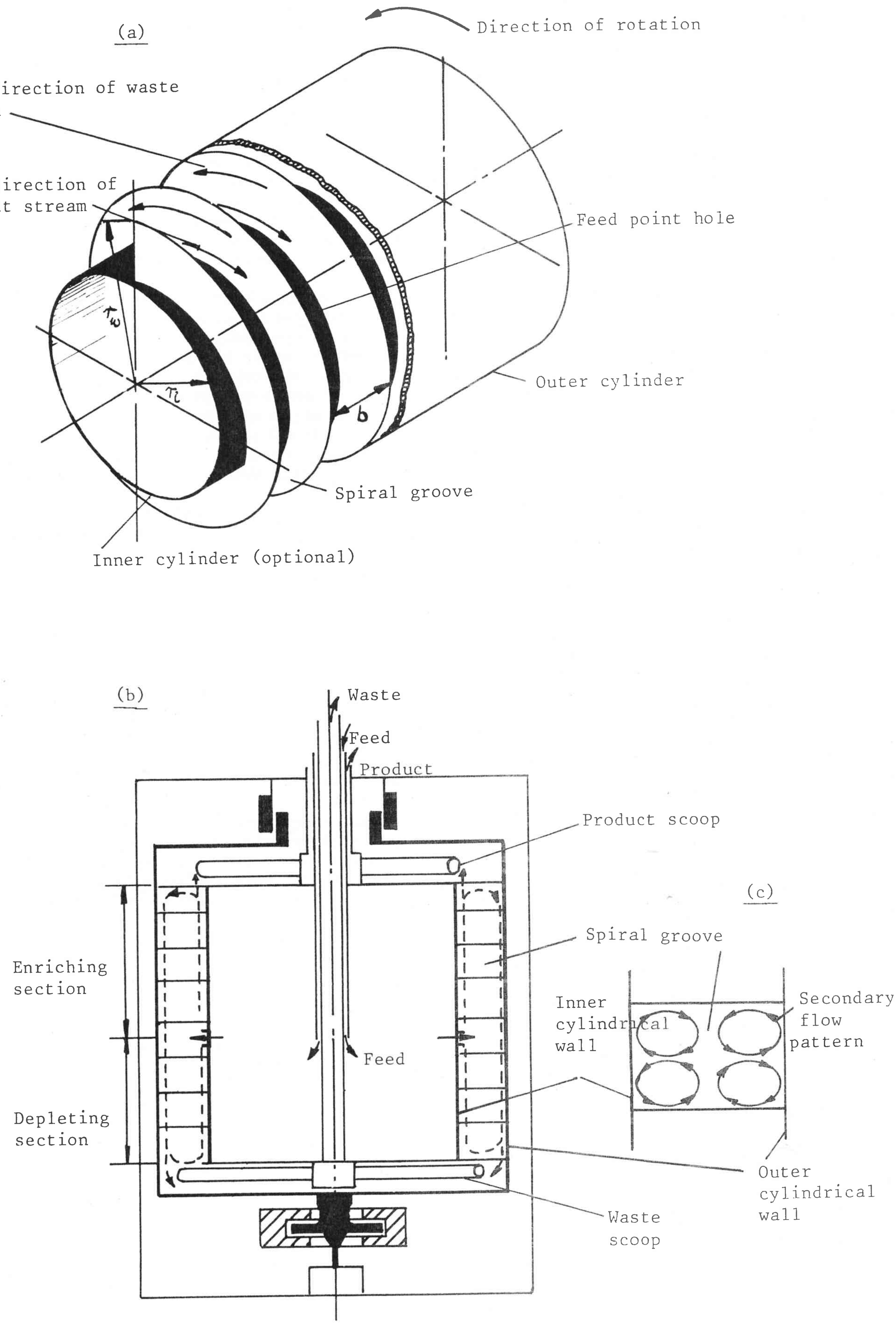

Figure 3 – Spiral centrifuge

The design and operation of the counter current spiral centrifuge

Two design possibilities are briefly analysed.

The first design option comprises an outer cylinder with endwalls containing an inner cylinder with a spiral groove in the annular space between the cylinders. See figures 3(a) and (b). If the inner cylinder is left out a second option results. Counter current flow occurs lengthwise along the spiral groove. In contrast to the conventional centrifuge, with mainly an axial flow pattern, the spiral flow is principally circumferential.

The function of the spiral is to provide a long pathlength for counter current flow, thus creating conditions for large separation factors.

Mechanisms for feed, product and waste extraction, and the bringing about of the counter current recirculating flow, e.g. scoop systems and temperature differences, are the same as for the conventional centrifuge and will not be discussed any further. It should be noted that the recirculation stream is much smaller in the spiral than in the conventional centrifuge. If an inner cylinder is utilised, simple holes may be provided at the feed point of the spiral. See figure 3(b). If there is no inner cylinder present, the feed stream will be introduced through the central feed pipe as closely possible to the theoretical feed point.

It is important that the flow directions of the gas streams are chosen correctly in relationship to the direction of rotation of the bowl in the case of the spiral centrifuge. The flow direction of the outer recirculating stream (at the largest radius) should be in the direction of rotation. This will force the flow against the outer wall of the centrifuge. The counter current on the inside, with flow direction opposite to that of the bowl, will experience a radial force in the direction of the centre line. This flow will therefore be forced against the inner cylinder.

The flow pattern, described above, should be stable. However, secondary radial flows can be expected as shown in figure 3(c). These would probably consist of laminar Goertler vortices. If the inner cylinder is not present, the dragging force and pumping action of the radial spiral wall would induce a similar pattern of laminar Goertler-type vortices. These vortices, besides the drag from the spiral wall, will increase the total flow resistance. One of the fundamental aspects therefore is to answer the question whether it will be possible to bring about the postulated counter current flow pattern, especially in long spirals. Utilising a high gas density would reduce velocities for a fixed mass flow. This will reduce flow resistance which is only a function of gas viscosity and velocity for a fixed geometry.

The validity of the simplified model will be affected by the fact that relative motion occurs between gas and centrifuge wall and that radial flows will probably be present.

Performance characteristics of spiral and conventional centrifuges

Illustrative examples demonstrate differences and similarities between performance characteristics of spiral and conventional centrifuges.

Independent parameters which influence the performance of the two types of centrifuges are the following:

(i) A, the peripheral speed parameter or spin number $= \dfrac{\Delta M v_w^2}{RT}$

(ii) B, the diffusion parameter or diffusion number $= \dfrac{\rho_w v_w r_w^2}{\rho D}$

(iii) Position of feed point characterised by z_{t_s}/z_t

(iv) θ, the cut

(v) Normalized product flow P′

(vi) x_c, the dimensionless radius ratio at the flow inflection point.

(vii) x_i, the dimensionless radius ratio defining the inner boundary of the flow region.

(viii) $Z/_{2r_w}$, the length to diameter ratio for the conventional centrifuge and n, the number of spirals in the case of the spiral centrifuge.

(ix) The internal recirculation, characterised by P/I_s for example. The values of the above parameters must be chosen.

Because

$$P/I_s = \frac{2P'}{(1 - x_c^2)K_1} \text{ for the conventional}$$

$$\text{and } = \frac{P'}{(1 - x_c)K_1} \text{ for the}$$

spiral centrifuge, it is only necessary to choose a value for the mass velocity ratio K_1, since P′ and x_c have been chosen.

Dependent parameters which were considered are the following:

(i) Total enrichment factor

$$\epsilon_{ps} = \alpha_{ps} - 1$$

(ii) Specific separative power
$\delta U/Z$ in SWU/m.a

where Z = Z for the conventional and
Z = nb for the spiral centrifuge

Specific parameters, chosen for both centrifuges, were the following:

θ = 0,5; 0,2; 0,05; 0,001
r_w = 0,075 m
v_w = 400; 600; 700 m/s
ρ_w = 0,175 kg/m³ for UF_6
$\rho D = 2{,}161 \times 10^{-5} \dfrac{kg}{m.s}$ for UF_6

K_1 varied between 10^{-2} and 10^{-5}
x_c varied between 0,7 and 0,97
x_i varied between 0,0 and 0,95
z_{t_s}/z_t = 0,5; 0,2; 0,05; 0,001

The value chosen for x_i was based on the assumption that the total useful pressure ratio in a centrifuge is of the order of 1 000. See [5]. A ratio of 3,0 was arbitrarily chosen for x_c. In general x will be given by:

$$x = \sqrt{1 - \frac{2RT}{Mv_w} \ln \frac{p_w}{p}}$$

where $P_{w/p}$ = relevant pressure ratio.

For the conventional centrifuge the product mass flow was varied between 10^{-8} and 10^{-2} kgU/s and $Z/2r_w$ between 5 and 30.

In the case of the spiral centrifuge b = 0,004 m and n varied between 1 and 80 while the product flow was varied between 10^{-10} and 10^{-10} kgU/s.

Results

Results include the flow pattern efficiency (which, for uniform mass velocity distribution, will be equal to one), influence of circulation and cascade ideality. The influence of product flow variation P, in kg uranium/s, on the specific separative power $\delta U/Z$, and separation factor ϵ for various values of K_1, the mass velocity ratio, is shown in figure 4 for the conventional and in figure 6 for the spiral centrifuge. Enveloping graphs were plot-

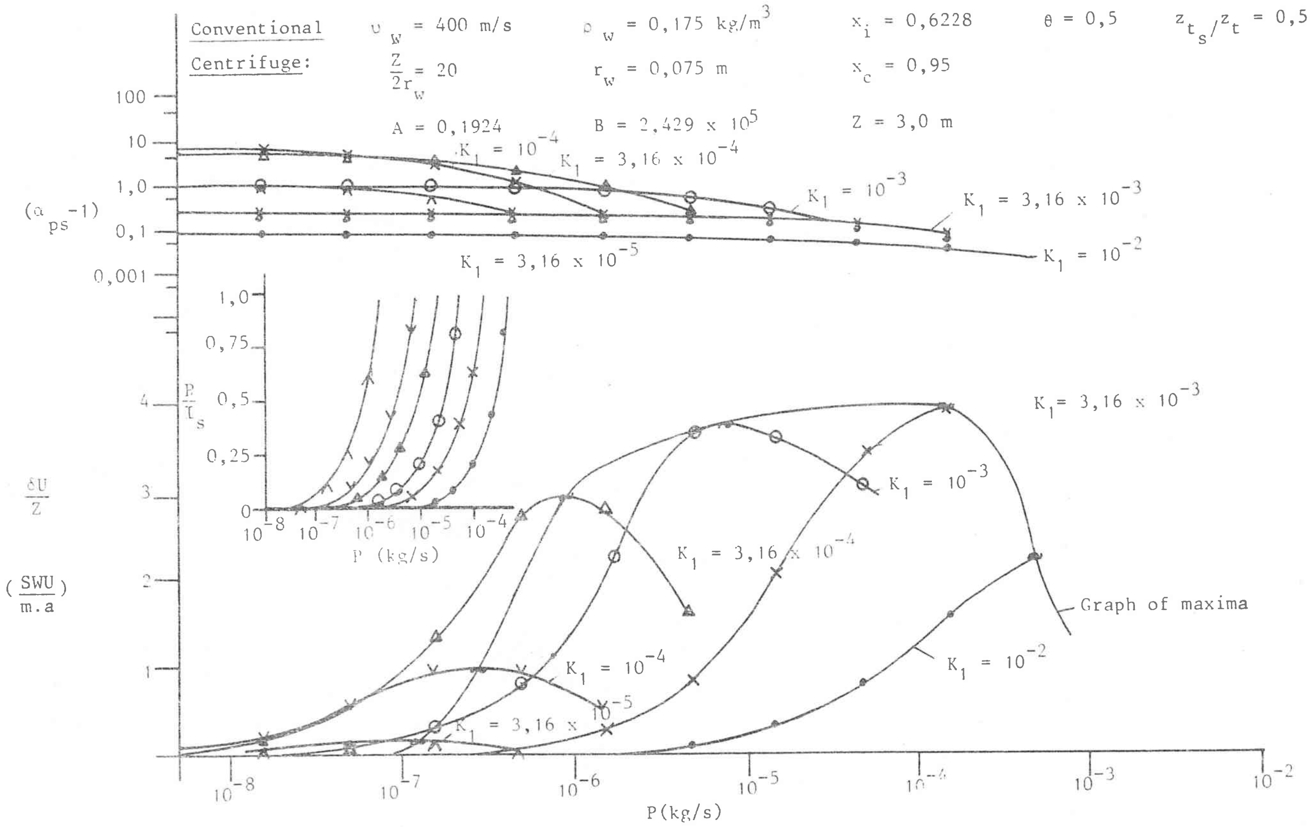

Figure 4 – Influence of the dimensionless mass velocity constant K_1 for conventional centrifuge

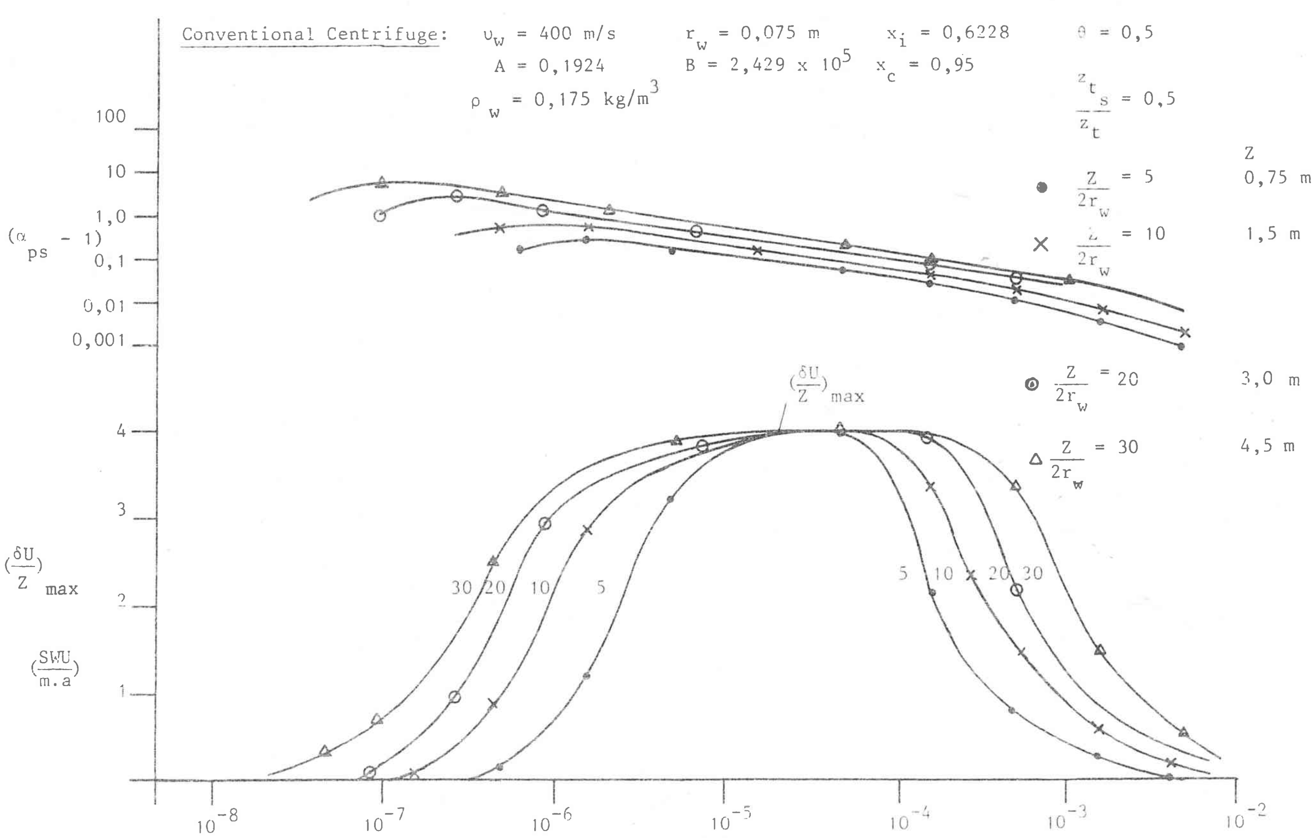

Figure 5 – Influence of length to diameter ratio

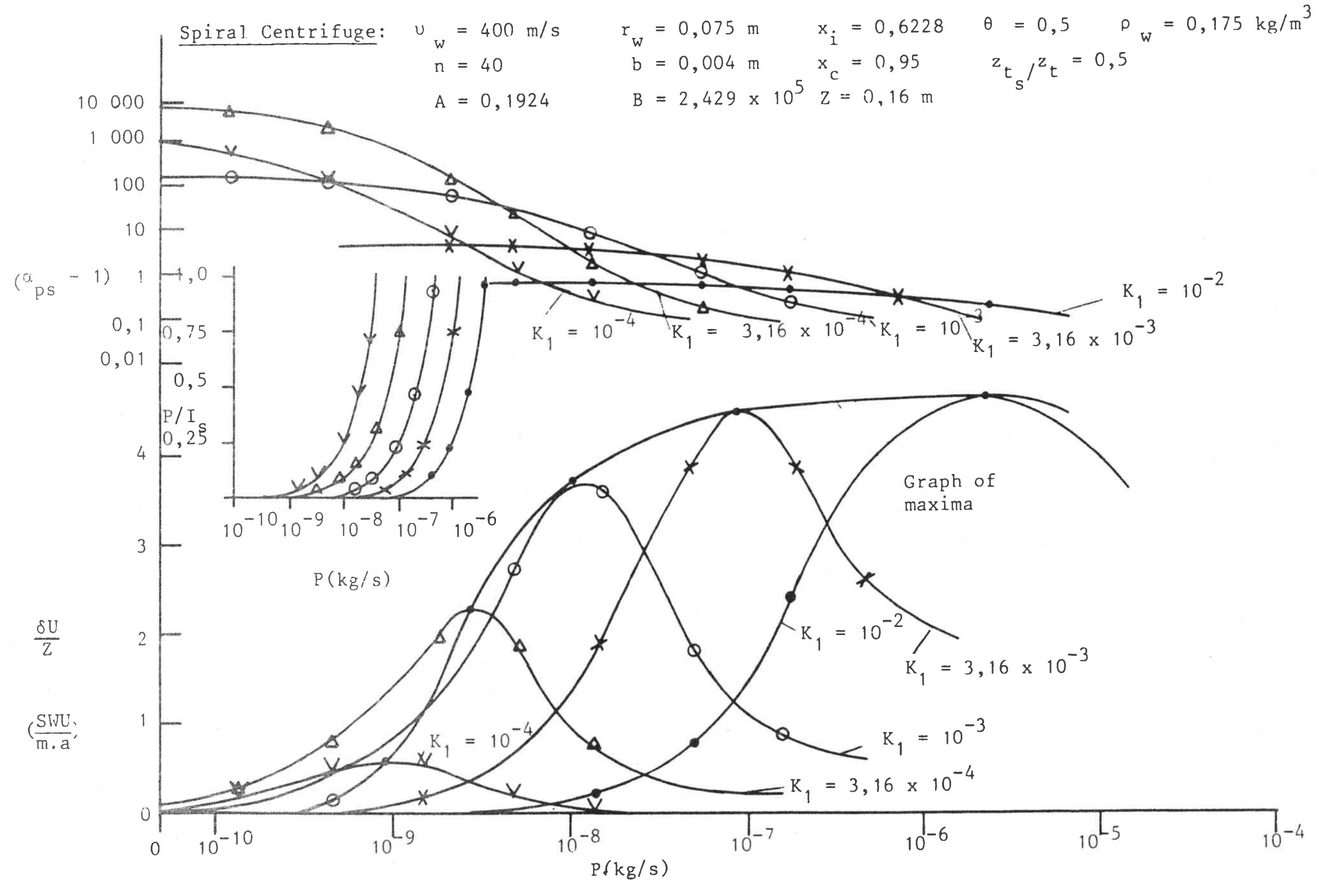

Figure 6 – Influence of the dimensionless mass velocity constant K_1 for spiral centrifuge

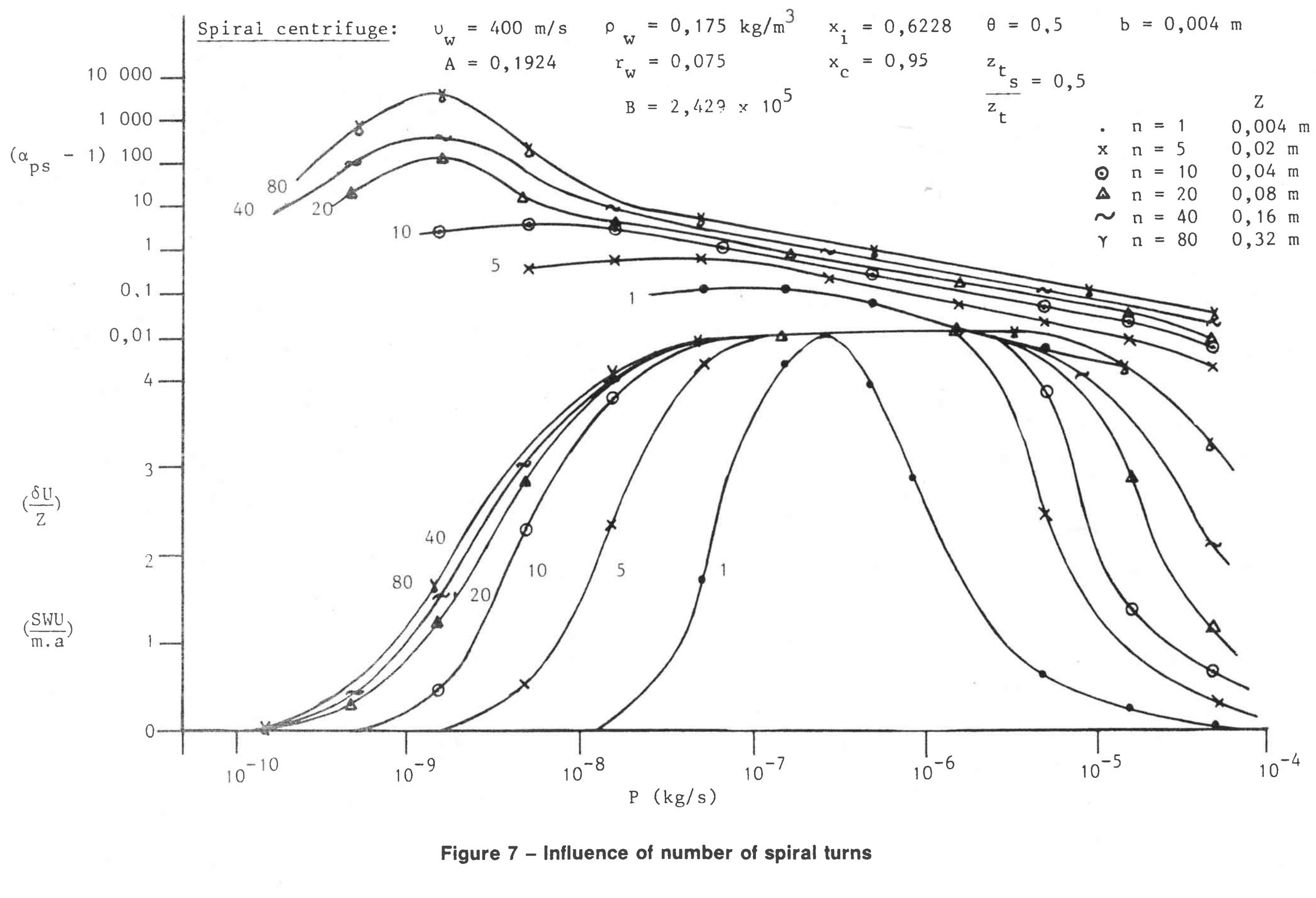

Figure 7 – Influence of number of spiral turns

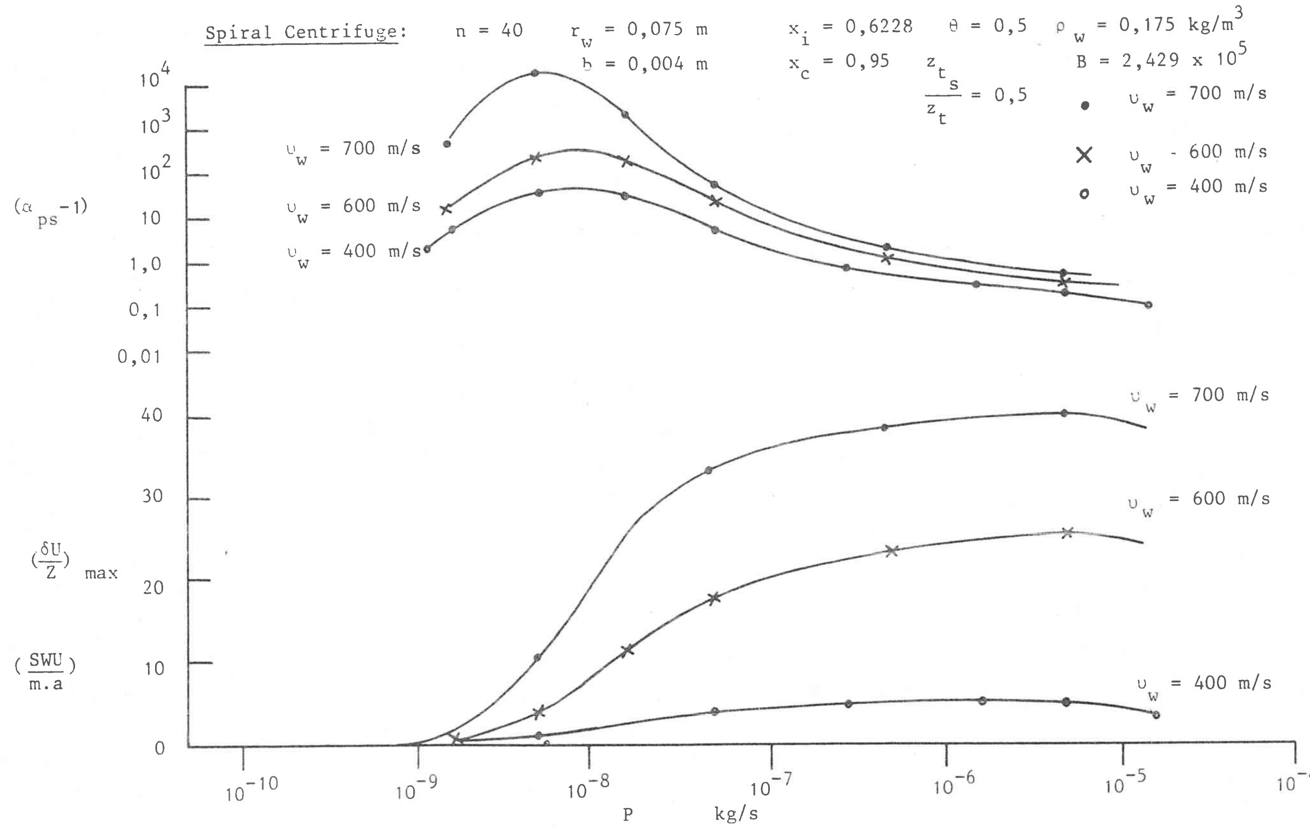

Figure 8 – Influence of wall speed (spiral centrifuge)

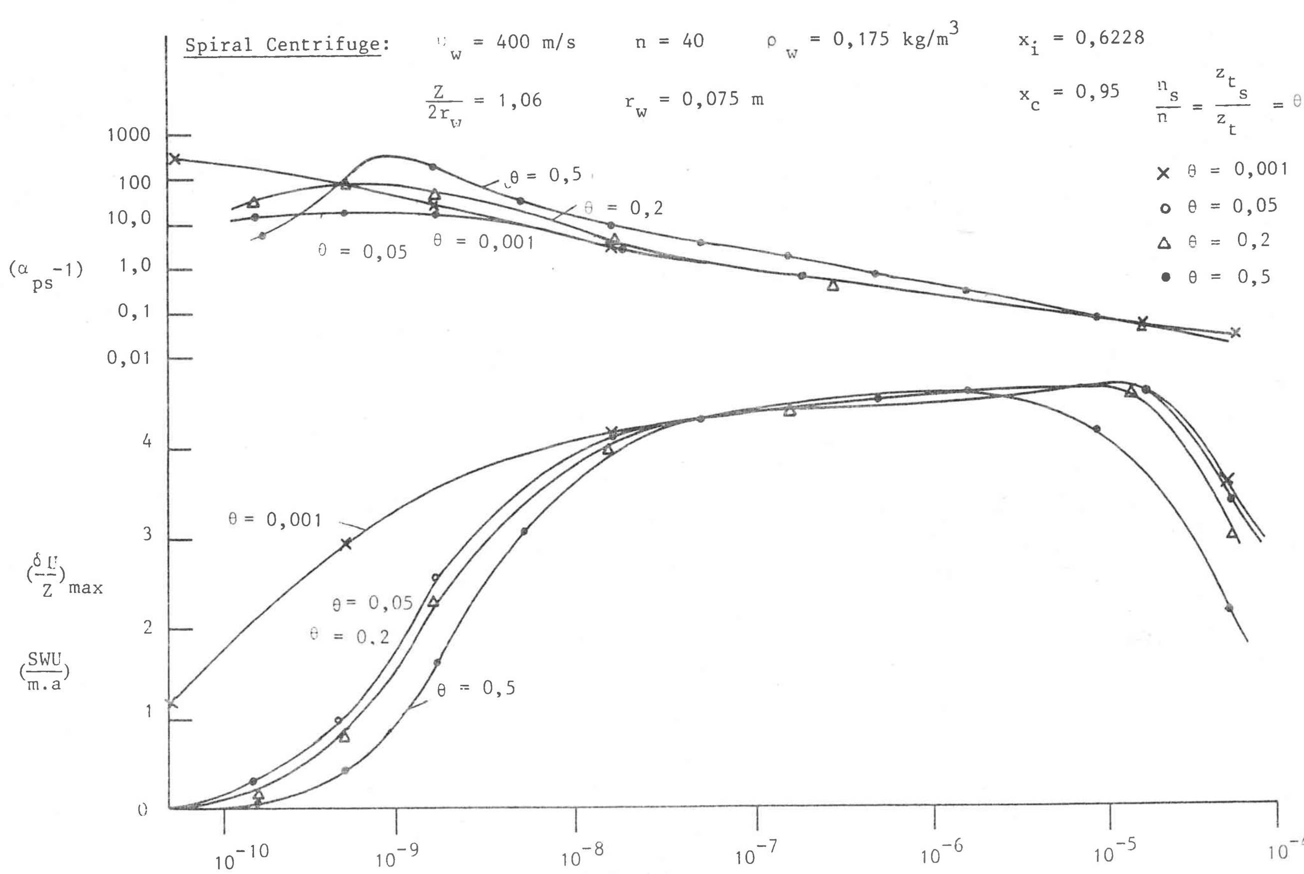

Figure 9 – Influence of cut and position of feed point

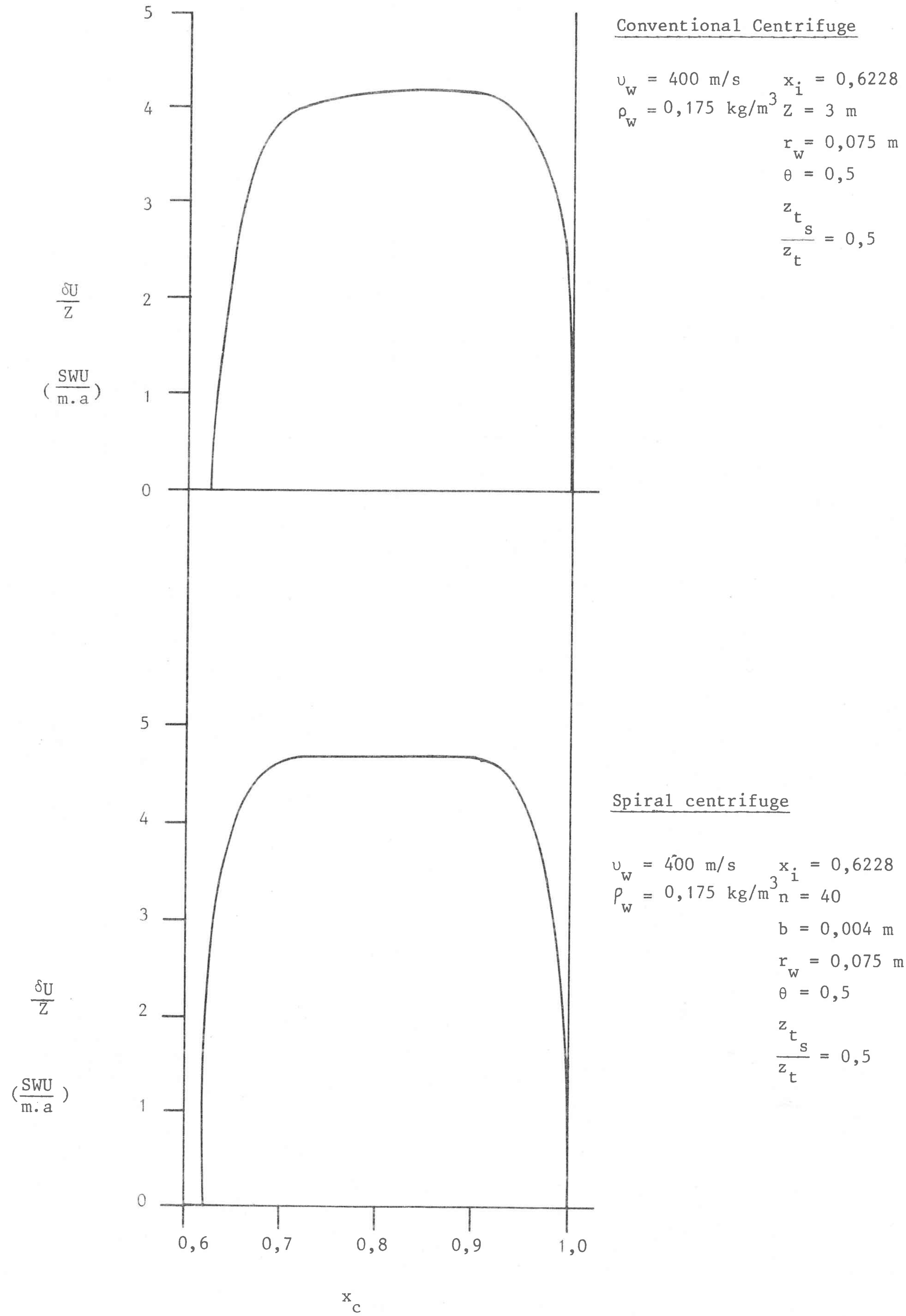

Figure 10 – Influence of distance of flow inversion point

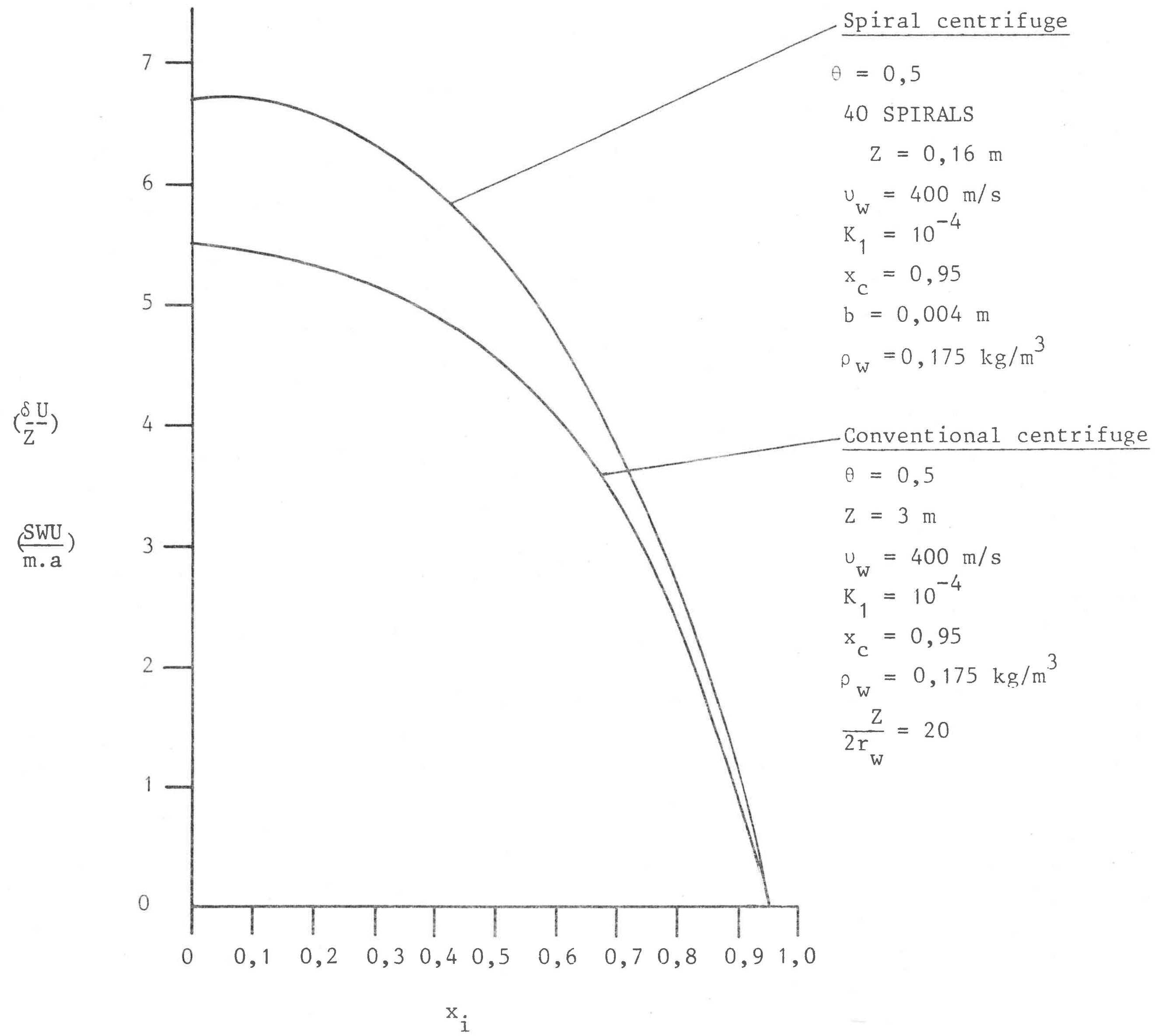

Figure 11 – Influence of inner flow radius

ted through the maxima of specific separative power for the various K_1 values as shown in figures 4 and 6. Corresponding values for the recirculation parameter P/I_s are also included.

Figures 5 and 7 display a set of graphs for optimum $\delta U/Z$ versus product flow. Corresponding values for separation factors are also shown. In the case of the conventional centrifuge the length to diameter ratio $Z/2r_w$ was varied and in the case of the spiral centrifuge the number of spirals. Both these figures demonstrate the well-known fact that by increasing the effective separation length of a centrifuge the separation factor can be increased.

The most striking difference between the spiral and conventional centrifuges is the very high separation factors which can be achieved in very short spiral bowls. The value of the spiral width b, was deliberately chosen small to demonstrate this fact more clearly. However, the maximum separative power per unit length for the spiral centrifuge is somewhat higher than for the conventional centrifuge at the same wall speed. In a practical case the value of b could conveniently be chosen to fit both a practical length and a desired enrichment.

The derived equations are convenient to use in a sensitivity analysis. Figure 8 shows the influence of a change in wall speed and figure 9 the influence of cut θ and simultaneous change of position of feedpoint where $n_s/n = \theta$. Other examples are shown in figures 10 and 11 where x_i is varied between 0 and 0,95 and x_c between 0,6 and 1,0 and its influence on $\delta U/Z$ for the spiral and conventional centrifuges is depicted.

Conclusions

(i) Theoretically it is possible to achieve one-step commercial enrichment (e.g. 3%) at good efficiency with a short spiral centrifuge (length-to-diameter ratio $< 1,0$) at a 400 m/s wall speed.

(ii) Very high separation factors are possible with a spiral centrifuge at 400 m/s wall speed but at a reduced efficiency.

(iii) Maximum separative power per unit length of centrifuge is slightly higher for the spiral centrifuge in comparison with conventional centrifuges at the same wall velocity. This is due to the simplifying geometric assumptions.

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