Allocation and Prediction of Structure Safety Reliability

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The disciplines of structural design and reliability engineering need to be brought in closer contact. Dynamic structures often require high strength and minimized mass. Classical design procedures based on the use of safety factors and given probabilities of failure are used for these structures and do not include reliability allocation or prediction. The paper explains the use of loadpaths and reliability block diagrams to model structure safety reliability. The use of structure functional entities is described. An interpretation is given of allocation and prediction of structure safety reliability as seen from a structural design point of view.

Structure safety reliability is the probability that a failure jeopardising the integrity of a structure and therefore its operational safety will not occur. Reliability, R, is the Probability that a failure will not occur, therefore it is the complement of P, the probability of failure, or $R = 1{,}0 - P$.

Reliability allocation and prediction should be part of the design phase of dynamic structures. It will enable the designer to determine the impact of structure safety reliability on maintenance and cost. Although this is not a well established approach to structural design yet, it can be applied to all dynamic structures which require high strength and minimized mass. Examples are mine hoists, sporting equipment and the structure of vehicles, boats and aircraft.

This design philosophy requires the disciplines of structural design and reliability engineering to be brought in closer contact. Classical design procedures are based on the use of safety factors and given probabilities of failure and do not incorporate reliability allocation and prediction.

In this paper the purpose is to bring the two disciplines together by giving an interpretation of structure safety reliability as seen from a structural design point of view. Towards this purpose a new concept will be introduced, design confidence defined as the reiliability of the design process.

To demonstrate the process of reliability allocation and prediction the airframe of a helicopter will be used as an example, see figure 1.

Structure safety reliability modelling

The philosophy behind the design of a structure is to provide structural integrity for carrying the various loads on it. Loads are carried and transferred by the structural components designed to form different loadpaths.

For the purpose of this paper the helicopter airframe will be divided into four main loadpaths, viz. main rotor, engine mount, tail rotor and landing gear loadpaths of which only the first two will be considered.

A safety reliability of a structure can be modelled with a reliability block diagram [1]. It is a series combination of the various loadpaths that can be distinguished in the structure. The safety reliability of the helicopter airframe is modelled with the block diagram in figure 2.

Series block diagrams can consequently be constructed for the components on each loadpath forming the second level of the reliability model, see figure 3. Some of the structural components form part of more than one loadpath. Their probability of failure has to be allocated in such a way that it takes their presence in each loadpath into account. A component on more than one loadpath makes a relatively larger contribution to the structure's probability of failure than when it occurs on only one. This assumption can be motivated from a stress point of view. More than one load, carried by different loadpaths, are transferred by such a component forming part of more than one loadpath. At times different loads may even be transferred simultaneously. The probability of component failure will be increased by the higher stresses induced by simultaneous loads or the increased number of loads, thus causing a larger contribution to the structure's probability of failure.

The relative structural importance of the items in each block of the block diagram is represented by weighting factors assigned to each item. The weighting factors for the items of any block diagram add to 100. Weighting factors are estimated for each component on a specific loadpath, relative to all other components in the same loadpath. Weighting factors representing the relation between the loadpaths are also estimated. Weighting factors assigned to the airframe are:

First level :	Main rotor	: 43
	Engine mount	: 21
	Tail rotor	: 21
	Landing gear	: 15
		100
Second level :		
Main rotor :	A-frame 1	: 20
	Connection to A-frame 1	: 15
	A-frame 2	: 20
	Connection to A-frame 2	: 15
	Connections to box frame	: 30
		100
Engine mount :	Pyramid 1	: 10
	Connection to pyramid 1	: 25
	Pyramid 2	: 10
	Connection to pyramid 2	: 25
	Connections to box frame	: 20
	Rear connection	: 10
		100

If any component on the second level of the reliability model forms part of more than one loadpath its relative weighting factor needs to be calculated first. The relative weighting factor of a component is calculated by multiplying the fraction of the loadpath (i.e. weighting factor/100) with the weighting factor (w.f.) of the specific loadpath. The resulting value is the weighting factor of the component relative to all other second level components in all the loadpaths. To obtain the final weighting factor for a component forming part of more than one loadpath, the weighting factors obtained for the component in each loadpath, are added. The resulting model is an all series representation of the second level with no component repeated.

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In the helicopter, for example:

The relative weighting factor of connections to box frame on the main rotor loadpath:

Component w.f. : 30
Fraction of loadpath : 30/100
Specific loadpath w.f. : 43
Final w.f. : (30/100) × 43 = 12,9

The relative weighting factor of connections to box frame on the engine mount loadpath:

Component w.f. : 20
Fraction of loadpath : 20/100
Specific loadpath w.f. : 21
Final w.f. : (20/100) × 21 = 4,2

The final weighting factor of connections to box frame = 12,9 + 4,2 = 17,1.

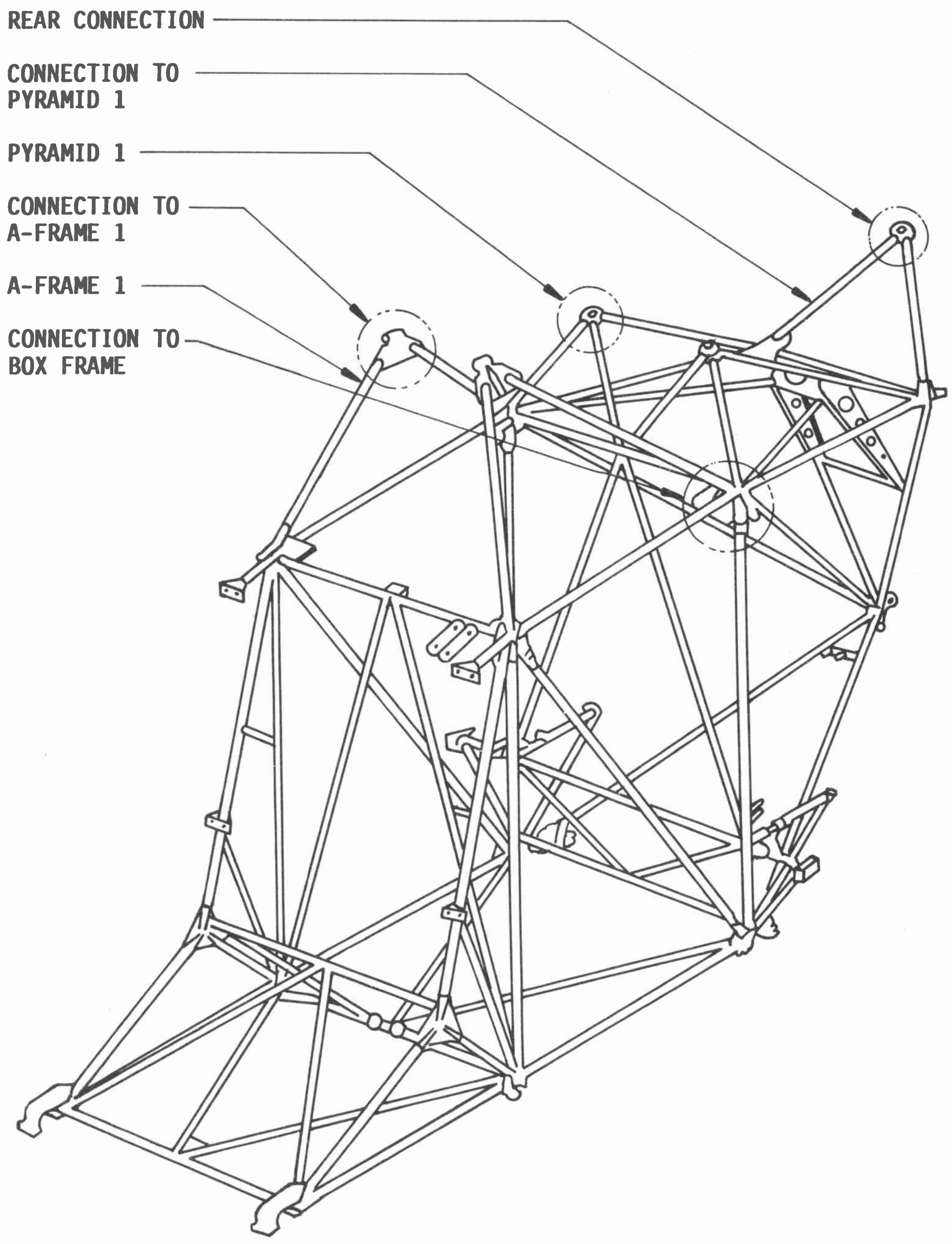

Figure 1 – The Airframe of a Typical Helicopter

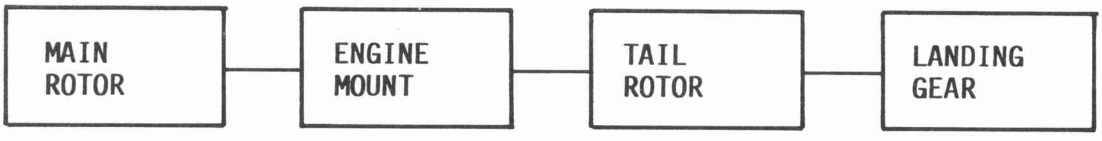

Figure 2 – Safety Reliability Block Diagram

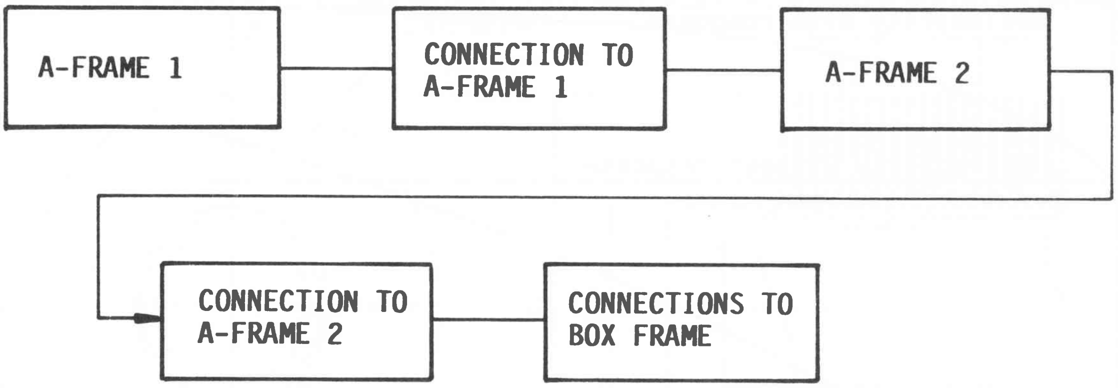

Figure 3 – Block Diagram for components in the Main Rotor Loadpath

To help structural engineers to choose weighting factors the concept of design confidence, i.e. the reliability of the design process, is introduced. The weighting factor for a structural component can be chosen more accurately by determining its design confidence. Design confidence is however difficult to calculate. It depends on various parameters: the design methods employed, design inputs, assumptions made in the design process and, if tests are performed to verify the design, the type of test and number of test samples. In order to determine design confidence for structures, graphs of design confidence as a function of the mentioned parameters have to be postulated for each part of the structure.

Design confidence is related to structure safety reliability. The allocated reliability may either be increased or decreased as the design confidence increases. For example, if there is uncertainty about the actual loads on a strut, the design confidence is low and a relatively large weighting factor, implying low reliability, has to be chosen. If during the design phase actual tests are performed, the design confidence will increase. If the tests indicate that the assumed loads were lower than the actual loads, a relatively low reliability should be allocated to the strut. Thus the large weighting factor is justified, see figure 4. If the tests indicate that the assumed loads were high, a higher reliability should be allocated to the designed strut. A smaller weighting factor is thus appropriate, figure 5.

This example only considers design confidence in relation to testing. Similarly design confidence is also influenced by the other parameters mentioned. These relationships also need to be postulated.

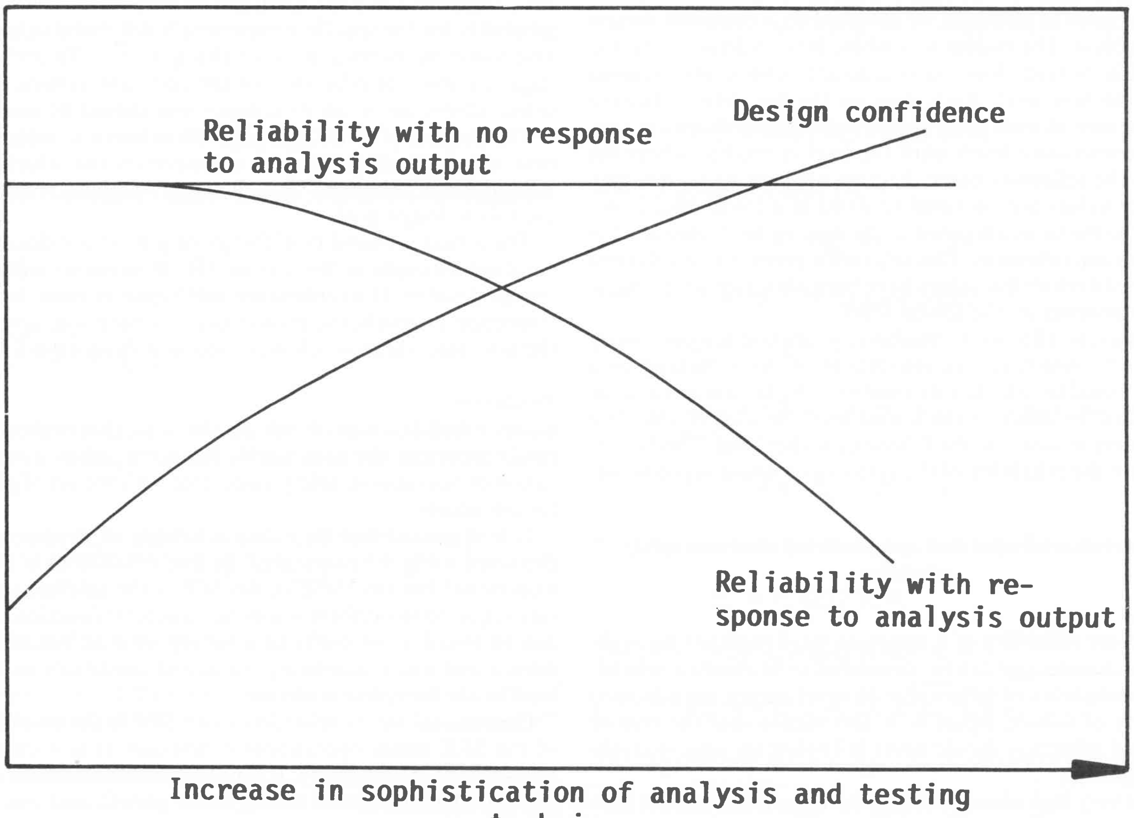

Figure 4 – Reliability/Design confidence relationship: Wall thickness decreased

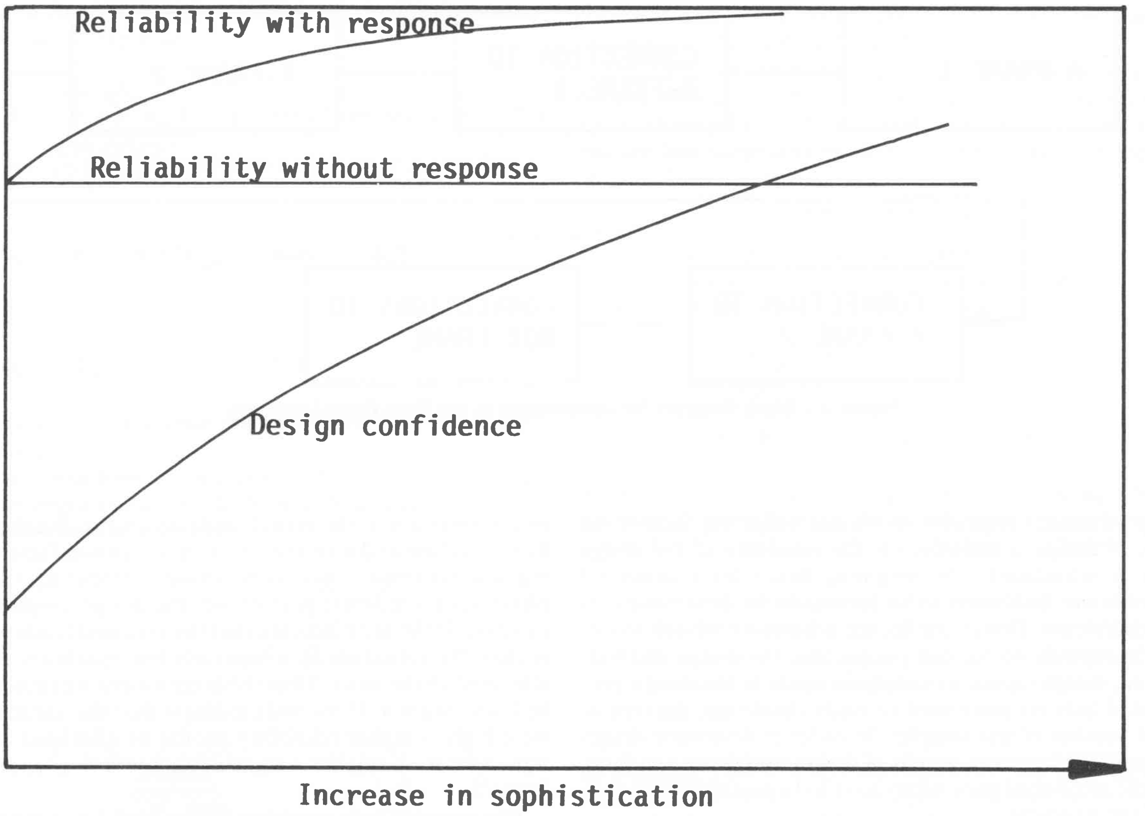

Figure 5 – Reliability/Design confidence relationship: Wall thickness increased

Reliability allocation and prediction

Reliability allocation is a top down procedure where a required reliability value is allocated or assigned to a complete system e.g. a structure. The system as a whole is considered as the top level and is broken down into different subsystems or items forming the first level. Each item on the first level is broken down into second level items and so forth. Reliabilities are allocated in consecutive levels until the level is reached where the blocks in the reliability block diagram are sufficiently detailed. Reliability values are allocated to items at a lower level in accordance to the value allocated to the item on the higher level of which they are sub-items. This top down procedure is followed through until reliability values have been allocated to the hardware components on the lowest level.

In contrast to allocation, reliability prediction is a bottom up procedure by which known reliabilities of items making up a system are used to calculate its reliability. Either known or allocated values for items on the lowest level are used to calculate the reliability of items on the following higher level. This is continued until the reliability of the system as a whole is predicted.

Interpretation of allocated and predicted structure safety reliability

Allocation

The allocated reliability of a structure stays constant throughout the service life, and can be considered as its absolute reliability. The probability of failure should never exceed the allocated probability of failure, figure 6. It also means that the true or operational reliability should never fall below the allocated reliability, figure 7.

When a very high absolute reliability value is allocated it indicates that the component should be designed for a life in excess of the design life of the structure. If the allocated reliability value is so high that it cannot be met, the component reliability should be increased through redesign. Alternatively the reliability of the components in a loadpath can be reallocated. The allocated reliability for the specific component is decreased to an acceptable value by increasing its weighting factor. To arrive at the required absolute reliability of the complete system, the allocated reliabilities of all its components should be met.

The purpose of safety reliability allocation is to assign or allocate absolute reliabilities to components for which the reliabilities are unknown. The allocated reliability value is then used as a design goal.

The actual probability of failure of a structure does not stay constant throughout the service life. It increases with time as cracks develop. If maintenance and repair is done at timeous inspection intervals, the probability of failure will never exceed the allocated value which was used as a design goal, figure 6.

Prediction

Safety reliability is predicted on the basis that it should accurately represent the real world. Predicted values are an indication of operational safety associated with the set of all critical failure points.

It is suggested that the safety reliability of structures can be predicted using the operational or true reliabilities of Structure Functional Entities (SFE's). An SFE is the smallest sub-structure required to perform a specific structural function. An SFE can be tested as an entity in a testing rig with boundary conditions and loads simulating the actual conditions as it would exist in the complete structure.

Operational safety reliability of an SFE is the true reliability of the SFE under operational conditions. It is a cumulative, time-dependent reliability. It should be calculated with an operational reliability model taking crack growth and crack detection into account. Operational reliability of a structure is determined by both the cumulative probability of failure and the

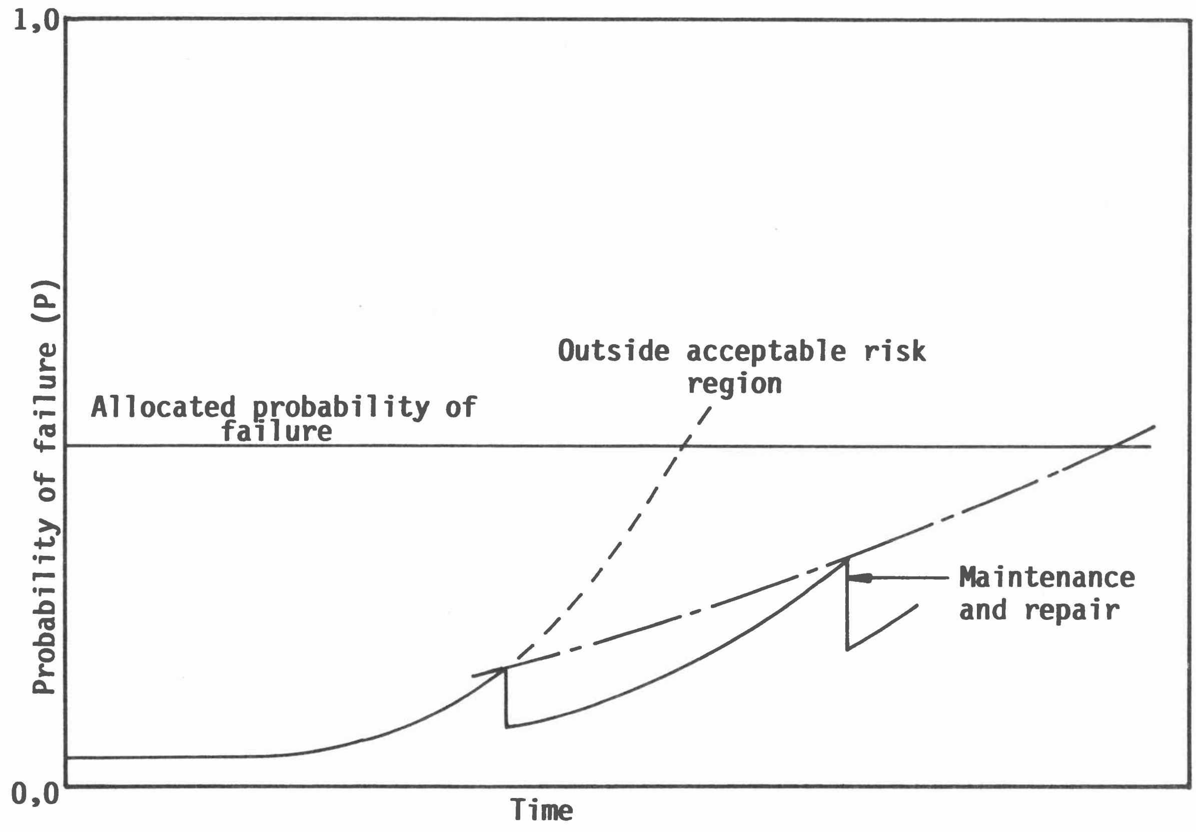

Figure 6 – Probability of failure versus Time

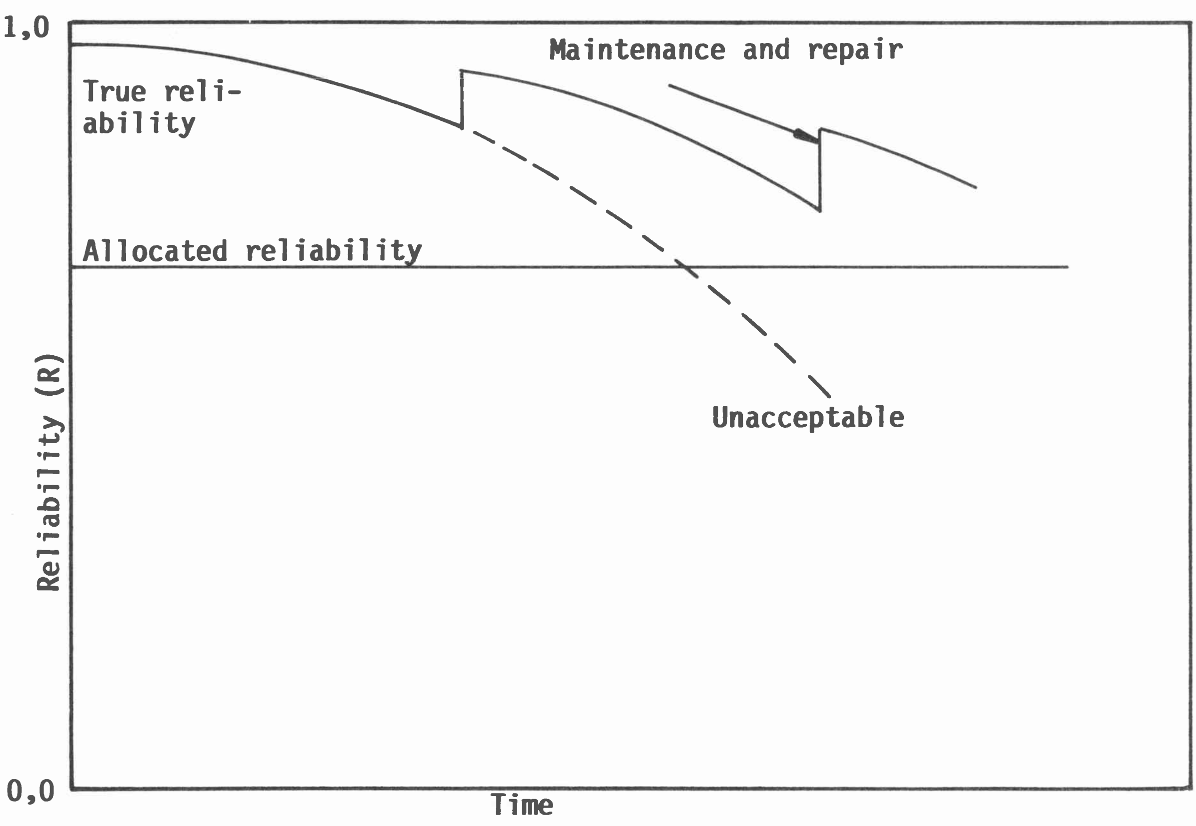

Figure 7 – Reliability versus Time

probability of not detecting a crack during the lifetime of the structure. A reliability model that can be used as a tool for scheduling of inspections, evaluating maintenance procedures and designing new structures is discussed in the literature [2, 3]. The reliability model includes parameterized distribution models for crack initiation, crack growth, fracture and crack detection. The parameters are determined by fracture mechanics, testing and analytical methods.

One of the purposes of safety reliability prediction is to determine early in the design phase whether the system will meet reliability requirements. At this stage operational safety reliability has not been determined for all SFE's. Absolute or allocated reliability is used instead. Designers therefore have to monitor reliability throughout the design phase of the complete structure. It is done by verifying predicted structure safety reliability in the laboratory through reliability development and qualification testing programs.

Conclusion

The techniques of reliability allocation and prediction has useful application in the field of structural design. Successful structural design previously depended on the completeness and accuracy of analytical methods. The design procedures were based on the use of safety factors and given probability of failure. This is not necessarily wrong, but is perceived by the reliability community to be insufficient. The described design philosophy puts the results of the analysis for all SFE's together to give the safety reliability of the complete system. It can also provide a tool for scheduling of inspections and determining the impact of structure safety reliability on maintenance and cost. Reliability allocation should therefore be part of the design phase of dynamic structures.

References

1. Billinton R. and Allan R. N. (1983). Reliability Evaluation of Engineering Systems, chapt. 5, pp 81-120. Pitman, London.
2. Kesling G. D. and Whittaker I. C. (1985). A simulation model of railroad reliability. *Simulation* 44 (4), 168-180.
3. Whittaker I. C. and Saunders S. C. (1973). Application of reliability analysis to aircraft structures subject to fatigue crack growth and periodic structural inspection. Air Force Materials Laboratory technical report AFML-TR-73-92, Dayton, Ohio.

Further reading

4. Whittaker I. C. and Besuner P. (1969). A reliability analysis approach to fatigue life variability of aircraft structures. Air Force Materials Laboratory technical report AFML-TR-69-65, Dayton, Ohio.
5. Whittaker I. C. and Saunders S. C. (1973). Exploratory development on application of reliability analysis to aircraft structures considering interaction of cumulative fatigue damage and ultimate strength. Air Force Materials Laboratory technical report AFML-82-283, Dayton, Ohio.