

# Dimensioning Heat Exchangers for Existing Dry Cooling Towers

by

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*A method is proposed whereby the optimum dimensions of finned tube bundles for installation in an existing cooling tower are determined, subject to certain practical constraints and for a given cost structure. The influence of variations in the independent parameters is quantified.*

### Nomenclature

|                  |   |
|------------------|---|
| A                | area m <sup>2</sup>                                   |
| C                | cost factor   |
| c <sub>p</sub>   | specific heat J/kg °C                                 |
| d                | diameter m  |
| E                | Euler number  |
| F                | temperature correction factor                         |
| f                | friction factor                                       |
| G                | mass velocity kg/s m <sup>2</sup>                     |
| g                | gravitational acceleration m/s <sup>2</sup>           |
| H                | height m  |
| K                | loss coefficient                                      |
| k                | thermal conductivity W/m °C                           |
| L                | length m  |
| m                | mass flow rate kg/s                                   |
| n                | number  |
| P                | pitch m, or power W                                   |
| p                | pressure N/m <sup>2</sup>                             |
| Δp               | pressure differential N/m <sup>2</sup>                |
| Pr               | Prandtl number  |
| Q                | heat transfer rate W                                  |
| R <sub>c</sub>   | thermal contact resistance m <sup>2</sup> K/W         |
| Re               | Reynolds number                                       |
| T                | temperature °C  |
| ΔT <sub>lm</sub> | logarithmic mean temperature difference °C            |
| U                | overall heat transfer coefficient W/m <sup>2</sup> °C |
| W                | width m   |
| v                | velocity m/s  |

|   |                            |
|---|----------------------------|
| p | passes                     |
| r | tube rows or root          |
| t | total, transversal or tube |
| w | water                      |

### Introduction

Although dry air-cooled condensers have been used in relatively small power plants for many years, an increasing number of larger units have been brought into operation more recently. At present plants having outputs of almost 4 000 MWe are under construction.

In this study the possibility of replacing the finned tube bundles of air-cooled systems with optimally dimensioned units is investigated. Replacement may be required where the original heat exchangers are damaged or have become ineffective due to corrosion, erosion or fouling. In addition to the fact that new types of finned tubes may be developed, the relative costs of materials, labour and energy change continually with time and location, resulting in correspondingly different optimised designs.

For purposes of illustration the proposed design procedure is applied to a hyperbolic natural draught cooling tower.

### Analysis

Consider the example of a large natural draught dry cooling tower as shown schematically in figure 1.

External to the tower the pressure difference in the stagnant

### Greek letters

|   |                           |
|---|---------------------------|
| ε | effectiveness             |
| η | efficiency                |
| ρ | density kg/m <sup>3</sup> |
| μ | dynamic viscosity kg/ms   |
| σ | contraction ratio         |
| θ | angle °                   |

### Subscripts

|    |                             |
|----|-----------------------------|
| a  | air                         |
| av | average                     |
| b  | bundles                     |
| ct | cooling tower               |
| e  | heat exchanger or effective |
| f  | fin                         |
| fr | frontal                     |
| i  | inlet or inside             |
| ℓ  | longitudinal                |
| o  | outlet or outside           |

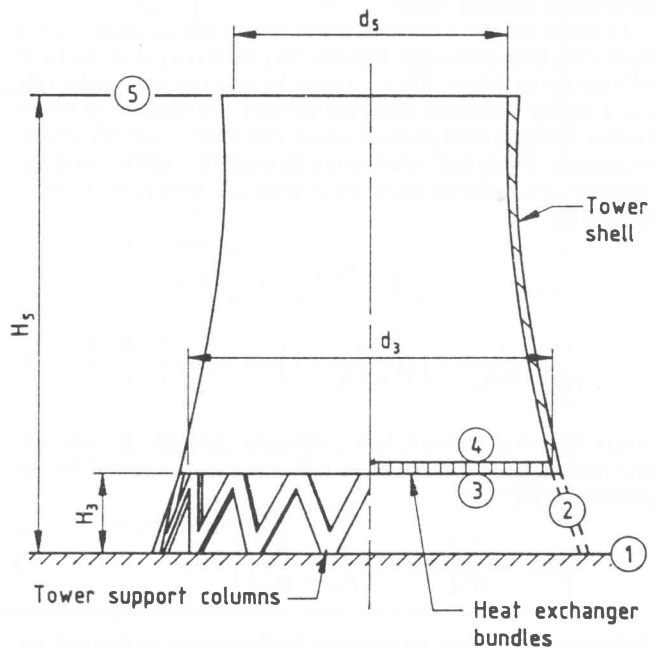


Fig. 1. Natural draft cooling tower

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air between ground level and the top of the tower, assuming constant density, is

$$p_{a1} - p_{a5} = \rho_{a1} g H_5 \quad (1)$$

Based on the assumption of the standard atmosphere, Montakhab [1] shows that this approximate equation deviates by less than 2% from the true pressure difference over a height of 213 m.

The air accelerates from stagnant ambient conditions at 1 to plane 4 after the heat exchanger bundles with a resultant change in pressure due to friction, form drag, inlet and outlet losses and flow acceleration. If  $H_3 \approx H_4$  a total pressure balance yields:

$$p_{a1} - (p_{a4} + 0,5 \rho_{a4} v_{a4}^2) = (K_{e3} + K_{ct}) 0,5 \rho_{a1} v_{a3}^2 - 0,5 \rho_{a4} v_{a4}^2 + \rho_{a1} g H_3 \quad (2)$$

where  $K_{e3}$  is the static pressure loss coefficient across the heat exchanger and  $K_{ct}$  is the tower pressure loss coefficient, both based on the tower cross-sectional inlet area  $A_3$ .

From 4 to 5 the air flows isentropically and with approximately constant density such that

$$(p_{a4} + 0,5 \rho_{a4} v_{a4}^2) - (p_{a5} + 0,5 \rho_{a4} v_{a5}^2) = \rho_{a4} g (H_5 - H_3) \quad (3)$$

Add equations (2) and (3) and equate the sum to equation (1)

$$(\rho_{a1} - \rho_{a4}) g (H_5 - H_3) = (K_{e3} + K_{ct}) 0,5 \rho_{a1} v_{a3}^2 + 0,5 \rho_{a4} v_{a5}^2 - 0,5 \rho_{a4} v_{a4}^2 \quad (4)$$

For  $d_3 = d_4$  this equation may be written in dimensionless form by dividing throughout by  $0,5 \rho_{a1} v_{a3}^2$  and by satisfying the continuity relation

$$\frac{(\rho_{a1} - \rho_{a4}) g (H_5 - H_3)}{0,5 \rho_{a1} v_{a3}^2} = (K_{e3} + K_{ct}) + \frac{\rho_{a1}}{\rho_{a4}} \left[ \left( \frac{d_3}{d_5} \right)^4 - 1 \right] \quad (5)$$

Equation (5) is known as the draught equation for the natural convection cooling tower.

In order to make more effective use of the available tower base area, heat exchanger bundles may be arranged in the form of V-arrays or deltas. The air stream leaving this heat exchanger has a highly turbulent distorted velocity distribution in which further losses in total pressure occur above the outlet of the heat exchanger. These and other losses through the inclined heat exchangers are included in the total pressure drop given by Mohandes [2],

$$\Delta p_{et0} = 0,5 \left( \frac{m_a}{A_{fr}} \right)^2 \left[ \frac{K_{ct}}{2} \left( \frac{1}{\rho_{a1}} + \frac{1}{\rho_{a4}} \right) + \frac{1}{\rho_{a1}} \left( \frac{1}{\sin \theta_{av}} - 1 \right) \left\{ \left( \frac{1}{\sin \theta_{av}} - 1 \right) + 2 K_{i0=90^\circ}^{0,5} \right\} + \frac{K_{dt}}{\rho_{a4}} \right] \quad (6)$$

where the total pressure loss coefficient through the heat exchanger under normal flow conditions can in general be expressed as [3]

$$K_{ct} = \frac{2}{\sigma^2} \left[ Eu + \left( \frac{\rho_{a1} - \rho_{a4}}{\rho_{a1} + \rho_{a4}} \right) \right] \quad (7)$$

Robinson and Briggs [4] propose the following correlation for the Euler number for bundles of staggered radially finned tubes:

$$Eu = \frac{\Delta p_e \rho_a}{G_{ca}^2} = \frac{18,93 n_r \left( \frac{d_r}{P_t} \right)^{0,927} \left( \frac{P_t}{P_t} \right)^{0,515}}{Re_a^{0,316}} \quad (8)$$

where  $Re_a = G_{ca} d_r / \mu_a = m_a d_r / \sigma \mu_a A_{fr}$

A static pressure drop coefficient for the V-array of bundles in the cooling tower, based on conditions at the inlet plane 3 is defined as

$$K_{e03} = \frac{\Delta p_{et0} - 0,5 \rho_{a1} v_{a3}^2 + 0,5 \rho_{a4} v_{a4}^2}{0,5 \rho_{a1} v_{a3}^2} = \frac{\Delta p_{et0}}{0,5 \rho_{a1} v_{a3}^2} + \left( \frac{\rho_{a1}}{\rho_{a4}} - 1 \right) \quad (9)$$

such that with equation (6) and the continuity relation

$$K_{e03} = \left( \frac{\pi d_3^2}{4 A_{fr}} \right)^2 \left[ \frac{K_{ct}}{2} \left( 1 + \frac{\rho_{a1}}{\rho_{a4}} \right) + \left( \frac{1}{\sin \theta_{av}} - 1 \right) \left\{ \left( \frac{1}{\sin \theta_{av}} - 1 \right) + 2 K_{i0=90^\circ}^{0,5} \right\} + \frac{\rho_{a1}}{\rho_{a4}} K_{dt} \right] + \left( \frac{\rho_{a1}}{\rho_{a4}} - 1 \right) \quad (10)$$

With  $K_{e3} = K_{e03}$  for the V-array layout, substitute equation (10) in equation (5) and find

$$\frac{(\rho_{a1} - \rho_{a4}) g (H_5 - H_3)}{0,5 \rho_{a1} v_{a3}^2} = \left( \frac{\pi d_3^2}{4 A_{fr}} \right)^2 \left[ \frac{K_{ct}}{2} \left( 1 + \frac{\rho_{a1}}{\rho_{a4}} \right) + \left( \frac{1}{\sin \theta_{av}} - 1 \right) \left\{ \left( \frac{1}{\sin \theta_{av}} - 1 \right) + 2 K_{i0=90^\circ}^{0,5} \right\} + \frac{\rho_{a1}}{\rho_{a4}} K_{dt} \right] + K_{ct} + \frac{\rho_{a1}}{\rho_{a4}} \left( \frac{d_3}{d_5} \right)^4 - 1 \quad (11)$$

The average flow incidence angle may be expressed by the following empirical relation:

$$\theta_{av} = 0,0019 \theta^2 + 0,9133 \theta - 3,1558 \quad (12)$$

where the apex semi-angle  $\theta$  is given in degrees.

Furthermore  $K_{i0=90^\circ}$  is the heat exchanger inlet contraction pressure loss coefficient under normal flow conditions ( $K_{i0=90^\circ} \approx 0,05$  for many industrial finned tube heat exchangers), while the downstream pressure loss coefficient can be expressed approximately by the following empirical relation

$$K_{dt} = \exp(5,83479 \cdot 10^{-4} \theta^2 - 0,1176997 \theta + 4,2817) \quad (13)$$

According to Hempel et al [5] the cooling tower pressure loss coefficient may be expressed as:

$$K_{ct} = 0,19 \left( \frac{d_3}{H_3} \right)^2 - (0,38 + 0,11 K_e^{0,7}) \frac{d_3}{H_3} + 4,9 \quad (14)$$

This equation includes a value of  $0,01 (d_3/H_3)^2$  which makes provision for the approximate resistance due to tower supports.

By introducing the gas law, equation (11) may be expressed approximately in terms of the ambient pressure, and temperatures before and after the heat exchanger as well as the air flow rate through the tower.

$$\frac{2 P_{a1}^2 \left( \frac{A_{fr}}{m_a} \right)^2 \left( 1 - \frac{T_{a1}}{T_{a4}} \right) g (H_5 - H_3)}{R^2 T_{a1}^2} = \frac{K_{ct}}{2} \left( 1 + \frac{T_{a4}}{T_{a1}} \right) + \left( \frac{1}{\sin \theta_{av}} - 1 \right) \left[ \left( \frac{1}{\sin \theta_{av}} - 1 \right) + 2 K_{i0=90^\circ}^{0,5} \right] + \frac{T_{a4}}{T_{a1}} K_{dt} + \left( \frac{4 A_{fr}}{\pi d_3^2} \right)^2 \left[ K_{ct} + \frac{T_{a4}}{T_{a1}} \left( \frac{d_3}{d_5} \right)^4 - 1 \right] \quad (15)$$

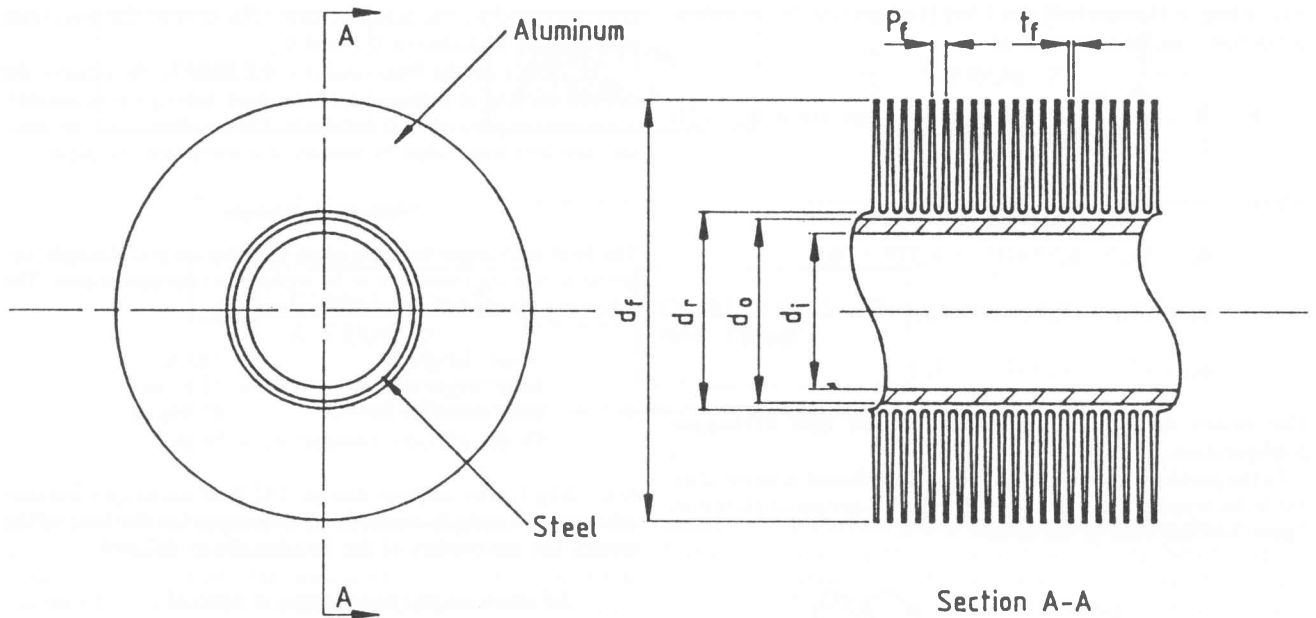


Fig. 2. Bi-metallic finned tube

The ultimate relation between the air flow rate through the tower and the outlet air temperature at given ambient conditions will be determined by the heat transfer characteristics of the finned tube bundles.

The heat transfer rate from the warm water to the air stream is

$$Q = m_a c_{pa} (T_{a4} - T_{a1}) = m_w c_{pw} (T_{w1} - T_{w0})$$

or

$$Q = UA F \Delta T_{tm} \tag{16}$$

where

$$UA = (1/h_{ac} A_a + 1/h_w A_w)^{-1}$$

For a clean bi-metallic radially finned tube as shown in figure 2,

$$h_{ac} A_a = \left[ \frac{1}{h_a \epsilon_f A_a} + \frac{\ln(d_o/d_i)}{2 \pi k_t L_{tt}} + \frac{\ln(d_r/d_o)}{2 \pi k_f L_{tt}} + \frac{R_c}{A_o} \right]^{-1} \tag{17}$$

According to Briggs and Young [6] the air-side heat transfer coefficient  $h_a$  through bundles of staggered radially finned tubes is given by

$$h_a = \frac{0,134 k_a Pr_a^{0,33} Re_a^{0,681}}{d_r} \left[ \frac{2(P_f - t_f)}{(d_f - d_r)} \right]^{0,2} \left[ \frac{(P_f - t_f)}{t_f} \right]^{0,1134} \tag{18}$$

where  $Re_a = G_{ca} d_r / \mu_a = m_a d_r / \sigma \mu_a A_r$

The effectiveness of the finned surface is expressed in terms of the fin efficiency, i.e.

$$\epsilon_f = 1 - A_f(1 - \eta_f)/A_a$$

According to Schmidt [7] the fin efficiency for radial fins can be determined approximately from

$$\eta_f = \frac{\tanh(b d_r \phi/2)}{(b d_r \phi/2)}$$

where  $\phi = (d_f/d_r - 1) [1 + 0,35 \ln(d_f/d_r)]$

and  $b = (2 h_a/k_f t_f)^{0,5}$

The water-side heat transfer coefficient is according to Gnielinski [8]

$$h_w = \frac{k_w f_w (Re_w - 1000) Pr_w [1 + (d_i/L_t)^{0,67}]}{8 d_i [1 + 12,7 (f_w/8)^{0,5} (Pr_w^{0,67} - 1)]} \tag{19}$$

where

$$Re_w = 4 m_w n_{wp} / \pi d_i \mu_w n_{tb} n_b$$

and for a smooth tube [9],

$$f_w = (1,82 \log_{10} Re_w - 1,64)^{-2} \tag{20}$$

The wetted inside tube surface area is

$$A_w = \pi d_i L_t n_{tb} n_b$$

If the water mass flow rate per unit cross-sectional inside tube area is given by

$$G_w = 4 m_w n_{wp} / \pi d_i^2 n_{tb} n_b$$

the pressure drop per unit length of finned tube is

$$\Delta p_w = f_w G_w^2 / 2 \rho_w d_i$$

The total water pumping power is thus

$$P_w = \pi d_i^2 \Delta p_w G_w L_t n_{tb} n_b / 4 \rho_w \tag{21}$$

The logarithmic mean temperature difference is defined as

$$\Delta T_{tm} = \frac{(T_{w1} - T_{a4}) - (T_{w0} - T_{a1})}{\ln [(T_{w1} - T_{a4}) / (T_{w0} - T_{a1})]} \tag{22}$$

According to Roetzel [10], the LMTD correction for crossflow conditions can be expressed as

$$F = 1 - \sum_{i=1}^4 \sum_{k=1}^4 a_{ik} (1 - \phi_p)^k \sin(2i \arctan \phi_a / \phi_w) \quad (23)$$

where

$$\phi_p = (\phi_a - \phi_w) / \ln [(1 - \phi_w) / (1 - \phi_a)]$$

$$\phi_a = (T_{a4} - T_{a1}) / (T_{wi} - T_{a1})$$

$$\phi_w = (T_{wi} - T_{wo}) / (T_{wi} - T_{a1})$$

The values of  $a_{ik}$  are individual to each heat exchanger configuration.

In the particular tower to be analysed the finned tube bundles are to be arranged radially in the form of V-arrays as shown in figure 3 in the base of the tower.

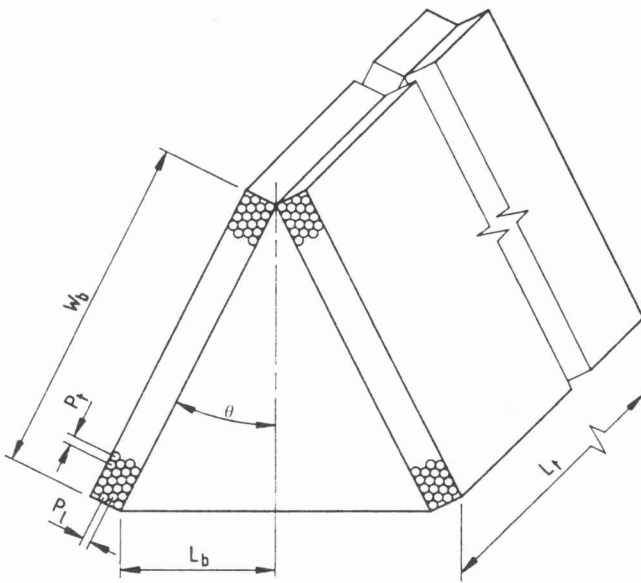


Fig. 3. Heat exchanger bundles

The final "optimised" retrofit finned tube geometry and layout within certain prescribed constraints is a function of material and other costs. For purposes of illustration it will be assumed that the total effective finned tube cost can be expressed as

$$C_{ft} = [C_{wf} (C_t + C_f) + C_{fx}] L_t n_{tb} n_b \quad (24)$$

where  $C_{wf}$  is a weighting factor

The core tube cost per unit length is

$$C_t = \frac{\pi}{4} \rho_t (d_o^2 - d_i^2) C_{tm}$$

where  $C_{tm}$  is the tube material cost per unit mass.

Similarly the cost of the fin material per unit tube length is

$$C_f = \frac{\pi P_f}{4 P_t} [(d_f^2 - d_t^2) t_f + (d_o^2 - d_o^2) (P_f - t_f)] C_{fm}$$

where  $C_{fm}$  is the fin material cost per unit mass.

The fixed cost of the tube per unit length  $C_{fx}$ , covers other

costs incurred by the manufacturer. The cost of the manifolds may also be included in  $C_{fx}$  and  $C_{wf}$ .

The object of the final analysis will thus be to achieve the desired cooling at a minimum total cost, taking into consideration certain prescribed constraints. The results of such an analysis are best illustrated by means of a numerical example.

### Numerical example

The heat exchanger bundles in an existing natural draught hyperbolic cooling tower are to be replaced at minimum cost. The following tower dimensions are specified:

|                                |            |
|--------------------------------|------------|
| Tower height $H_s$             | = 120 m    |
| Inlet height $H_3$             | = 13,67 m  |
| Inlet diameter $d_3$           | = 82,958 m |
| Outlet (throat) diameter $d_5$ | = 58 m.    |

According to the original design, 142 heat exchanger bundles are arranged radially in the form of V-arrays in the base of the tower. The particulars of the bundles are as follows:

|  |           |
|--|-----------|
| Effective length (finned tube) of bundle $L_t$ | = 15 m    |
| Effective width of bundle $W_b$                | = 2,262 m |
| Bundle semi-apex angle $\theta$                | = 30,75°  |
| Number of water passes $n_{wp}$                | = 2       |
| Number of tube rows $n_r$                      | = 4       |

To minimise cost it is recommended that the original bundle base support be employed and that no changes be made to the water pipe layout to and from the bundles or the water pump. The bundle support structure imposes the restriction that  $L_b = W_b \sin \theta = 1,157$  m.

The existing pump provides a water flow rate of 4 390 kg/s at a pressure drop of 16 302 N/m<sup>2</sup> through the finned tubes and deviations from these values are not permitted. In effect this also means that the pumping power  $P_w$  remains unchanged.

Due to structural considerations the tube wall and the fin root thickness are assumed constant at 1,655 mm and 1,08 mm respectively. The contact resistance  $R_c$  is assumed to be negligible.

The cooling tower is required to remove 365,08 MW at an inlet water temperature of 61,45°C, an ambient air pressure at ground level of 84 600 N/m<sup>2</sup> and a corresponding air temperature of 15,6°C.

In order to achieve an optimum design, the independent parameters  $t_f$ ,  $d_f$ ,  $\theta$ ,  $d_i$ ,  $P_t$ ,  $P_f$ ,  $P_t$  and  $n_{wp}$  may be varied subject to the constraints specified. A fin pitch of 2,35 mm is arbitrarily chosen as the design reference value. Where fouling is of significance very small fin pitches are avoided. The values of  $C_{wf}$ ,  $C_{tm}$ ,  $C_{fm}$  and  $C_{fx}$  are taken as 2, 0,8\$/kg, 4,2\$/kg and 2\$/m respectively.

The cost minimisation problem may be solved numerically with a general purpose algorithm for non-linear constrained optimisation. A Generalised Reduced Gradient algorithm [11] and a Constrained Variable Metric algorithm [12], were investigated and it was found that the Variable Metric algorithm was very efficient when applied to this problem. All results presented in this paper were obtained using the FORTRAN routine VMCWD [13]. Derivatives of the cost function and constraints were calculated numerically. Variables and constraints were carefully scaled, since they differ greatly in magnitude. All calculations were performed in double precision (approximately 16 decimal digits) in order to obtain reasonably accurate values for the derivatives. On average, routine VMCWD called the routines which calculate the cost function and constraints and their gradients, 6 to 10 times for each complete optimisation.

The results of the cost minimisation are presented in table 1. The values of the independent parameters corresponding to the minimum cost are taken as reference values (first line of table 1).

In order to investigate the dependence of the minimum cost on each parameter, one parameter is then perturbed and a minimisation performed with respect to the other parameters. The influence of various parameters on the total cost are also shown graphically in figure 4.

**Conclusions**

According to the results shown in table 1 the optimum fin thickness is only 0,0903 mm while the fin diameter is 61,95 mm.

In practice somewhat thicker fins are preferred resulting in a corresponding increase in system cost as shown in table 1.

According to the analysis the finned tubes are closely packed at optimum conditions if a triangular pattern is assumed. For larger fin thicknesses a more open arrangement is preferable. The influence of the longitudinal pitch is shown for both a triangular tube layout and for the case where  $P_l$  is not tied to the transversal pitch. As  $P_l$  increases the system cost is reduced. This reduction is however only applicable within certain limits in which the finned tube correlations apply.

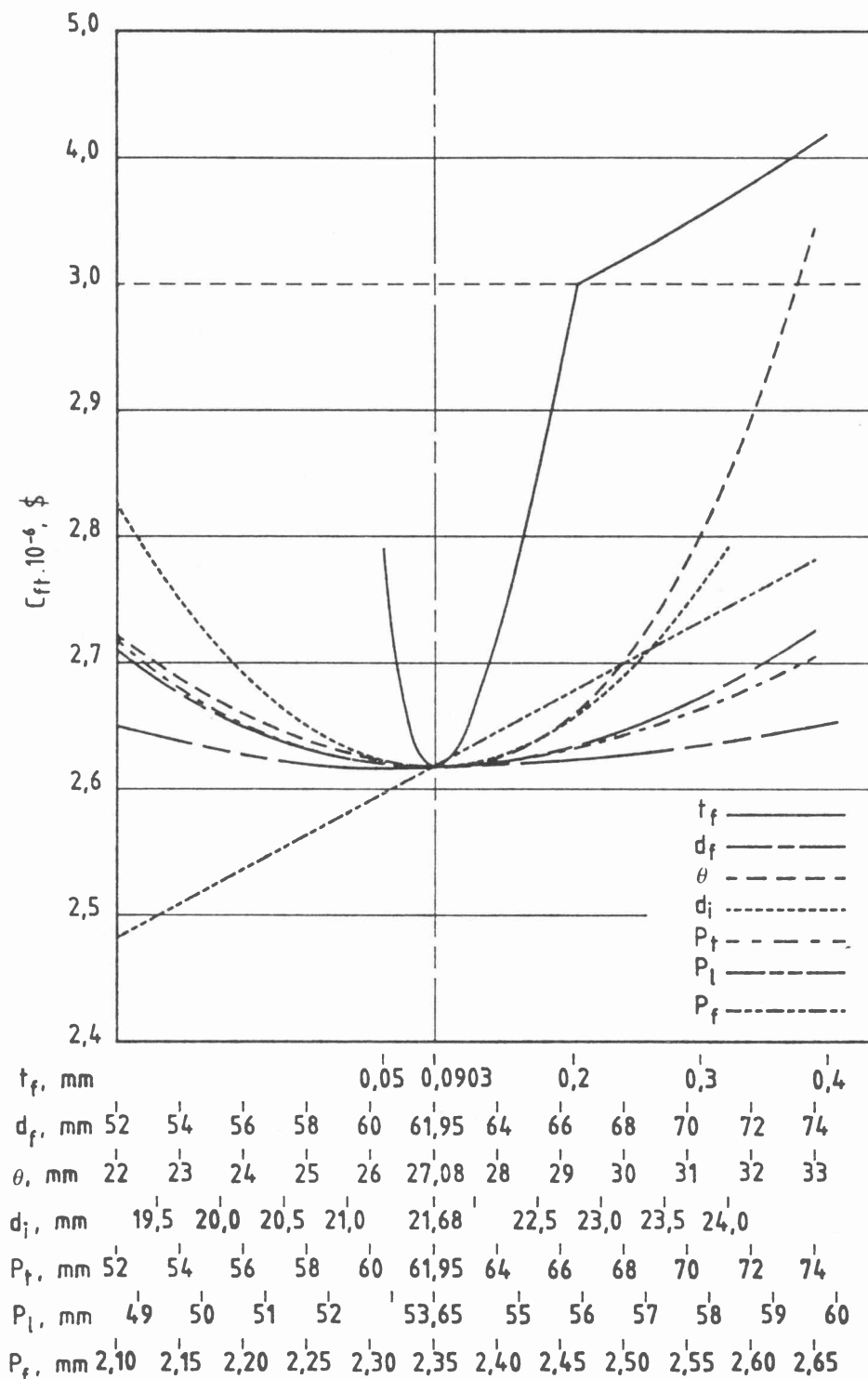


Fig. 4. Influence of changes in independent variables on cost

Table 1 Optimised finned tube bundles ( $n_{wp} = 2$ )

| Parameter                          | $t_r$<br>(mm) | $d_r$<br>(mm) | $\theta$<br>(°) | $d_i$<br>(mm) | $P_t$<br>(mm) | $P_t$<br>(mm) | $P_r$<br>(mm) | C<br>(\$) |
|------------------------------------|---------------|---------------|-----------------|---------------|---------------|---------------|---------------|-----------|
| (Optimum)                          | 0,0903        | 61,95         | 27,08           | 21,68         | 61,95         | 53,65         | 2,35          | 2 617 867 |
| $t_r$                              | 0,05          | 55,67         | 22,68           | 19,57         | 55,67         | 48,21         | 2,35          | 2 789 929 |
|                                    | 0,1           | 62,79         | 27,88           | 22,01         | 62,79         | 54,37         |               | 2 623 508 |
|                                    | 0,2           | 64,71         | 30,35           | 23,46         | 68,86         | 59,63         |               | 2 984 172 |
|                                    | 0,3           | 64,40         | 30,61           | 23,70         | 70,19         | 60,78         |               | 3 545 850 |
|                                    | 0,4           | 63,22         | 30,00           | 23,48         | 69,75         | 60,40         |               | 4 181 762 |
| $d_r$                              | 0,0768        | 50            | 25,20           | 19,85         | 52,40         | 45,37         | 2,35          | 2 756 065 |
|                                    | 0,0831        | 55            | 26,24           | 20,70         | 56,37         | 48,81         |               | 2 661 283 |
|                                    | 0,0878        | 60            | 27,04           | 21,41         | 60,00         | 51,96         |               | 2 621 230 |
|                                    | 0,0940        | 65            | 27,08           | 22,07         | 65,00         | 56,29         |               | 2 625 553 |
|                                    | 0,0996        | 70            | 26,90           | 22,64         | 70,00         | 60,62         |               | 2 668 214 |
| $\theta$                           | 0,0734        | 60,38         | 21              | 20,60         | 68,68         | 59,48         | 2,35          | 2 762 512 |
|                                    | 0,0793        | 60,78         | 24              | 21,13         | 64,73         | 56,06         |               | 2 659 733 |
|                                    | 0,0895        | 61,94         | 27              | 21,65         | 61,94         | 53,64         |               | 2 617 917 |
|                                    | 0,1328        | 64,69         | 30              | 22,82         | 64,69         | 56,02         |               | 2 702 217 |
|                                    | 0,2016        | 72,15         | 33              | 24,56         | 72,15         | 62,48         |               | 3 044 095 |
| $d_i$                              | 0,039         | 44,41         | 22,16           | 18            | 45,46         | 39,37         | 2,35          | 3 080 373 |
|                                    | 0,079         | 58,57         | 26,26           | 21            | 58,57         | 50,73         |               | 2 632 745 |
|                                    | 0,139         | 74,33         | 29,82           | 24            | 74,33         | 64,37         |               | 2 791 658 |
| $P_t$                              | 0,0732        | 50,00         | 26,11           | 19,75         | 50            | 43,30         | 2,35          | 2 767 791 |
|                                    | 0,0878        | 60,00         | 27,04           | 21,41         | 60            | 51,96         |               | 2 621 230 |
|                                    | 0,1011        | 67,16         | 26,07           | 22,40         | 70            | 60,62         |               | 2 660 119 |
| $P_t$<br>( $P_t = 0,866P_t$ )      | 0,0763        | 51,96         | 26,39           | 20,12         | 51,96         | 45            | 2,35          | 2 718 706 |
|                                    | 0,0922        | 63,51         | 27,09           | 21,88         | 63,51         | 55            |               | 2 619 920 |
|                                    | 0,1075        | 69,86         | 25,14           | 22,70         | 75,06         | 65            |               | 2 719 226 |
| $P_t$<br>( $P_t \geq P_{t \min}$ ) | 0,0863        | 58,89         | 26,99           | 21,25         | 58,89         | 51            | 2,35          | 2 626 282 |
|                                    | 0,0870        | 58,89         | 27,36           | 21,34         | 58,89         | 55            |               | 2 602 379 |
|                                    | 0,0876        | 58,77         | 27,71           | 21,42         | 58,77         | 59            |               | 2 580 370 |
|                                    | 0,0881        | 58,66         | 28,04           | 21,50         | 58,66         | 63            |               | 2 559 978 |
| $P_r$                              | 0,0828        | 61,91         | 28,17           | 22,22         | 63,82         | 55,27         | 2,1           | 2 482 365 |
|                                    | 0,0854        | 62,07         | 27,83           | 22,00         | 62,83         | 54,41         | 2,2           | 2 537 426 |
|                                    | 0,0884        | 62,09         | 27,40           | 21,79         | 62,09         | 53,77         | 2,3           | 2 591 074 |
|                                    | 0,0922        | 61,81         | 26,76           | 21,57         | 61,81         | 53,53         | 2,4           | 2 644 808 |
|                                    | 0,0959        | 61,53         | 26,14           | 21,36         | 61,53         | 53,29         | 2,5           | 2 699 153 |
| $C_{im}$                           | 0,0857        | 60,66         | 26,77           | 21,42         | 60,66         | 52,53         | 2,35          | 2 418 587 |
|                                    | 0,0933        | 62,78         | 27,28           | 21,84         | 62,78         | 54,36         |               | 2 748 466 |
|                                    | 0,1005        | 64,74         | 27,73           | 22,22         | 64,74         | 56,06         |               | 3 067 823 |
| $C_{im}$                           | 0,1039        | 65,64         | 27,94           | 22,39         | 65,66         | 56,84         | 2,35          | 2 204 230 |
|                                    | 0,0921        | 62,45         | 27,20           | 21,78         | 62,45         | 54,08         |               | 2 550 819 |
|                                    | 0,0843        | 60,26         | 26,67           | 21,34         | 60,26         | 52,19         |               | 2 880 734 |
| $C_{wf}$                           | 1,0           | 0,1118        | 67,71           | 28,41         | 22,79         | 67,71         | 2,35          | 1 630 395 |
|                                    | 3,0           | 0,0818        | 59,55           | 26,50         | 21,20         | 59,55         |               | 3 571 350 |
|                                    | 5,0           | 0,0744        | 57,35           | 25,95         | 20,75         | 57,35         |               | 5 449 476 |
| $C_{fix}$                          | 1,0           | 0,0773        | 58,21           | 26,17         | 20,93         | 58,21         | 2,35          | 2 256 577 |
|                                    | 3,0           | 0,1016        | 65,04           | 27,81         | 22,28         | 65,04         |               | 2 950 093 |
|                                    | 5,0           | 0,1211        | 70,07           | 28,93         | 23,23         | 70,07         |               | 3 554 603 |
| $n_{wp} = 4$                       |               |               |                 |               |               |               |               |           |
| (Optimum)                          | 0,0826        | 77,39         | 26,72           | 36,18         | 83,87         | 72,63         | 2,35          | 2 611 998 |
| $t_r$                              | 0,1           | 78,88         | 27,63           | 37,10         | 86,91         | 75,27         | 2,35          | 2 629 576 |
|                                    | 0,2           | 82,08         | 29,97           | 39,42         | 94,67         | 81,99         |               | 3 051 830 |
|                                    | 0,3           | 81,59         | 30,10           | 39,61         | 95,50         | 82,71         |               | 3 666 459 |
|                                    | 0,4           | 79,70         | 29,37           | 39,00         | 93,77         | 81,20         |               | 4 354 121 |

Where fouling is not serious, a reduction in fin pitch will generally result in a lower cost system.

The influence of variations in the cost structure are also shown in table 1.

The possibility of installing a four-pass heat exchanger was also investigated. The resultant optimum design is only fractionally less costly than the two-pass system. For the thicker fin the converse is true. Furthermore it is noted that the inside diameter of the tubes is considerably larger to ensure a pressure drop on the water-side that does not exceed that specified for the pump.

The authors have extended the above optimisation method to include the designs of complete cooling systems for new plants.

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