

# Modelling gas flow in a direct injection diesel engine: II – Turbulence

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## Abstract

*Turbulence in the combustion chamber of an engine has a strong influence on combustion characteristics and heat transfer rates. The objective of this investigation was to develop a gas turbulence model applicable to direct injection diesel engines. The model was designed to generate the expected trends that could be ascertained from the limited experimental data available. In addition the model was kept as simple as possible without sacrificing a practical level of accuracy. The output from the model compared favourably with published data in terms of trends and orders of magnitude.*

## Introduction

The air motion within the combustion bowl of a direct injection (DI) diesel engine has a considerable bearing on the combustion characteristics and therefore on emissions and efficiency. In addition the gas velocities adjacent to the cylinder walls have a significant influence on convective heat transfer rates.

The measurement of air flow in the DI diesel engine has provided a major challenge owing to restricted access to the combustion chamber. Nevertheless, non-intrusive techniques of carrying out velocity measurements such as laser doppler anemometry have been applied with a reasonable degree of success. In parallel with the development of these techniques, the modelling of air motion has received considerable attention and has helped considerably in the understanding and interpretation of the in-cylinder flow processes.

The flow patterns are normally divided into mean flow components which are represented by squish and swirl, and fluctuating flow components, which are referred to as turbulence. While it is possible to model the mean flow components fairly accurately [1], the subject of turbulence remains somewhat nebulous because of the lack of experimental data.

In spite of its complexity, turbulence cannot be ignored when characterising the flow processes in a diesel engine as it contributes significantly to heat transfer and to combustion. Tabaczynski [2] reported that two of the major features of turbulent flow were its definite structure in the flow field and its tendency to be governed by chamber geometry near top dead centre of compression. Turbulence models presently in use provide reasonable levels of accuracy in terms of expected trends. However, they generally employ procedures which provide ensemble-averaged solutions and can only be validated by ensemble-averaged velocity data [3].

The objective of this work was to develop a gas turbulence model applicable to DI diesel engines, while maintaining simplicity without sacrificing a practical level of accuracy.

## Literature review

It has been generally accepted that the shear flow past the intake valve is the major source of turbulence in engine cylinders and that this turbulence persists throughout the cycle [2,4]. After inlet valve closing (IVC) viscous shear stresses perform deformation work which increases the internal energy of the fluid at the expense of kinetic energy of the turbulence. Turbulence therefore requires a continuous supply of energy to make up for these viscous losses [5]. Viscous dissipation therefore tends to reduce turbulence intensity during the cycle.

An example of the relative magnitude and trends that can be expected for a diesel engine with bowl-in-piston is illustrated in Figure 1 representing the output from the model of Morel and Karibar [6] in which the combustion chamber was divided into three regions. These regions comprised the bowl, the chamber volume directly above the bowl and the remaining volume bordered by the piston crown, cylinder liner and an imaginary cylinder of the same radius as the bowl. The dominating peak of turbulence generated early in the intake stroke shown in Figure 1 decays fairly rapidly for the remainder of the intake stroke and approximately half way into the compression phase as a result of viscous dissipation. As the intake valve closes the sharp shear layers disappear but the turbulence generated by them remains [7]. The initial conditions of turbulence for the subsequent compression process are established at this point as for swirl.

The piston bowl to bore diameter ratio and the piston crown-head clearance at TDC are factors which have a significant effect on the in-bowl flow field and hence on the generation of turbulence. Production and dissipation of turbulence during compression is governed by four main factors excluding the initial generation of turbulence during the intake stroke. Shear within the combustion chamber is created by angular velocity gradients caused by swirl deviating from solid body rotation. This shear process is a mechanism for generating and dissipating turbulence. Shear at the boundary layer adjoining the chamber walls has a similar mechanism and is also taken into account.

A third contributor to production and dissipation of turbulence is the result of compression. During the com-

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pression and combustion processes the turbulent kinetic energy is amplified due to the rapid distortion that the cylinder charge undergoes with rising cylinder pressures. This effect is illustrated in Figure 1 by the small rise in turbulence intensity towards the end of compression. Ac-

cording to Wong and Hault [8] piston motion during compression caused an amplification of turbulence based on the rapid distortion theory. This effect was also referred to by Arcoumanis and Whitelaw [3] as turbulence production by compressive strain.

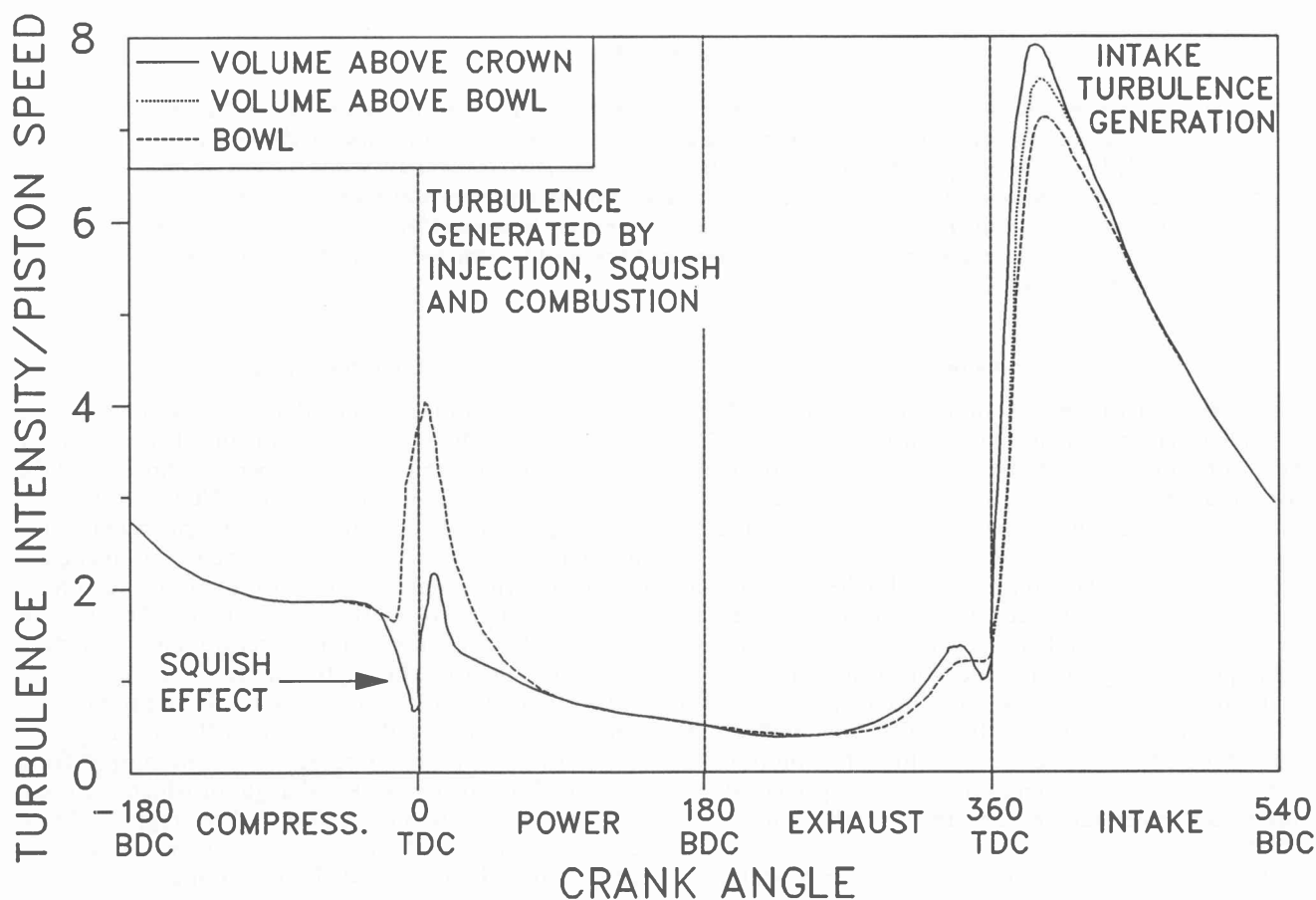


Figure 1 - Turbulence intensity in the three flow regions (after [6]).

The last factor is squish which can have a major effect on the production of turbulence as shown in Figure 1. In the cup region, Morel and Keribar [6] catered for a large increase in turbulence resulting from the inflow of gases as well as injection and combustion contrary to the findings of Rao and Bardon [4]. In Figure 1 turbulence in the squish region above the piston crown decreases abruptly close to TDC because of the small clearance between head and piston increasing viscous dissipation. However, immediately after TDC there is a sharp rise in turbulence caused by reversed squish. It should be noted that the turbulence remains uniform for all regions of the chamber over most of the compression stroke with differences occurring from 20-30° BTDC onwards. The curves emerge again further on in the expansion stroke.

The turbulence also tends to be isotropic during the compression and expansion strokes. Fansler [9] concluded that near-TDC peaks in rms fluctuation intensity were dominated by squish-induced velocity fluctuations which tended to decrease with increasing bowl-bore diameter ratio.

The model most extensively used in engines for turbulence is the  $k - \epsilon$  model which takes into account the rate

of turbulent kinetic energy production and the rate of viscous dissipation. Davis and Tabaczynski [10] provided an equation for the production and dissipation of turbulence kinetic energy which included variables responsible for production due to compression, squish and intake generated turbulence respectively. These terms were offset by a dissipation variable.

As the intention of this project is to examine and model the events taking place from the point of IVC through the compression and expansion strokes to the point of exhaust valve opening (EVO), a factor that had to be considered was a starting value for the turbulence intensity. Rao and Bardon [4] indicated that reported values for turbulence intensity at intake valve closure varied between 20-50% of mean piston speed. They suggested a representative value for the turbulence kinetic energy/unit mass at the start of the compression stroke of 12.5% of the square of the mean piston speed. These observations were largely based on measurements in cylindrical chambers with flat pistons having relatively low compression ratios.

For chambers with flat pistons and no squish, turbulence levels of 0,3-0,5 times the mean piston speed have

been measured at TDC under different conditions with tendencies towards homogeneity and isotropy [3]. Hayder, Varna and Bracco [11] concluded from computations and measurements in engines with pancake chambers that the maximum value of the TDC turbulence intensity was about one half the mean piston speed when the intensity of turbulence was defined as the intensity of the high frequency components of the velocity fluctuations.

It was apparent from the research published that turbulence remained a variable that was difficult to measure and model accurately. However, some guidelines for models had been established which afforded acceptable levels of accuracy and which provided expected trends.

### Formulation of model

The method adopted for calculating turbulence intensity was similar to that of Davis and Borgnakke [12]. The method relied on the classical  $k$  and  $\epsilon$  turbulence approach. The model of Davis and Borgnakke [12] was an extension of the work done by Launder and Spalding [13] and followed on the model reported by Borgnakke, Davis and Tabaczynski [14]. This model had subsequently demonstrated its reliability in applications by Belaire, Davis, Kent and Tabaczynski [15], Davis, Tabaczynski and Belaire [16] and Davis and Tabaczynski [10].

The combustion chamber was divided into two volumes. The inner volume of the chamber represented a cylindrical volume of the same diameter as the piston bowl and the outer volume comprised the remaining volume above the piston crown. To maintain simplicity in the model the production of turbulence due to shear at the interface between the inner and outer demarcated volumes in the combustion chamber was ignored. In addition production of turbulence due to shear at the walls was disregarded. The graphs of angular momentum flux provided by Murakami, Arai and Hiroyasu [17] illustrated the relatively small contribution of these factors compared to squish, which, according to Fansler [9] dominated as a producer of turbulence near TDC. The equation for production and dissipation of turbulent kinetic energy given by Davis and Tabaczynski [10] in fact did not include terms for production due to shear within the gas flow and at the walls.

The production of turbulence due to compression was also included in the model as a number of researchers had indicated that it was significant [6, 8, 14, 18]. The  $k - \epsilon$  model consisted of two coupled differential equations describing the variation of turbulent kinetic energy and its dissipation rate:

$$\frac{dk}{dt} = P_{com}^k + P_{sq} - \rho_c \cdot \epsilon \quad (1)$$

$$\frac{d\epsilon}{dt} = P_{com}^\epsilon + C_1 \cdot \frac{\epsilon}{k} \cdot P_{sq} + \rho_c \cdot \frac{k^2}{L^2} - C_2 \cdot \frac{\epsilon^2}{k} \quad (2)$$

where  $k$  = turbulent kinetic energy,  $m^2/s^2$   
 $\epsilon$  = dissipation rate of  $k$ ,  $m^2/s^3$   
 $\rho_c$  = density of gas in cylinder,  $kg/m^3$ .

The P terms represent the volumetric production of  $k$  and  $\epsilon$  due to compression and squish:

$$P_{com}^k = \frac{2}{3} \cdot k \cdot \frac{d\rho_c}{dt}, \text{ kg/m} \cdot \text{s}^3$$

$$P_{com}^\epsilon = \frac{4}{3} \cdot \epsilon \cdot \frac{d\rho_c}{dt}, \text{ kg/m} \cdot \text{s}^3$$

$$P_{sq} = C_3 \cdot \frac{d\rho_c}{dt} \cdot \frac{1}{2} \cdot V_r^2, \text{ kg/m} \cdot \text{s}^3$$

where  $C_3$  = constant

$V_r$  = squish velocity at bowl lip, m/s

Also in equation (2) the constants  $C_1$  and  $C_2$  were 1.45 and 1.9 respectively as specified in the equations of Borgnakke *et al.* [14] and provided originally by Launder and Spalding [13]. The term  $k^2/L^2$  in equation (2) was not specified in any of the equations applied by Borgnakke *et al.* [14], Davis and Borgnakke [12], Belaire *et al.* [15], Davis *et al.* [16] and Davis and Tabaczynski [10]. However, Morel and Keribar [6] included it as an *ad hoc* representation of the boundary layer effects on the bulk dissipation rate. The value of  $L$  was set by Morel and Keribar [6] at twice the minimum geometrical dimension of the region in the combustion chamber.

In the model for this study a global turbulence was determined for each of the two volumes previously defined for the calculation of swirl. Hence the instantaneous turbulent kinetic energy and its dissipation rate were computed for each region. The value of  $L$  in equation (2) was set equal to twice the instantaneous piston-to-head clearance for the outer volume, and twice the sum of this same clearance and the depth of the bowl for the inner volume.

In the application of these equations it was assumed that a negative production of turbulence was impossible as in the cases of turbulence production from both compression and squish where  $d\rho_c/dt$  was negative after TDC during the expansion stroke. For the compression term,  $d\rho_c/dt$  was equated to zero after TDC. The production of turbulence from squish was treated as an inflow source of turbulence and therefore, for the inner volume, the squish production term was positive during the compression stroke and zero during expansion. For the outer volume the reverse was true with the production being initially zero and then positive for the reverse squish effect.

As the model was formulated to include the period between IVC and EVO only, turbulence production during the intake stroke was excluded. As a result it was necessary to provide a starting value for  $K$  and  $\epsilon$  at IVC. Details of a procedure for calculating a starting value for  $k$  are provided in the next section. For the initial dissipation rate the following expression applied by Belaire *et al.* [15] was used:

$$\epsilon = C_4 \cdot k^{1.5}/l$$

where  $C_4$  = adjustable constant

$l$  = integral length scale set equal to the sum of the piston-to-head clearance at IVC and the depth of the bowl.

In the application of this model ordinary differential equations (1) and (2) were solved in the same way as the

two coupled equations for the swirl velocities. The equations were therefore of the form:

$$\frac{dk}{dt} = f(k, \epsilon)$$

$$\frac{d\epsilon}{dt} = f(k, \epsilon)$$

Hence these equations were solved for each of the two volumes using the predictor-corrector method, the solution being initiated with the Runge-Kutta method. The resulting turbulence kinetic energies were then used in the calculation of a resultant velocity at each wall in the combustion chamber.

### Verification

The achievement of an absolute indication of turbulence intensity via modelling is impossible without extensive measurements in the engine on both a spatial and temporal basis to verify the model. However, for this work it was important that acceptable trends and orders of magnitude were reflected in the output of the model rather than the attainment of accurate turbulence intensities.

If the turbulence intensities in the vicinity of each of the six walls was examined in conjunction with the mean vel-

ocities generated by swirl and squish and piston velocities at these walls, certain expected trends could be predicted as the piston approached TDC. The flow effects close to TDC were of particular concern as these had a significant bearing on peak heat transfer rates and on fuel-air mixing. The model of Morel and Keribar [6] produced a prominent peak in turbulence intensity within the bowl at TDC due to kinetic energy of injection and of squish motion leaving the clearance space between the piston crown and head. The turbulence in this latter space was first reduced by increased viscous dissipation caused by the small clearance and then increased with the reversed squish. Turbulence generation due to compression caused the rise in turbulence intensity during the compression stroke. The predictions of Assanis and Heywood [18] also showed this increase, however, there was no second peak close to TDC reflecting the increased turbulence in the cup from squish.

The measurements by Fansler [9] of turbulence indicated that as the squish and swirl flows reached their maximum velocities, the rms fluctuations representing turbulence at the radii close to the side of the cup showed a sharp peak. Nearer the centre of the cup this peak was considerably less pronounced but still present. Moving axially from the cup entrance to the bottom of the cup Fansler [9] measured a decrease in turbulence close to TDC (see Figure 2).

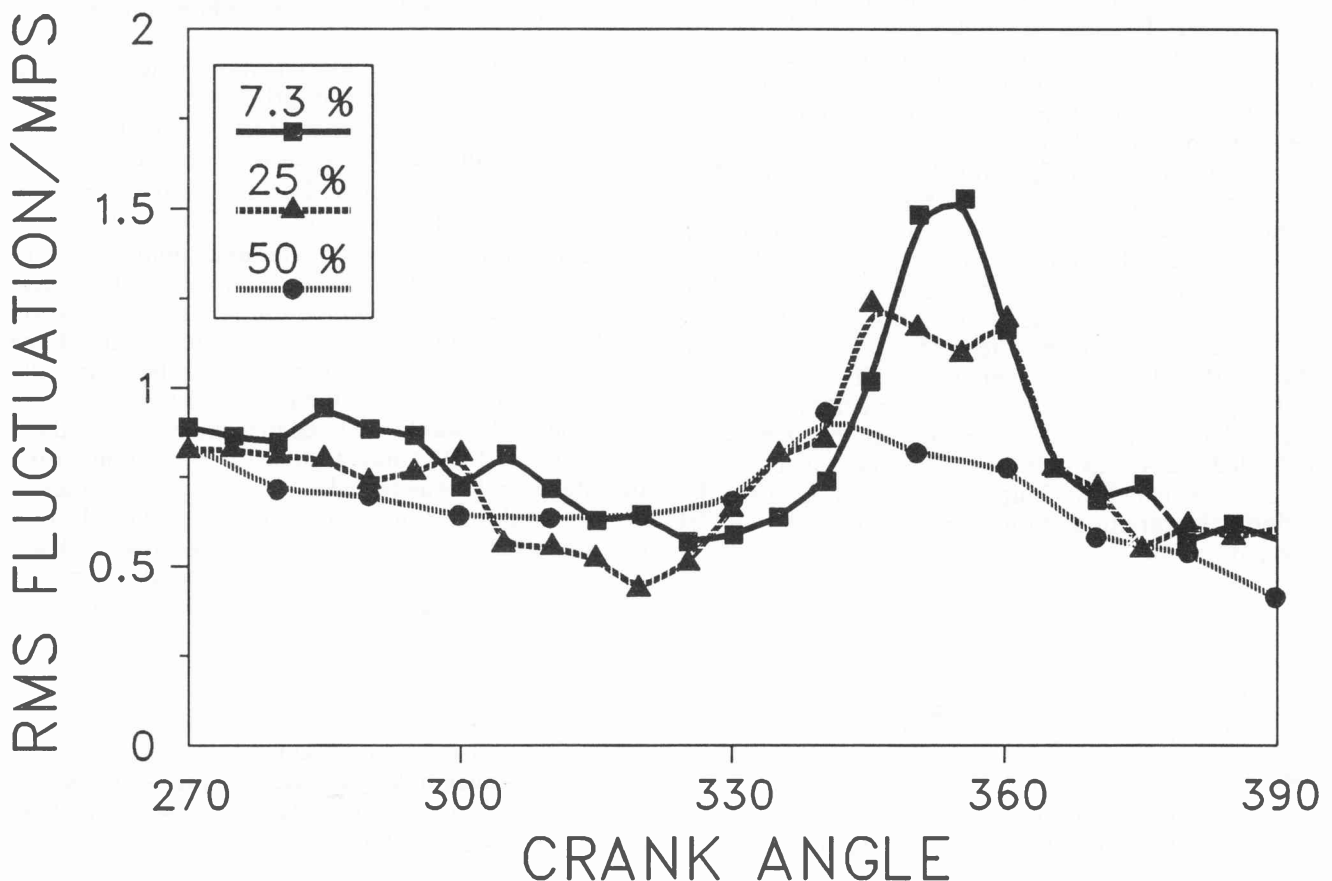


Figure 2 - Ensemble-averaged rms velocity fluctuations/mean piston speed measured at distances of 7.3%, 25% and 50% of the bowl depth from the cylinder head and at a radius equal to 80% of the bowl radius. Engine speed = 600 r/min (after [9]).

Further examination of Figure 2 shows no evidence of the increase in turbulence due to compression which Morel and Keribar [6] generated with their models in the range of 280-350° CA. Rao and Bardou [4] mentioned that measurements by different investigators did not consistently support this phenomenon.

With reference to the problem of providing a starting value for turbulence at IVC, Hayder *et al.* [11] cited the expressions used by Grasso and Bracco [19] which related the initial values of turbulence kinetic energy and its dissipation rate to the engine speed in r/min and volumetric efficiency,  $e_v$ :

$$k = A \cdot e_v^2 \cdot (\text{rpm})^2$$

$$\epsilon = B \cdot e_v^3 \cdot (\text{rpm})^3$$

where A and B = dimensional empirical constants.

Hayder *et al.* [11] modified these expressions by assuming that the initial turbulence intensity was proportional to a characteristic intake velocity and introduced an explicit initial reference length scale based on the maximum open intake area. The final equations also included chamber cross-sectional area, crank angle of intake opening and mean piston speed.

It was evident from these approaches to determining initial turbulence values that mean engine speed and a parameter reflecting the average intake air velocity should be considered in formulating a suitable equation for turbulence at IVC. In order to obtain an indication of the range of turbulence intensities that could be expected at IVC in practice a study of published, measured and modelled values was carried out. The results are assembled in Table 1. As can be seen both flat pistons and bowl-in-piston configurations were examined as, at IVC, the bowl has very little effect on the generation of turbulence.

Turbulence models such as those of Morel and Keribar [6] and Davis and Tabaczynski [10] determined turbulence kinetic energy during the intake stroke based on

port and valve geometry and lift, flow rates and discharge coefficient. These same factors are used to determine swirl ratio at IVC. It is therefore proposed that turbulence intensity should be a function of swirl ratio at IVC and mean piston speed. The swirl ratio is governed by the intake characteristics while the mean piston speed combines engine stroke and speed.

After an examination of the turbulence intensity values and their variation with other variables the following dimensionally correct expression was formulated for turbulence intensity, TI:

$$TI = \frac{\sqrt{(\text{MPS}, \text{TV})}}{2}, \text{ m/s} \tag{3}$$

where MPS = mean piston speed, m/s  
 TV = tangential velocity in the cylinder at bore diameter and at IVC, m/s.

From Table 1 it can be seen that equation (3) provides values of the right order of magnitude in most cases although the value predicted for the engine of Morel and Keribar [6] is approximately half their modelled value. However, Morel and Keribar [6] did not verify their modelled values directly with measurements.

In spite of the differences obtained between equation (3) and the actual values in Table 1 it was concluded that this equation provided credible values for turbulence intensity at IVC. In addition the portion of the engine cycle of greatest importance was close to TDC with both valves closed. For this period it had been shown that the generation of turbulence was largely independent of the starting value at IVC being primarily determined by engine geometry [3, 11].

The selection of published data to verify the turbulence model was very limited. Researchers such as Fansler [9] and Saito, Daisho, Uchida and Ikeya [21] had published experimental data for turbulence within the bowl. However, owing to the problem of the confined space at TDC no measurements in the zone above the piston crown had been performed.

**Table 1** Modelled and measured turbulence intensities for different engines and conditions

Ref.	Measured/ Modeled	Piston Type	Stroke (m)	Bore (m)	Comp. Ratio	Engine Speed (r/min)	MPS (m/s)	Swirl Ratio	ITV (m/s)	MTV (m/s)	ITI (m/s)	ETI (m/s)	MTI (m/s)
1	Model	Flat	0.0794	0.096	9	1500	3.97	1.5	11.3	11.5*	3.8	3.35	2.62
1	Model	Flat	0.0794	0.096	9	1500	3.97	1.9	14.3	14.4*	2.5	3.76	2.15
2	Meas.	Flat	0.083	0.076	5.4	460	1.273	18.25	33.4	33.4*	1.97	3.26	0.7
3	Model	BIP	0.144	0.133	14.8	1800	8.64	3.6	45.1	77**	20.2	9.88	29.5
4	Meas.	BIP	0.11	0.125	16	1000	3.67	3.4	22.2	24.4**	2.9	4.5	5.1
4	Meas.	BIP	0.11	0.125	16	1000	3.67	2.1	13.7	13.3**	2.9	3.54	3
5	Meas.	BIP	0.108	0.0984	11	600	2.16	1.41	20.4	16.3**	1.81	3.32	3.79
5	Meas.	BIP	0.108	0.0984	11	300	1.08	2.81	10.2	8.15**	1.31	1.65	1.23

BIP = bowl-in-piston  
 MPS = mean piston speed  
 ITV = tangential gas velocity calculated at bore diameter and at IVC  
 MTV = maximum tangential velocity in the cylinder after IVC  
 \* - Tangential velocity calculated at bore diameter  
 \*\* - Tangential velocity calculated at bowl diameter  
 ITI = turbulence intensity at IVC

ETI = turbulence intensity at IVC estimated from equation (3)  
 MTI = maximum turbulence intensity in the cylinder after IVC  
 Ref. 1 - Davis and Tabaczynski [10]  
 2 - Johnston, Robinson, Rorke, Smith and Witze [20]  
 3 - Morel and Keribar [6]  
 4 - Saito, Daisho, Uchida and Ikeya [21]  
 5 - Fansler [9]

While the model applied in this present study was based on the models of Borgnakke *et al.* [14], Davis and Borgnakke [12], Belaire *et al.* [15], Davis *et al.* [16] and Davis and Tabaczynski [10], none of these researchers provided suitable results for calibrating or validating the present model. Hence the results of Morel and Keribar [6] were used as they had provided curves representing turbulence in the region above the piston crown as well as the bowl. While their curves were generated from a model, the general trends and relative magnitudes of turbulence were representative of what was expected in practice.

The present model required the adjustment of the two constants,  $C_3$  and  $C_4$  governing the magnitude of turbulence production from squish and the initial dissipation rate. Using the same baseline engine of Morel and Keribar [6] that was discussed earlier in this section the two constants were adjusted until reasonable agreement was achieved in terms of trends and relative magnitudes. The

starting value for turbulence intensity used in this comparison was calculated from equation (3) and therefore was somewhat lower than the value obtained by Morel and Keribar [6]. However, it was not clear from the contents of their paper as to how they calculated the turbulence intensity from the turbulent kinetic energy. For the purpose of the present study the turbulent kinetic energy was determined from the turbulence intensity using the relationship  $k = 3 (TI)^2/2$  representing three dimensional kinetic energy.

To circumvent this discrepancy in starting values, the turbulence intensity output from the present model was multiplied by a constant which then provided identical starting values for turbulence intensity. Figure 3 shows the output from the model against the curves of Morel and Keribar [6]. The constant  $C_4$  was given a value of three which resulted in fairly close agreement between the curves during the early stages of the compression stroke.

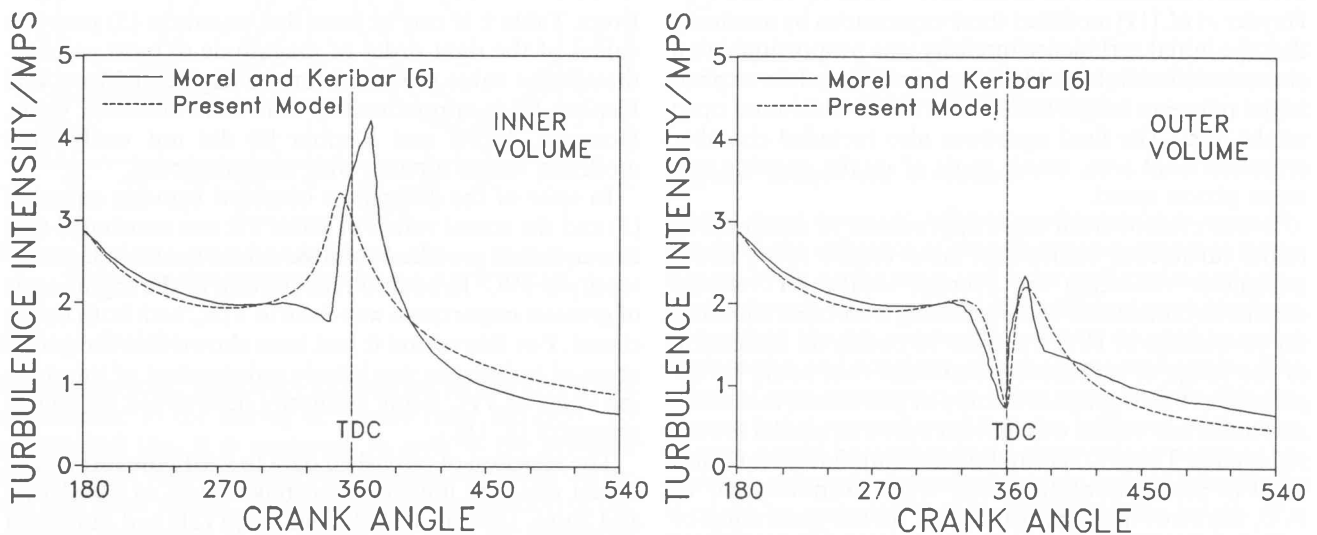


Figure 3 – The variation of non-dimensional turbulence intensity for the present model compared to the model output of Morel and Keribar [6].

For turbulence generation in the inner volume of the cylinder constant  $C_3$  was set equal to one. Figure 3 shows that the turbulence intensity starts to rise earlier than the curve of Morel and Keribar [6] as TDC is approached. The peak value of turbulence is also lower than that of Morel and Keribar [6] and the subsequent rate of decay in turbulence is less. However, the model of Morel and Keribar [6] was applied to a fired engine and included a term for production of turbulence from injection. Some of the terms were also influenced by gas properties. These factors would cause greater turbulence just after TDC as illustrated by the position of peak turbulence in Figure 3.

In the case of the volume above the piston crown, setting  $C_3$  equal to one resulted in the turbulence after TDC being somewhat lower than obtained by Morel and Keribar [6]. As this region is largely unaffected by turbulence from injection it was decided that  $C_3$  should be increased to achieve better agreement between the two curves. It was found that a value of three provided approximately the same peak of turbulence from reverse squish as illustrated in Figure 3.

It was noticed in the implementation of the model that the term  $k^2/L^2$  in equation (9) for the dissipation rate that was originally applied by Morel and Keribar [6], was largely responsible for the significant drop in turbulence in the piston crown region as TDC was approached. In Figure 3 the small rise in turbulence during the compression stroke for the outer volume is the result of turbulence production from compression. This effect cannot be seen in Figure 3 for the inner volume as the turbulence production from squish starts to dominate from a point earlier on in the compression stroke. This dominance of turbulence from squish induced velocity fluctuations is in agreement with the measurements of Fansler [9] and Saito *et al.* [21].

From the results obtained and illustrated in Figure 3 it was concluded that the model of turbulence was generating turbulence levels with acceptable trends and magnitudes. It was also necessary to take into account the difficulties of measuring turbulence intensity accurately and with adequate spatial resolution when using published data for verification purposes. From Table 1 it can be

seen that the maximum turbulence intensity is generally less than 40% of the maximum tangential velocity. In the case of the measured intensities for the bowl-in-piston engines the maximum turbulence intensity was of the order of 1/4 of the maximum tangential velocity measured at the bowl lip. Hence it was concluded that, while the contribution of turbulence was significant in relation to the other velocity components, it did not dominate the resultant velocities.

### Conclusions

A turbulence model was formulated with sufficient terms to generate expected trends during the engine cycle. Credible starting values for turbulence at inlet valve closure were provided by taking into consideration the swirl ratio and mean piston speed. After the adjustment of two constants the output from the model compared favourably with published data with reference to trends and orders of magnitude.

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