Pulsed Ruby Holography in Vibration Studies

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Abstract

It is demonstrated that pulsed ruby time average holographic interferometry can be considered seriously as a vibration analysis technique.

The evidence is provided in the form of an experiment performed with a cantilever beam which is subjected to a number of vibrational modes by a variable frequency vibratory apparatus.

Good agreement is noted between the theoretically predicted and experimental results.

Introduction

Although the classical structural dynamics analysis techniques have wide applicability, there are situations where their usage is precluded because of geometry, size of the structure etc. The vibrations that are of interest to the structural engineer are those that can cause damage and invariably structures excited at their natural frequencies will resonate and eventually self-destruct. Resonant vibrations can be studied by holographic interferometry with considerable ease provided the source of illumination in the production process of the hologram has sufficient energy and can be pulsed at high rates. The most commonly used technique is called time-average holographic interferometry. In this technique a hologram of the vibrating surface is recorded with an exposure time which is long compared to the period of vibration. The hologram captures all fields of displacements that existed during the exposure time in proportion to the fraction of time during their existence. However since a vibrating object spends most of its time at the positions where the velocity is zero, the emerging picture is one that reconstructs two fields from the object. These two fields finally interfere and present us with a fringe pattern which permits ready identification of the vibration modes as well as an accurate measurement of the vibration amplitude. This paper demonstrates the usefulness of holographic analysis and measurement by the example of a vibrating cantilever beam.

Analysis

The particular example of a vertically mounted cantilever beam which is being excited sinusoidally at resonant conditions has been chosen because the mathematical analysis is fairly straight-forward and the model is small. Ordinarily the traditional method of measurement such as the use of an accelerometer would be precluded because the structure being small the loading mass of the accelerometer would affect its modal parameters.

Consider Figure 1 depicting a vertically mounted cantilever beam which is transversely excited. It can be shown that the equation of motion is

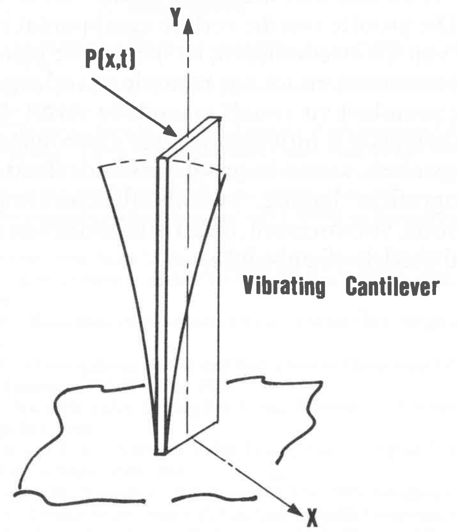

Figure 1 – Vertically mounted cantilever beam transversely excited

$$\frac{d^2}{dx^2}\left[\mathrm{EI}\frac{d^2v}{dx^2}\right] + \rho\mathrm{A}\frac{d^2v}{dt^2} = \mathrm{P}(x,\ t) \qquad (1)$$

If one assumes a harmonic motion given by

$$v(x,\ t) = \mathrm{V}(x)\cos(\omega t - \phi) \qquad (2)$$

and bearing in mind that the beam's resonant frequency is approached when the forced vibration frequency matches the natural frequency, we solve equation (1) and obtain (for the case of EI not a function of x)

$$\frac{d^4(x)}{dx^4} - \lambda^4 v(x) = 0 \qquad (3)$$

where

$$\lambda^4 = \rho\frac{\mathrm{A}\omega^2}{\mathrm{EI}}$$

The general solution of equation (3) can be written according to Craig (1) in the form

$$\mathrm{V}(x) = \mathrm{C}_1\mathrm{Sinh}\lambda x + \mathrm{C}_2\mathrm{Cosh}\lambda x + \mathrm{C}_3\mathrm{Sin}\lambda x + \mathrm{C}_4\mathrm{Cos}\lambda x \qquad (4)$$

which when the appropriate boundary conditions are in-

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troduced yields the equation which describes the mode shapes

$$V_r(x) = C\{\cosh(\lambda_r x) - \cos(\lambda_r x) - K_r(\sinh(\lambda_r x) - \sin(\lambda_r x))\} \tag{5}$$

$$\text{where } K_r = \left[\frac{\cosh(\lambda_r L) + \cos(\lambda_r L)}{\sinh(\lambda_r L) + \sin(\lambda_r L)}\right]$$

and C is an arbitrary amplitude constant.
Finally the natural frequency can be obtained from

$$\omega_r = \left(\frac{EI}{\rho A}\right)^{\frac{1}{2}} \frac{(\lambda_{(r)}L)^2}{L^2} \tag{6}$$

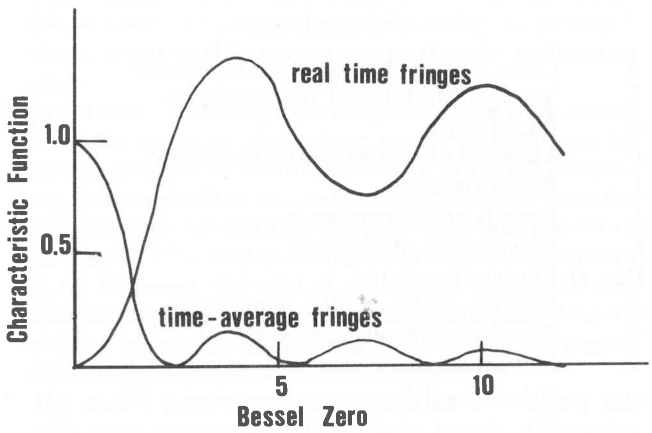

Figure 2 – Zeros of Bessel function

The product $\lambda_{(r)}L$ has been worked out by Chang (2) for a large number of vibrational modes.

According to Hariharan (3) when using time-average holographic interferometry the complex amplitude of the vibrating surface is known as the characteristic function that contains J_o (the zero-order Bessel function of the first kind). If the vibration amplitude varies across the object, it gives rise to contours of equal amplitude (fringes). The dark fringes correspond to the zeros of the Bessel function (see figure 2) and the bright fringes correspond to its maxima. The first maximum corresponds to the nodes and is the brightest of the bright fringes. It is possible to determine the amplitude of the motion by simply counting the dark fringes from the bright zero order $J_o(o)$ and solve the equation

$$Z = \frac{J_o(n)l}{4\pi \sin \alpha_1 \cos \alpha_2} \tag{7}$$

where Z = vibration amplitude
$J_o(n)$ = zero order Bessel function associated with a given fringe count
l = laser light wavelength
α_1 = the half angle between the illumination and observation vectors
α_2 = the angle between observation and the displacement of the object

Often it is reasonable to assume that the motion of the object is directed normal to its surface, particularly when the object is a thin plate or shell. Thus by examining the photograph of a hologram of a vibrating object it is possible to establish its vibration state.

Experimental Procedure and Results

A cantilever beam having the dimensions of 125 mm length, 25 mm wide, and 1,0 mm thick made of mild steel was clamped on a vice facing the output shaft of a vibratory apparatus which is capable of producing a large range of frequencies of sinusoidal vibration from 3Hz to 15KHz as well as amplitudes from μ-metres to centimetres. The beam was set to vibrate at resonant conditions which were easily established audibly and/or visibly once their value had been calculated from equation (6).

Using a 1,0 joule pulsed ruby laser in a non-Q switched mode (open lasing) holograms were obtained of the vibrating beam. In this mode the laser pulse is in the order of 1 millisecond duration well sufficient to isolate any environmental influences in the exposure.

The resulting holograms were photographed and scrutinized on the basis of establishing the mode of vibration, locating the nodal points and calculating the amplitude of vibration.

The location of the nodes and antinodes as well as the amplitude correspond very well when compared with the analytical solution. Figure (3) are typical photographs of the holograms obtained showing the vibration mode, nodal points and antinodes of the vibrating cantilever. Figure (4) shows the comparison of the theoretical results with those obtained through the holographic analysis.

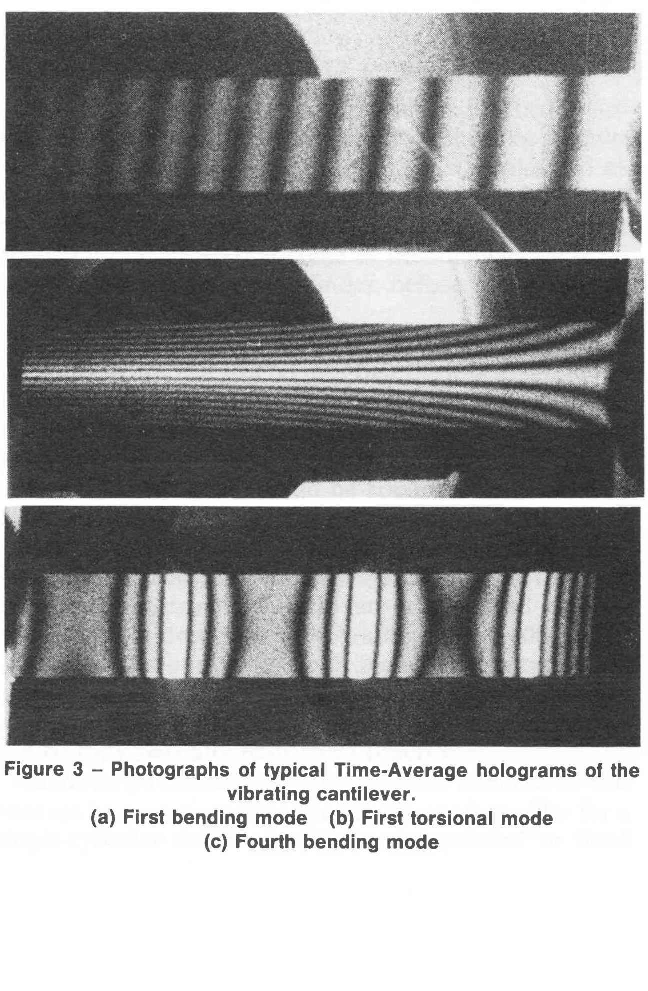
Figure 3 – Photographs of typical Time-Average holograms of the vibrating cantilever.
(a) First bending mode (b) First torsional mode
(c) Fourth bending mode

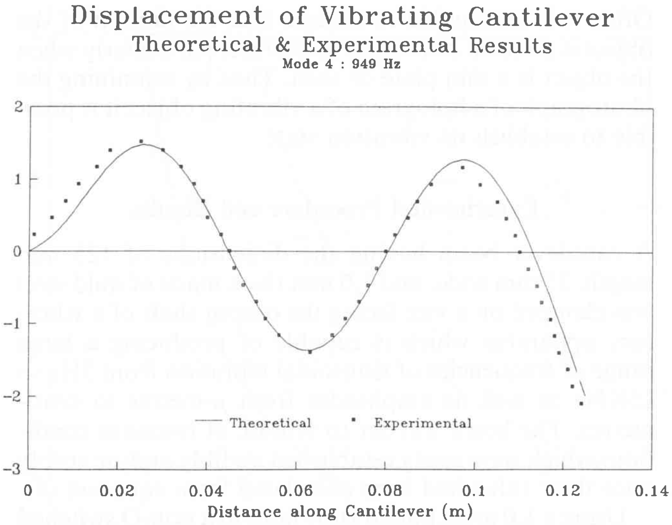

Figure 4 – Comparison of the predicted, to the experimental 4th bending mode of the vibrating cantilever.

Closure

It is demonstrated that holographic interferometry and in particular time-average pulse holography is a powerful tool in vibration analysis and measurement. The method is of particular interest to the designer of structures and machine components that are destined to operate in dynamic situations, for example gyros, inertia guidance systems etc. Apart from the fact that the fringes can only be seen after processing the holograms, the only other limitation of the techniques is that it does not give information about the relative phases of the vibration, i.e. it does not distinguish between portions of the object bending in or out of the surface at any instant of time. However, the advantages of a single holographic exposure yielding information about the mode and amplitude of vibration, the localisation of vibratory nodes, as well as the fact that the technique is applicable to non-sinusoidal motions by far outweigh disadvantages.

References

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Nomenclature

A	Cross sectional area of the cantilever
E	Youngs Modulus of the cantilever
ϕ	Phase angle
I	Moment of inertia of the cantilever
J_o	Bessel function
L	Length of the cantilever
l	Wavelength of the light
$P(x, t)$	Forcing function
ρ	Density of the cantilever material
v	Transverse displacement
V_r	Rth mode shape
ω_r	Rth natural frequency