Application of Modal Control to Flexible Structures

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Abstract

The application of a classical control technique to a cantilevered beam has been considered. A model of the beam was developed using experimental modal analysis. This model was applied to develop a velocity feedback control law to control the first three modes of vibration of the beam. The importance of modal coupling in a continuous system is demonstrated by theoretical evaluation of the control law for an initial tip deflection, where it is demonstrated that neglect of the coupling degrades the controller performance. The performance of the velocity feedback control law in controlling flow induced flutter is then experimentally evaluated in a wind tunnel test. The control law, while simple, is shown to reduce the energy of vibration of the beam, in its first mode, by nearly 150 times indicating that classical feedback control on flexible structures can be very effective.

Nomenclature

a_r The principal co-ordinate corresponding to the r^{th} mode
c_i The velocity feedback constant from a sensor located at x_i
f_j Generalised force at position x_j
J The performance parameter
ψ_i^r The i^{th} mass normalised mode shape coefficient of the r^{th} mode
u_i Beam response at x_i
ω_r The undamped natural frequency of the r^{th} mode
ς_r Equivalent modal damping coefficient applied to the r^{th} mode

Introduction

The active control of flexible engineering structures has become an area of increasing interest to researchers and structural engineers. The trend in construction technology towards using lightweight materials, and the existence of refined structural design methods such as the finite element method can lead to light, flexible structures with low damping. Consequently these structures may exhibit undesirable dynamic characteristics. The application of active control may remedy this situation since it facilitates the modification of the dynamic characteristics of the structure to external disturbances or excitation. The successful implementation of active control necessitates a study encompassing the following aspects.

- The identification of the structural parameters to enable the mathematical definition of the system and hence the determination of suitable controller coefficients.
- The unconditional stability of the controlled structure to all forms of disturbance.
- The experimental verification of the system performance.

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The purpose of this study is to present an application illustrating the methodology as well as experimental results confirming the significant advantage which can be derived from applying active control to a structure. The objective of the control system presented is the minimisation of the response of the first three modes of vibration of a simple cantilevered beam. Velocity feedback forms the basis of the control law and is considered both theoretically and experimentally. The system is illustrated in Figure 1.

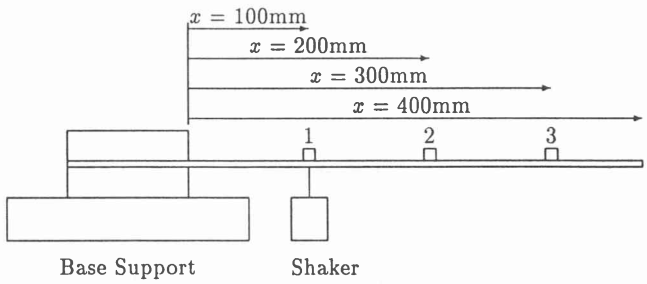

Figure 1: A schematic representation of a cantilever beam.

Theory

Mathematical Model of The Structure

The development of a suitable mathematical model of the beam can be achieved by one of two methods. The governing partial differential equations can be derived and solved using standard mathematical techniques [1]. In the classical approach, the beam equation admits a variable separable solution, where the solution is found to consist of a product of temporal and spatial functions. Alternatively the beam may be discretised, and an n degree of freedom discrete representation of the beam developed. The latter method is usually applied, since discrete effects, such as concentrated mass or stiffness can be accounted for directly in the equations of motion. If the latter method is pursued, the equivalent discrete beam structure considered is illustrated in Figure 2.

The result of the discretisation is a spatial model of the beam of the form:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{f\} \quad (1)$$

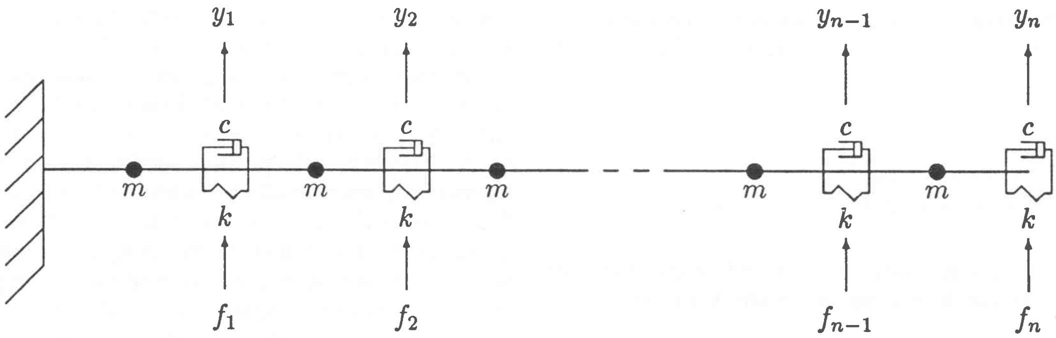

Figure 2: The equivalent discrete system.

Where $[M]$ is the mass matrix
$[C]$ is the damping matrix
$[K]$ is the stiffness matrix

By assuming proportional Rayleigh damping and transforming the equations of motion into principal co-ordinates, it can be shown that the frequency response function of the beam relating the input force at co-ordinate j to the response at co-ordinate i, can be written in terms of the eigenvectors and eigenvalues of the equation of motion as follows[2]:

$$\frac{\mathcal{F}(u_i)}{\mathcal{F}(f_j)} = H_{ij} = \sum_{r=1}^{n} \frac{\psi_i^r \psi_j^r}{\omega_r^2 - \omega^2 + i2\varsigma\omega_r\omega} \tag{2}$$

Where
- ω_r is the undamped natural frequency of the r^{th} mode.
- ς_r is the equivalent modal damping damping coefficient applied to the r^{th} mode.
- ψ_i^r is the i^{th} mass normalised mode shape coefficient of the r^{th} mode.

An alternative approach to developing a mathematical model of the structure, and extracting the modal parameters ω_r, ς_r, ψ analytically, is to develop an analytical model from experimental data directly, by applying an experimental modal analysis technique. In this way the modal parameters ω_r, ς_r and ψ are determined experimentally. Through the definition of these parameters, a realistic analytic model of the system evolves. Since the parameters are determined experimentally by direct measurement, the influence of boundary conditions and the structural properties of the beam are inherently accounted for.

Modal parameter extraction

In this particular application, the control force will be applied to the structure through a modal shaker with a current feedback control amplifier. The current feedback control amplifier provides an armature current proportional to the amplifier input voltage, and hence a force which is proportional to this voltage. In order to relate the amplifier input voltage directly to the structural response, the frequency response functions relating the input voltage signal to the output displacements were evaluated. In this way the calibration between the voltage signal applied to the amplifier, and the structural displacement is contained directly in the frequency response functions. In addition, the dynamic characteristics of the structure and shaker are accounted for.

The frequency response functions relating the structural response to the voltage input are then formulated as:

$$\frac{\Phi(u_i)}{\Phi(V)} = H_{i1} = \sum_{r=1}^{n} \frac{\phi_1^r \phi_i^r}{\omega_r^2 - \omega^2 + i2\varsigma\omega_r\omega} \tag{3}$$

Where
- ω_r is the undamped natural frequency of the r^{th} mode.
- ς_r is the equivalent modal damping damping coefficient applied to the r^{th} mode.
- ϕ_i^r is the i^{th} mode shape coefficient of the r^{th} mode, normalised to the voltage/force calibration.
- ϕ_1^r is mode shape coefficient at the shaker location of the r^{th} mode, normalised to the voltage/force calibration.

The Frequency response Functions relating the shaker input voltage to the acceleration at the three sensor locations on the beam (100 mm, 200 mm, 300 mm from the base) were obtained experimentally. A Genrad 2515 modal analyser was used to generate a sinusoidal input voltage to the shaker over the desired frequency range while the acceleration was measured with accelerometers. The SDRC I-DEAS package was used to extract the modal parameters ω_r, ς_r, and ϕ_i^r. A circle fit technique [3] proved to be sufficiently accurate, since the natural frequencies of the structure were well separated. The natural frequency and modal damping coefficients extracted are listed in Table 1.

Table 1: Modal parameters obtained using the circle fit technique on the experimental data

Modal parameters		
Mode	ω (Hz)	ς (%)
1	8.04	1.067
2	43.365	2.151
3	80.877	0.919

The modal parameters extracted allow the mathematical model of the system to be written in terms of the principal co-ordinates:

$$\ddot{a}_1 + 2\varsigma_1\omega_1\dot{a}_1 + \omega_1^2 a_1 = \phi_1^1 V(t) \tag{4}$$
$$\ddot{a}_2 + 2\varsigma_2\omega_2\dot{a}_2 + \omega_2^2 a_2 = \phi_1^2 V(t) \tag{5}$$
$$\ddot{a}_3 + 2\varsigma_3\omega_3\dot{a}_3 + \omega_3^2 a_3 = \phi_1^3 V(t) \tag{6}$$

These principal co-ordinates are related to the physical co-ordinates by the following mode shape matrix:

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{matrix} \phi_1^1 & \phi_1^2 & \phi_1^3 \\ \phi_2^1 & \phi_2^2 & \phi_2^3 \\ \phi_3^1 & \phi_3^2 & \phi_3^3 \end{matrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \tag{7}$$

The mode shape matrix, normalised to include the voltage/force calibration was obtained as:

$$[\phi] = \begin{matrix} 0.032 & 2.855 & 3.182 \\ 3.763 & 4.495 & -5.367 \\ 8.918 & 1.744 & -6.393 \end{matrix}$$

Control techniques

The purpose of a non-predictive classical control technique is to utilise feedback information to assimilate desirable structural parameters in such a manner as to improve the structural response to disturbances. The task is greatly simplified if the disturbance is known. This is true since the bandwidth of the excitation can be identified and the controller parameters selected to eliminate resonant frequencies in this bandwidth by applying a control action to modify the modal stiffness or mass of the structure accordingly. Nevertheless, dramatic improvements may still be obtained even if the disturbance is unknown (for example the stochastic disturbance generated by free stream turbulence) by applying velocity feedback to increase the damping action in the principal modes of the structure. In this study, it was assumed that an arbitrary excitation may be encountered, and consequently a velocity feedback control action was applied.

The greatest difficulty associated with the control of continuous vibration systems is the consequence of a phenomenon known as modal coupling. Modal coupling results from the fact that while it is the principal co-ordinates that must be controlled, it is the physical response at discrete locations that is measured by the sensors. The implications of modal coupling can be quite dramatic and can result in:

- Complication of the control algorithm
- A reduction in the controller performance.
- Instability in the uncontrolled modes.

The source of the problem can be easily demonstrated by examining the control of a single mode of a continuous system. In order to implement the control action, velocity feedback is obtained from a single sensor placed on the structure. Since the structure is continuous, it will not be possible to place the sensor at a location such that the response represents only motion in the mode to be controlled. If this were possible, it would require that the sensor was located at an antinode of the controlled mode whilst being a node of the uncontrolled modes. For this reason the response will in general contain *parasitic* components due to the truncated or *parasitic* modes. Similarly, the placement of the actuator will not excite a pure mode. Consequently positive feedback may occur in the parasitic or uncontrolled modes leading to instability in these modes due to the control action introduced. This is simply demonstrated by considering the implementation of a control action of the first mode whilst neglecting the intermodal coupling to the truncated or parasitic modes. In this case the control action is assumed to be proportional to the velocity measured at location 1, where c_1 represents the derivative coefficient chosen to achieve the desired level of damping action in the first mode, ie:

$$V(t) = c_1\dot{u}_1 \tag{8}$$

Since the physical displacements are related to the principal co-ordinates in the following way:

$$u_1 = \phi_1^1 a_1 + \phi_1^2 a_2 + \dots \phi_1^n a_n \tag{9}$$

The equation governing the response of the first principal mode then becomes:

$$\ddot{a}_1 + 2\varsigma_1\omega_1\dot{a}_1 + \omega_1^2 a_1 = c_1\phi_1^1(\phi_1^1\dot{a}_1 + \phi_1^2\dot{a}_2 + \dots \phi_1^n\dot{a}_n) \tag{10}$$

The equations governing the parasitic response in the n^{th} truncated mode is then:

$$\ddot{a}_n + 2\varsigma_n\omega_n\dot{a}_n = c_1\phi_1^n(\phi_1^1\dot{a}_1 + \phi_1^2\dot{a}_2 + \dots \phi_1^n\dot{a}_n) \tag{11}$$

It is evident that in order to achieve stability in the first mode, $c_1\phi_1^1\phi_1^1 < 2\varsigma_1\omega_1$. For stability of the higher modes, $c_1\phi_1^n\phi_1^n < 2\varsigma_n\omega_n$, a condition which may not be satisfied. Thus in general, inter-modal coupling will occur between parasitic modes which have been neglected in the reduced modal model. Indeed it is quite possible that this coupling can lead to positive feedback on the uncontrolled modes resulting in an unstable system. One way of overcoming this difficulty is to use a notch filter to ensure that the control action does not excite transients in the parasitic modes, thus reducing the possibility of a self-excitation or instability mechanism from arising.

In view of the complications arising from inter-modal coupling, which would be unavoidable when controlling more than one mode, it is pertinent to consider the available options in determining the feedback constants required to ensure adequate controller performance.

Control in the presence of modal coupling

Three techniques of determining the feedback constants in the presence of modal coupling will be considered here. These account for inter-modal coupling in the controlled modes, but a filter is still required to ensure stability in the uncontrolled modes.

1. **Neglecting the coupling between the controlled modes.** In this case the modal coupling is neglected and the coupling terms appearing in the equations governing

the principal co-ordinates are treated as external disturbances. This will be acceptable if the natural frequencies are well separated since the beam behaves as a cascade of parallel filters. Any disturbance not in the passband of each filter will be severely attenuated which means that the coupling disturbances will be strongly filtered. The feedback constants are selected to provide the desired modal damping coefficients in each of the controlled modes, which ensures the stability of the controlled modes.

2. **Eigenvalue placement.** Here the truncated model of the beam together with negative feedback is used to achieve optimum placement of the system eigenvalues. In order to achieve this, the modal model is cast into the state space by defining a new vector which has twice the dimension of the number of modes retained in the modal model:

$$q = \begin{Bmatrix} a \\ \dot{a} \end{Bmatrix} \tag{12}$$

If velocity feedback from different sensor positions is used then the state space model becomes:

$$\{\dot{q}\} + [A]\{q\} = \{f_{\dot{a}}\} \tag{13}$$

Where $f_{\dot{a}}$ is the external disturbance force and the components of the matrix $[A]$ will depend on the feedback constants.

By orthogonalising the matrix $[A]$, the feedback constants can be determined so as to achieve some desired distribution of the eigenvalues of $[A]$. This is in effect the same as root placement for single input single output systems. This has the advantage that the system stability can be checked by observing that absolute stability requires the eigenvalues of the matrix $[A]$ to have positive real parts. A typical approach would be to ensure that the dominant root (the eigenvalue with smallest real part) lies as far to the left of the imaginary axis as possible.

3. **Using performance parameters.** If the nature of the disturbance is known, it may be possible to optimise the beam response by minimising a suitably defined performance parameter. A good example is where the disturbance is an initial tip deflection, in this case a suitable performance parameter becomes:

$$J = \int_O^T \Sigma u_i^2 dt \tag{14}$$

Where T is sufficiently large to allow the vibration to die out.

u_i is the displacement at the i^{th} sensor location.

Clearly minimisation of this performance parameter will account for the modal coupling and is equivalent to finding the optimum eigenvalue distribution that will minimise the settling time for the given initial conditions.

Theoretical implementation of velocity feedback

To investigate the effect of velocity feedback on vibration attenuation, the beam response to a tip deflection is considered. Two cases are investigated, in the first case the modal coupling is neglected and in the second the modal coupling is accounted for by minimising the performance index defined in equation 14. In both cases the feedback constants from the sensors at posisitons $z_1 = 100$ mm, $z_2 = 200$ mm and $z_3 = 300$ mm are c_1, c_2 and c_3 respectively. The control action is applied as the superposition of the velocity from each sensor:

$$V = c_1\dot{u}_1 + c_2\dot{u}_2 + c_3\dot{u}_3 \tag{15}$$

Where $\dot{u}_i = \phi_i^r \dot{a}_r$

Neglecting modal coupling

In this case the governing differential equations for the truncated modal model become:

$$\ddot{a}_1 + 2\varsigma_1\omega_1\dot{a}_1 + \omega_1^2 a_1 = \phi_1^1(Z_1\dot{a}_1 + Z_2\dot{a}_2 + Z_3\dot{a}_3) \tag{16}$$
$$\ddot{a}_2 + 2\varsigma_2\omega_2\dot{a}_2 + \omega_2^2 a_2 = \phi_1^2(Z_1\dot{a}_1 + Z_2\dot{a}_2 + Z_3\dot{a}_3) \tag{17}$$
$$\ddot{a}_3 + 2\varsigma_3\omega_3\dot{a}_3 + \omega_3^2 a_3 = \phi_1^3(Z_1\dot{a}_1 + Z_2\dot{a}_2 + Z_3\dot{a}_3) \tag{18}$$

Where
$$Z_1 = c_1\phi_1^1 + c_2\phi_2^1 + c_3\phi_3^1$$
$$Z_2 = c_1\phi_1^2 + c_2\phi_2^2 + c_3\phi_3^2$$
$$Z_3 = c_1\phi_1^3 + c_2\phi_2^3 + c_3\phi_3^3$$

If the modal coupling is neglected then the coupling forces are modelled as disturbances and the constants c_1, c_2 and c_3 are selected to achieve an effective modal damping factor of unity. This ensures that the initial values of the principal co-ordinates decay at the maximum rate and also that the system is stable. The required values of the feedback constants are easily determined as:

$$\begin{Bmatrix} c_1 \\ c_2 \\ c_3 \end{Bmatrix} = \begin{Bmatrix} -437 \\ 446 \\ -542 \end{Bmatrix} \tag{19}$$

The effectiveness of velocity feedback in attenuating an initial tip deflection of 10 mm was determined by numerically stimulating the governing equations where the neglected modal coupling terms were retained as disturbance forcing functions. The initial conditions for the principal co-ordinates were determined by using standard bending theory which gives the initial deflection of the beam as:

$$u = \frac{u_o x^2}{2l^3}(3l - x) \tag{20}$$

These initial displacements were transformed into principal co-ordinates via the mode shape matrix. Displacement vs time plots for sensor 3 located 300 mm from the base are given in Figure 3 for the uncontrolled case and in Figure 4 for the controlled case.

It is apparent from these figures that even when modal coupling is neglected, the decay rate is greatly improved.

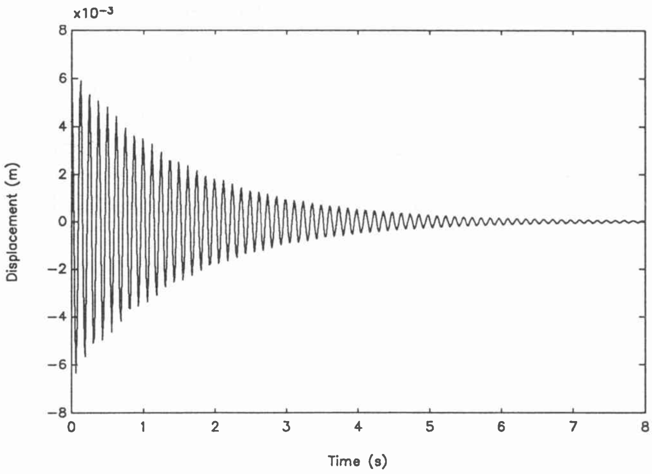

Figure 3: Uncontrolled beam response at position 3.

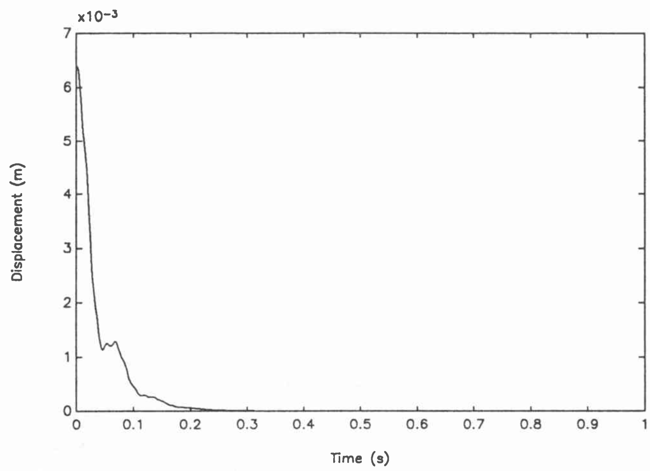

Figure 4: Controlled beam response at position 3, neglecting modal coupling.

Accounting for modal coupling

The effect of modal coupling can be incorporated into the solution of c_1, c_2 and c_3 by minimising the performance index defined by equation 14. It is evident that:

$$J = J(c_1, c_2, c_3) \tag{21}$$

Minimisation of this index was performed by using the maximum descent root finding technique [4]. The maximum descent technique makes use of the fact that for a given set of feedback constants $\tilde{c}_0 = (c_1, c_2, c_3)$, the direction of maximum decrease in J is give by:

$$\tilde{u} = -\nabla J = -(\frac{\partial J}{\partial c_1}, \frac{\partial J}{\partial c_2}, \frac{\partial J}{\partial c_3}) \tag{22}$$

The technique then requires minimisation of the performance index along the line

$$\tilde{c} = \tilde{c}_o - \alpha \nabla J \tag{23}$$

Once the value of the parameter α that minimises J along this line has been determined, the process is repeated until the global minimum of J is found. Application of the steepest descent technique to the governing differential equations was performed using a fourth order Runge-Kutta solver and for the given initial conditions, the optimum feedback constants were determined as:

$$\left\{ \begin{array}{c} c_1 \\ c_2 \\ c_3 \end{array} \right\} = \left\{ \begin{array}{c} -471 \\ 393 \\ -425 \end{array} \right\} \tag{24}$$

Simulated Response

Two sets of control parameters have been determined, the first set is determined by neglecting modal coupling and the control constants are selected on the basis of achieving critical damping and hence the minimum settling time in each principal coordinate independently. The second set is determined as an optimum set of coefficients in the presence of modal coupling with respect to a performance criterion of minimum physical response settling time. The response at the three sensor locations, to an initial tip deflection of 10 mm is simulated with respect to these sets in figures 5, 6, 7.

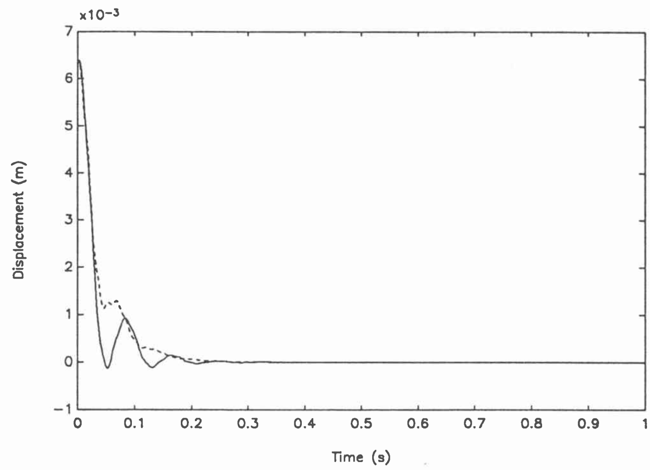

Figure 5: Beam response at position 3 (solid line respresents optimal control).

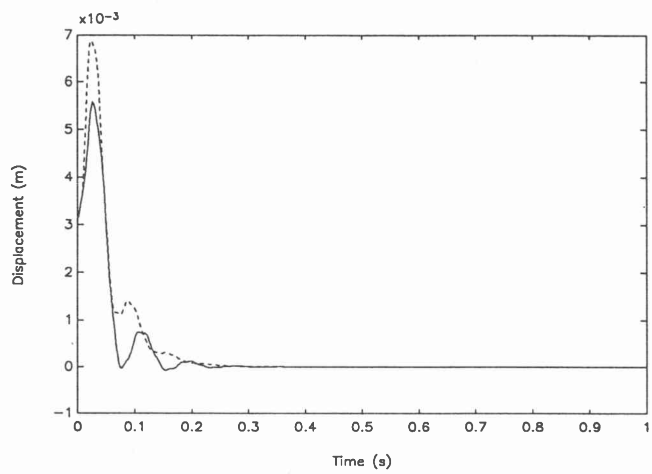

Figure 6: Beam response at position 2 (solid line respresents optimal control).

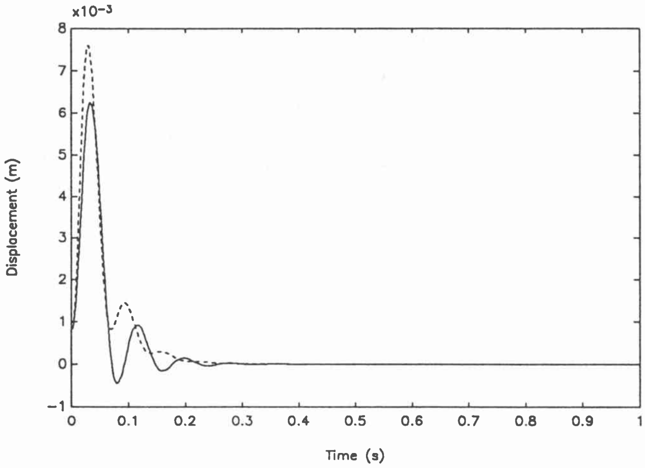

Figure 7: Beam response at position 1 (solid line respresents optimal control).

It is evident from these plots that both sets of constants dramatically influence the beam response, as expected. However, it is also evident that the optimal parameters achieve the shorter settling time. Figure 8 presents a plot comparing the voltage signal required to implement the control action. It is interesting to note that the voltage requirement is less in the case of optimal control, and hence so is the energy input. This emphasises the importance of modal coupling. Both cases have been treated theoretically; practical limitations on the control voltage would limit the degree of feedback achievable.

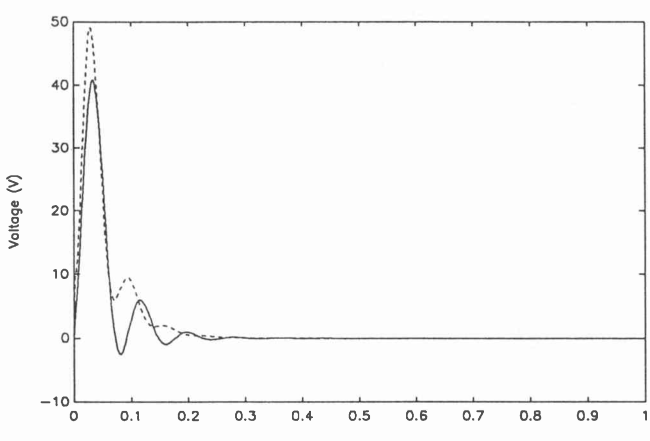

Figure 8: Control voltage input to the shaker (solid line represents optimal control).

7 Practical implementation of the control system

Although the study considered the active control of three modes of the beam, which was demonstrated numerically, experimental implementation of the control was limited to a single mode for simplicity.

The components of the control system

Figure 9 shows the general representation of the closed loop system used to implement velocity feedback in controlling a flexible beam.

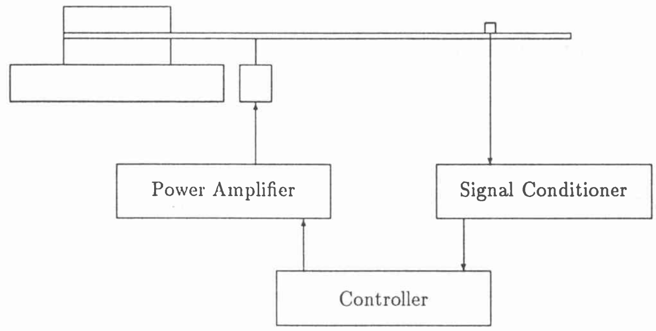

Figure 9: A schematic representation of the control loop.

The accelerometer located 300 mm from the base of the beam was used to measure the response of the beam. An analogue controller was employed to implement the control law. The signal entering the controller was filtered to eliminate the high frequency noise or signals produced by parasitic modes. The controller consisted of a single amplifier which multiplied the incoming signal by the required constant (accounting for the calibration constant of the accelerometer). This signal was then passed through an analogue integrator, which converted it into the required velocity format.

An electrodynamic shaker was attached at location 1, close to the base of the beam by a stinger, and applied the control force. Since the power amplifier of the shaker employed a current feedback device which produced an output current and hence force proportional to the input voltage, the shaker performed directly as a force actuator.

Experimental evaluation of velocity feedback

Observation of the beam in the wind tunnel revealed two important facets of the flutter problem:

- The flow induced vibration was aperiodic due to the highly nonlinear aerodynamic interaction between free- stream and structure. The motion of the beam was characterised by small oscillations with intermittent large amplitude excursions.
- The interaction between the beam and the free stream excited a dominant response in the first mode of vibration. This was observed to be the case visually and confirmed by obtaining a power spectrum of the signal from the accelerometer.

Consequently, it was decided that control of the first mode should yield significant advantage and provide a reasonable test of the single mode controller developed.

Adequate filtering was applied to account for the neglect of modal coupling, thus only one control constant need be evaluated. This constant was selected so as to achieve the maximum physically realisable damping in the first mode, ensuring the most rapid decay of a disturbance.

The feedback constant required to achieve critical damping of the first mode, can be evaluated from

$$V(t) = c_3 \dot{u}_3 \tag{25}$$

Furthermore, since the natural modal damping is very small, the value of the feedback constant required to achieve critical damping is

$$c_3 = \frac{-2\omega_1}{\phi_1^1 \phi_3^1} \tag{26}$$

This gives $c_3 = -358.4$. However, the input voltage to the amplifier is limited to 10 volts, and a reduced constant of $c_3 \approx -35$ was finally employed. This represents an induced damping factor of approximately 10%.

Experimental Results

The autospectrum of the accelerometer voltage record for the uncontrolled system and controlled system is presented Figure 10. In this figure, the solid line represents the uncontrolled structure, whilst the dotted line represents the structural response with control.

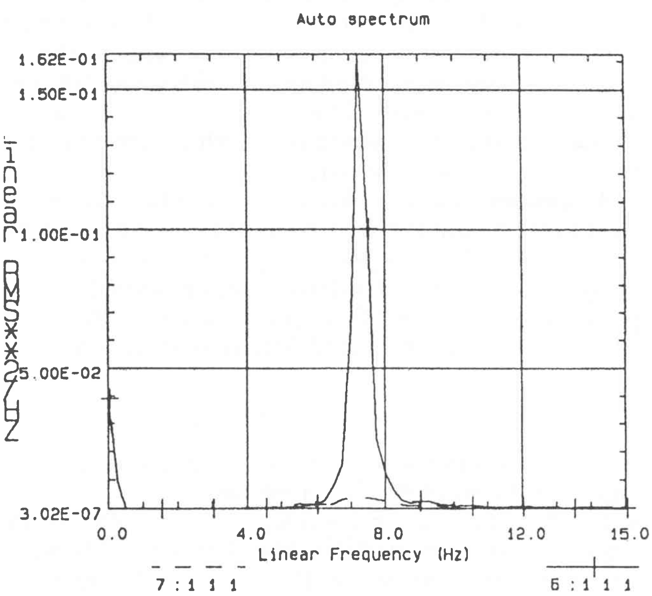

Figure 10: The autospectrum of the controlled and uncontrolled data.

The reduction of the amplitude of response in the controlled system is estimated to represent an attenuation of 150 times in terms of the energy of the beam. Although the overall result of aplying active control to the beam in this circumstance represents a significant reduction in vibration amplitude and energy content, it is pertinent to observe that this effect is not purely due to the control action. The non-linear interaction between the free stream and the structure induces the excitation to the structure. Attentuation of the excitation, or the mechanism by which it arises will result in dramatic attenuation of the response. Thus the controller action not only influences the transient response of the beam to aerodynamic disturbances, but it effectively influences the generation of these disturbances resulting in a dramatic reduction in the observed response. The effect of reducing the large amplitude excursions is to reduce the energy of the vibrating beam by approximately 150 times. This is where the effectiveness of the controller is most noted, whilst it is apparent that the controller is not effective in controlling the small amplitude vibrations.

Conclusion

This study has demonstrated the process involved in applying active velocity feedback control to a cantilever structure. It has been demonstrated numerically that it is possible to apply the principle of modal control to a continuous structure, and consequently control more than one mode. Inclusion of the modal coupling significantly improves the effectiveness of the controller. Neglect of modal coupling without adequate consideration of the parasitic modes, may lead to instability in these modes.

An experiment was conducted to assess the benefits of applying active control to reduce the flow induced response of a cantilever beam placed in a wind tunnel. Dramatic improvements were observed, even though the uncontrolled system behaved in a highly nonlinear fashion. This application clearly demonstrates that in such a situation, applying active control not only achieves attenuation of the transient structural response, but as a result also reduces the severity of the excitation mechanism, resulting in a gearing effect in terms of the observed reduction in amplitude of vibration.

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