Laminar Crossflow Through Prismatic Porous Domains

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Abstract

The phase averge Navier-Stokes equation presents the foundation for the present quantification of high Reynolds number crossflow through prismatic porous media. A representative unit cell (RUC) model, based on a microstructure typification of the porous media, is discussed. A new interpretation of the model leads to an improvement in the closed form quantification of the pore scale fluid-solid interaction in the Darcy-Forchheimer transitional flow region, as well as the full Forchheimer flow region. The presented unified flow model renders the phase average Navier-Stokes equation a powerful tool for numerical analysis of transport of momentum transverse to prismatic porous media, over the whole laminar flow region. Validity of the model interpretation is demonstrated through comparison with numerical flow solutions at the pore scale, as well as experimental results for flow across heat transfer tube banks.

Nomenclature

A	area,
A_p	cross-sectional pore area,
c_d	drag coefficient,
D	tube diameter,
d	microscopic characteristic length,
d_s	solid width,
F	microscopic shear factor,
F_o	low Reynolds number asymptotic microscopic shear factor,
F_∞	high Reynolds number asymptotic microscopic shear factor,
f'	empirical friction factor,
G_{max}	maximum pore mass velocity,
$\boldsymbol{g}$	gravitational body force per unit mass,
$\boldsymbol{I}$	vector integral expression,
K	hydrodynamic permeability,
l	pore length,
N	number of transverse tube rows,
p	pressure,
Δp	pressure difference,
p_f	$\langle p \rangle_f$,
$\boldsymbol{q}$	specific discharge $\langle \boldsymbol{v} \rangle$,
q	magnitude of $\boldsymbol{q}$,
Re	pore Reynolds number, $2\rho v_p(d-d_s)/\mu$,
Re_{qD}	tubular prism Reynolds number, $\rho q D/\mu$,
Re_{qs}	particle Reynolds number, $\rho q d_s/\mu$,
S_{fs}	fluid solid interface,
T	tortuosity,
t	time,
V_f	fluid filled 'void' volume,
V_s	solid volume,
V_o	total volume,
$\boldsymbol{v}$	fluid velocity within V_f,
$\boldsymbol{v}_f$	$\langle \boldsymbol{v} \rangle_f$,
$\boldsymbol{v}_p$	mean pore velocity within pore section,
v_p	magnitude of $\boldsymbol{v}_p$,
x	coordinate,
β	empirical inertia parameter,
ε	porosity (void fraction),
λ	second coefficient of fluid dynamic viscosity,
μ	fluid dynamic viscosity,
$\boldsymbol{\nu}$	normal vector on S_{fs} pointing into V_s,
ρ	fluid mass density,
ϕ	generic variable,
$\langle \phi \rangle$	volumetric phase average of ϕ,
$\langle \phi \rangle_f$	volumetric intrinsic phase average of ϕ,
$\mathring{\phi}$	deviation of ϕ

Introduction

The physics of transport in porous media presents a challenging research field that finds widespread practical application. The diversity of this is illustrated by reasearch areas such as groundwater seepage, cellular bone behaviour, crushable foams, air pollution and composite material processes.

The first significant attempt at addressing fluid discharge through porous media was postulated by Darcy[1]. The vectorized Darcy equation (1) relates the specific discharge, $\boldsymbol{q}$, linearly to the pressure gradient through the hydrodynamic permeability, K, of the porous medium.

$$\nabla p = \rho \boldsymbol{g} - \frac{\mu}{K}\boldsymbol{q}. \qquad (1)$$

Although the Darcy relationship is useful, early experimental observations indicated that the pressure gradient is related to the square of the specific discharge at higher ($> 10^2$) Reynolds number flow. This lead to an improved empirical relationship postulated by Forchheimer[2]. The

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Forchheimer equation (2) consists of the Darcy equation with an additional term which is quadratic in specific discharge and contains an empirical inertia parameter, β.

$$\nabla p = \rho g - \frac{\mu}{K}q - \beta\rho q q. \tag{2}$$

Equation (2), and similar empirical formulae, have two major shortcomings with regard to meaningful flow analysis in porous media. The first is the lack of macroscopic diffusive and/or macroscopic convective terms, which relates to an inability to capture macroscopic phenomena such as boundary conditions. The second shortcoming is their empirical basis. Empirical coefficients are expensive to generate and are often only applicable over limited ranges of Reynolds numbers, due to the incorrect parametric interpretation of the microstructure-flow interaction.

An averaging theorem developed by Slattery[3] and Whitaker[4] facilitated the theoretical solution of the macroscopic problem. The theorem, which expresses the volumetric average of a spatial derivative of a tensorial quantity as the spatial derivative of the volumetric average of the said tensoral quantity, presented the opportunity for the rigorous mathematical phase averaging of the Navier-Stokes equation. The result of this is a rigorous governing equation for transport of a fluid phase in a porous continuum.

The phase average Navier-Stokes equation has two terms in addition to the characteristic terms of the micro Navier-Stokes equation. The first is an additional convective term generated by the phase averaging of the Navier-Stokes convective term, and the second is a surface integral term related to microscopic shear and inertial effects.

Du Plessis and Masliyah[5] presented a Representative Unit Cell (RUC) concept, which lead to the characterisation of the porous microstructure in terms of only two independent parameters, namely porosity and characteristic length. Their approach has been proved[5], [6], [7] suitable and accurate for quantifying the integral term in the low Reynolds number ($Re_{qD} < 10^2$) flow region, in various types of porous media.

Du Plessis[6] proposed a rectangular solid microstructure RUC for prismatic porous media. This model exhibits good experimental agreement for crossflow in the region $0 \leq Re_{qD} \leq 25$. The model, however, underpredicts the microscopic momentum loss for higher Reynolds numbers.

The present paper formulates a new approach to quantifying the surface integral term for saturated two-dimensional crossflow through prismatic media and presents a unified flow model which is applicable over a wide range of Reynolds numbers. A semi-empirical asymptotic expression based on form drag is established for laminar flow with pore-scale recirculation. This asymptote is then matched to a low Reynolds number asymptote based on fully developed flow through rectangular prismatic RUC pore sections. The proposed model is valid over the whole porosity range from zero through one.

A practical and common example of this type of prismatic porous media is found in heat transfer tube banks. Validation of the present approach is by comparison of the model's predicted flow losses with the empirical loss factors presented by Jakob[8] for crossflow through staggered tube banks. Good agreement is demonstrated for $0 \leq Re_{qD} \leq 10^4$.

The presented results confirms the microstructure interpretation of Du Plessis and Masliyah[5] as physically sound. It also presents further evidence that the Forchheimer effect is due to microscopic inertial flow phenomena and does not stem from the additional macroscopic convective term generated by phase averaging the Navier-Stokes equation.

Transport Equations

The Navier-Stokes-Duhem equation governing the transport of momentum in a continuum is given by

$$\frac{Dv}{Dt} = g - \frac{\nabla p}{\rho} + \frac{\mu + \lambda}{\rho}\nabla(\nabla \cdot v) + \frac{\mu}{\rho}\nabla^2 v, \tag{3}$$

for an isotropic homogeneous Newtonian fluid. If the flow is also incompressible and steady, equation (3), can be simplified to

$$\rho\nabla \cdot (vv) = \rho g - \nabla p + \mu\nabla^2 v. \tag{4}$$

The above Navier-Stokes equation is specialised for flow in porous media by phase averaging the equation over a Representative Elementary Volume (REV). A schematic REV is shown in figure (1), for which local porosity is defined as the ratio of fluid volume within the REV to total volume enclosed by the REV.

The phase average of an extensive tensoral property ϕ of the fluid phase within the REV, is defined as

$$\langle\phi\rangle \equiv \frac{1}{V_o}\int_{V_f} \phi dV. \tag{5}$$

Similarly the intrinsic phase average, $\langle\phi\rangle_f$, and the phase average deviator, $\mathring{\phi}$, of the fluid property ϕ, is defined as

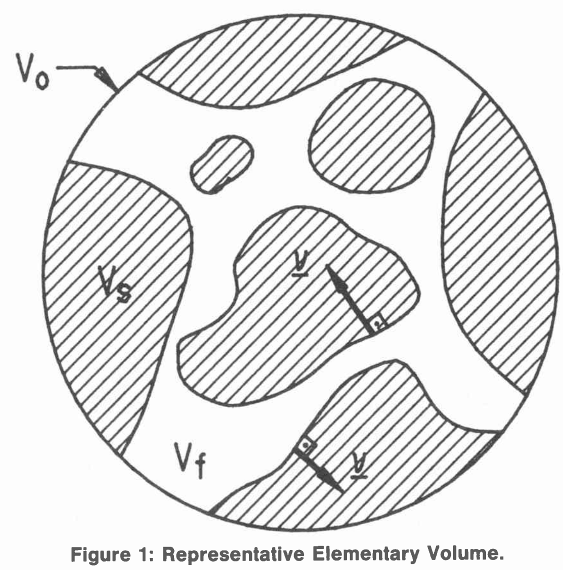

Figure 1: Representative Elementary Volume.

$$\langle\phi\rangle_f \equiv \frac{1}{V_f}\int_{V_f}\phi dV, \tag{6}$$

and

$$\mathring{\phi} \equiv \phi - \langle\phi\rangle_f. \tag{7}$$

Rigorous mathematical averaging of equation (4) was shown by Du Plessis and Masliyah[5], to culminate in the following governing equation for transport of momentum in porous media.

$$\rho\nabla\cdot(\varepsilon v_f v_f) = \rho\varepsilon g - \varepsilon\nabla p_f + \mu\nabla^2(\varepsilon v_f) -$$

$$\rho\nabla\cdot(\varepsilon\langle\mathring{v}\mathring{v}\rangle_f) + \frac{1}{V_o}\int_{S_{fs}}(\mu\nu\cdot\nabla v - \mathring{p}\nu)dS. \tag{8}$$

The left hand side of the above equation, as well as the first three terms on the right hand side, closely resembles the standard Navier-Stokes equation (4).

The fourth term on the right hand side of the equation is the divergence of the porosity times the intrinsic phase average of the square of the velocity deviator. It can be shown that this term vanishes for the case of zero gradient in porosity and zero gradient in average velocities. In cases where such gradients do exist, this term tends to smear out the gradients.

The surface integral term of equation (8) captures the effect of the fluid-solid momentum transfer within the pores. The meaningful simplistic closed form quantification of this term is a prerequisite to unlocking the potential of the equation. To facilitate this, an explicit interpretation of prismatic porous microstructures are introduced in the next section.

Prismatic Microstructure

The RUC is a schematic representation of a hypothetical control volume containing a single pore, which captures the locally averaged essence of the microstructural parameters in a physically plausible manner.

Du Plessis[6] proposed a rectangular prismatic solid representation as shown in figure (2) for prismatic porous media. This RUC clearly captures the media's porosity, characteristic solid-to-solid length and porous medium type. It also has other notable characteristics, for example (i) solid-fluid interfaces suitable for simplistic area integration, and (ii) applicability in the total porosity range. Cylindrical primatic solid representation would not have these characteristics.

In addition to the geometrical interpretation embedded in the RUC, the following assumptions are made to relate the RUC to physical saturated crossflow in prismatic porous media:

(a) There exists no velocity component parallel to the prism axes of the RUC.

(b) The RUC is rotationally invariant and can always be orientated with one face of the prismatic solid parallel to the local specific discharge $\boldsymbol{q}$. Since the solid representation is hypothetical, this RUC characteristic does not add any restrictions to the actual physical porous medium.

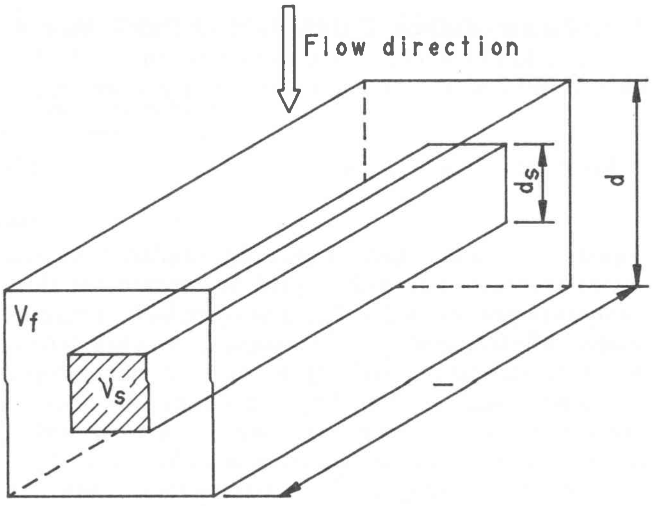

Figure 2: RUC for Prismatic Porous Media.

(c) The principle of maximum staggering applies, which means that adjacent prisms are conceptionally staggered in such a manner that the fluid is forced to traverse through all transverse duct sections. The effects of channeling is therefore not included in the present work.

For the RUC of figure (2), of length l and solid-to-solid distance d, the total volume V_o is given by

$$V_o = ld^2. \tag{9}$$

A rectangular prismatic solid representation with a square cross sectional area of d_s^2, leaves the remaining fluid volume, V_f, as

$$V_f = l(d^2 - d_s^2) \tag{10}$$

and the locally averaged porosity, ε, as

$$\varepsilon = 1 - \left(\frac{d_s}{d}\right)^2. \tag{11}$$

The average streamwise pore velocity, v_p, which is the average streamwise velocity of the fluid phase at the minimum pore cross sectional flow area, A_p, is given by

$$v_p = \frac{qA}{A_p} = \frac{qd}{d - d_s}. \tag{12}$$

The presence of solid matter in porous media forces fluid particles to follow a tortuous route through the media. This characteristic of a porous medium is expressed as the tortuosity of the medium, and is defined as the ratio of the total tortuous path length, d_e, to the characteristic length, d, where the tortuous path length is the ratio of the fluid volume enclosed by the RUC, to the minimum pore cross sectional flow area in the RUC. The present definition of tortuosity, which is the inverse of that used by Du Plessis[6], has bigger interpretive appeal, since an increase in tortuous path length, d_e, is reflected by an increase in tortuosity, T.

From equations (10) and (12) it follows that

$$T = \frac{d_e}{d} = 1 + \frac{d_s}{d}. \tag{13}$$

Straightforward combination of equations (11) and (13) yields the characteristic functional relationship between porosity and tortuosity for prismatic porous media

$$T = 1 + \sqrt{1 - \varepsilon}. \tag{14}$$

A rigorous mathematical link between the RUC model and the phase averaged Navier-Stokes equation (4), can be established by relating the fluid velocity, $\boldsymbol{v}$, the average pore velocity, $\boldsymbol{v}_p$, and the specific discharge, $\boldsymbol{q}$, to each other.

An expression for the cross-sectional pore area, A_p, is found by combining the definitions of tortuous path length and tortuosity, with the expression for V_o of equation (9).

$$A_p = \frac{\varepsilon l d}{T}. \tag{15}$$

Considering the volumetric flow rate through the RUC, and using the pore area expression of equation (15), the following relationships are easily verified.

$$\boldsymbol{q} = \langle \boldsymbol{v} \rangle = \varepsilon \langle \boldsymbol{v} \rangle_f = \boldsymbol{v}_p \frac{A_p}{A} = \boldsymbol{v}_p \frac{\varepsilon}{T}. \tag{16}$$

Microscopic Fluid-Solid Interaction

The surface integral over the fluid-solid interface contained in (4), can be quantified by considering the momentum transfer between the two phases at the microscopic level. For saturated crossflow through prismatic media, the hypothetical RUC described in the previous section will be employed to find a solution of the following form for the whole laminar flow region:

$$\boldsymbol{I} = \frac{1}{V_o} \int_{S_{fs}} (\mu \boldsymbol{\nu} \cdot \nabla \boldsymbol{v} - \mathring{p} \boldsymbol{\nu}) dS \equiv -\mu \boldsymbol{q} F. \tag{17}$$

The above shear factor F, is a physical shear resistance coefficient per unit cross area for the porous domain and is equal to the quotient of porosity, ε, and hydrodynamic permeability, K, for fully developed flow.

Low Reynolds Number Flow

As Reynolds numbers approach zero, fully developed laminar flow between parallel plates is assumed as representative of the RUC fluid-solid interaction in the axial direction of a pore. This follows from the fact that a pore will be flanked by the two parallel edges of the solid representations in adjacent RUCs. The assumption, which is similar to the approach of Du Plessis[6], holds for both streamwise and transverse pores, due to the principle of maximum staggering.

To facilitate the closed form solution of equation (17), the first term of the equation can be rearranged in terms of the rate of shear strain, i.e.

$$\mu \boldsymbol{q} F_o = \frac{1}{V_o} \int_{S_{fs}} \left(\mathring{p} \boldsymbol{\nu} - \mu \frac{\partial \boldsymbol{v}}{\partial \nu} \right) dS. \tag{18}$$

Consider now fully developed laminar flow in the axial direction of a streamwise orientated pore. The shear stress generated between the solid and the fluid in the axial direction of this pore, i.e. the second term in the integral expression of equation (18), leads to a momentum loss which has been quantified by Shah and London[11] in terms of the following friction factor:

$$f = \frac{24}{Re}. \tag{19}$$

Integration of the pressure deviation term of equation (18) along fluid-solid interfaces oriented parallel to the macroscopic streamwise direction, has no net result due to the symmetry of the RUC. The shear rate term of equation (18), integrated along solid faces perpendicular to the flow direction of figure (2), has no component in the flow direction. Integration of the pressure deviation term along these surfaces, will however contribute to the streamwise momentum loss. Denoting front and rear perpendicular solid faces with respect to the flow direction, and considering the pressure deviation integral over these faces, it is clear that the pressure deviation integral must equal the shear stress induced momentum loss along the streamwise orientated pore. With $\boldsymbol{\nu}$ orientated inwards into the solid representation, as well as taking the definition of friction factor into account, the second term of equation (18) can be quantified for the RUC as

$$\int_{S_{fs}} (-\mu \frac{\partial \boldsymbol{v}}{\partial \nu}) dS = d_s l \rho v_p^2 \frac{24}{Re} \left(\frac{\boldsymbol{q}}{q} \right). \tag{20}$$

Transformation of the pressure deviation term of equation (18) follows in a similar way, according to the arguments above, yielding

$$\int_{S_{fs}} (\mathring{p} \boldsymbol{\nu}) dS \equiv d_s l \rho v_p^2 \frac{24}{Re} \left(\frac{\boldsymbol{q}}{q} \right). \tag{21}$$

The closed form solution for the factor F_o, of equation (18), can now be established as follows through the combination of equations (9), (13), (20), (21), and the expression for Reynolds number.

$$F_o = \frac{24 T d_s}{d^2 \varepsilon (d - d_s)}. \tag{22}$$

It is convenient and informative to express F_o only in terms of the two fundamental independent characteristics of the porous medium, namely porosity and characteristic length. Manipulation of equations (11), (13), (14), (15), and (22), as well as noting that the minimum pore cross sectional area, A_p, is given by $l(d - d_s)$, yields the required result:

$$F_o d_s^2 = \frac{24(1 - \varepsilon)^{\frac{3}{2}}}{(1 - \sqrt{1 - \varepsilon})^2}. \tag{23}$$

Recirculation

An asymptotic solution for equation (18) will be sought based on the assumption that form drag dominates shear strain rate effects at high Reynolds number flow. Form drag stems from pore-scale recirculation on the lee side of prismatic solid matter in the porous domain. The resultant streamwise pressure deficit can be expressed in terms of the pore velocity magnitude v_p, and a pressure drag coefficient c_d, as

$$\Delta p \equiv \tfrac{1}{2}\rho v_p^2 c_d. \tag{24}$$

The high Reynolds number asymptotic value of equation (18) is therefore empirically quantified for the RUC through equation (24) as

$$\mu \boldsymbol{q} F_\infty = \frac{\rho v_p^2 c_d l d_s}{2V_o}\left(\frac{\boldsymbol{q}}{q}\right). \tag{25}$$

F_∞ can be isolated in terms of porosity and characteristic length, through manipulation of equations (9), (11), (14) and (25), yielding

$$F_\infty d_s^2 = \frac{Re_{qs} c_d}{2\varepsilon^2}(1 + \sqrt{1 - \varepsilon}\,)^2(1 - \varepsilon) \tag{26}$$

The numerical value of the drag coefficient, c_d, will depend on the actual prism cross section shape and smoothness, most closely resembling the specific physical porous medium. Although the true pore-scale flow is very complex, it is proposed that this simplistic approach will capture the essence of recirculation effects through the physically sound RUC pore velocity description.

Unified Model

A semi-empirical unified solution to the surface integral term of equation (8) is constructed through asymptotic matching of the low Reynolds number asymptote of equation (23) and the high Reynolds number asymptote of equation (26). The superposition technique described by Churchill and Usagi[12] will be used. A unit shifting exponent is selected for simplicity.

$$F d_s^2 = \frac{24(1-\varepsilon)^{\frac{3}{2}}}{(1-\sqrt{1-\varepsilon})^2} + \frac{Re_{qs} c_d}{2\varepsilon^2}(1+\sqrt{1-\varepsilon}\,)^2(1-\varepsilon) \tag{27}$$

Substitution of the surface integral term of the phase average Navier-Stokes equation (8), by the result of the above equation, renders the phase average Navier-Stokes equation a powerful tool for numerical solution of macroscopic momentum transport in prismatic porous domains.

Discussion

Typical examples of prismatic porous domains are found in heat transfer tube banks, although in the latter case the tubes are arranged in arrays rather than isotropical and therefore do not comply with the maximum staggering condition. The present results are compared with empirical pressure drop factors for such tube banks, in order to gain confidence in the listed assumptions and proposed simplifications. Due to the mentioned non-isotropy of the tube banks it is expected that the theoretical results presented will overpredict tube bank pressure drops.

Holman[9] presented the pressure drop due to the flow of gases over a bank of tubes as

$$\nabla p = \frac{2f' G_{max}^2 N}{\rho}\left(\frac{\mu_w}{\mu_b}\right)^2. \tag{28}$$

In this equation f' is an empirical friction factor, N represents the number of transverse rows and subscripts w and b refer to wall and bulk fluid properties respectively. For a large number of transverse rows, zero heat transfer and steady state flow in the global x-direction, equation (28) can be simplified to

$$-\frac{dp}{dx} = \frac{2f' G_{max}^2}{\rho d}. \tag{29}$$

Comparison of the solid volume representation of the RUC with the tube bank cylindrical tubes of diameter D, implies that

$$d_s^2 \equiv \frac{\pi}{4} D^2. \tag{30}$$

G_{max} is the mass velocity at the minimum flow area of the RUC and is equal to ρv_p. This relationship, together with equations (11), (14), (16) and (30), facilitates the rewriting of equation (29) in terms of the porosity and a characteristic length, as follows:

$$-\frac{dp}{dx}\left(\frac{\varepsilon D^2}{\mu q}\right) = \frac{4f' Re_{qD}}{\sqrt{\pi}\varepsilon}(1 + \sqrt{1 - \varepsilon}\,)^2\sqrt{1 - \varepsilon}. \tag{31}$$

Jakob[8] constructed the empirical staggered tube bank friction factor, f', of equation (32).

$$f' = \left(0.250 + \frac{0.118}{[(d - D)/D]^{1.08}}\right) Re_{max}^{-0.16}. \tag{32}$$

Equations (11) and (30) facilitates the expression of the above factor in terms of the present RUC nomenclature.

$$f' = \left[0.25 + \frac{0.249(1-\varepsilon)^{0.54}}{[1.77 - 2(1-\varepsilon)^{0.5}]^{1.08}}\right](Re_{qD})^{-0.16}\left[\frac{\varepsilon}{T}\right]^{0.16} \tag{33}$$

Direct comparison of results requires the specialization of the phase average Navier-Stokes equation (8) for the flow conditions empirically quantified by equations (31) and (33). This is accomplished through the following considerations in terms of equation (8):

(i) The macroscopic flow is one-dimensional (x-direction) and fully developed.

(ii) The left hand side of the equation equates to zero due to assumption (i).

(iii) Macroscopic boundary effects are considered insignificant resulting in the omission of the third and fourth terms on the right hand side of the equation.

(iv) Gravity does not participate in the solution of horizontal flow.

(v) The gradient of the intrinsic phase average pressure is equal to the measured pressure gradient over the tube bank.

Equation (8) may consequently be simplified to

$$FD^2 = -\frac{dp}{dx}\left(\frac{\varepsilon D^2}{\mu q}\right). \qquad (34)$$

Consecutive application of equations (17), (27) and (30) transforms equation (34) into a format suitable for the straight forward calculation of pressure drops over staggered tube banks, based on the presented unified model.

$$\frac{dp}{dx} = -\left(\frac{\mu q}{\varepsilon D^2}\right) \times$$

$$\left[\frac{96(1-\varepsilon)^{\frac{3}{2}}}{\pi(1-\sqrt{1-\varepsilon}\,)^2} + \frac{c_d Re_{qD}}{\sqrt{\pi}\varepsilon^2}(1+\sqrt{1-\varepsilon}\,)^2(1-\varepsilon)\right]. \qquad (35)$$

The above quantifcation of the surface integral term in the phase average Navier-Stokes equation is graphically compared with the empirical relations of Holman[9] and Jakob[8] in figures (3), (4) and (5). The graphs explore porosities of 0.43, 0.61 and 0.81 respectively. Numerical solutions for low Reynolds number flow by Coulaud et al[10], as well as the flow development model of Du Plessis[6], are included in the graphs for completeness.

The high Reynolds number laminar flow asymptotic drag coefficient for smooth circular cylinders subjected to free stream crossflow conditions, falls in the range of 0.8 to 1.2. The unified model shows good experimental agreement for lower drag coefficient values (c_d = 0.5), which may be attributed to a recovery of pressure behind each cylinder caused by the geometrical arrangement. Such an effect will be more pronounced at lower porosities, which is confirmed by figures (3) to (5).

Conclusion

A semi-empirical quantification of the surface integral term of the phase average Navier-Stokes equation is presented for laminar crossflow through prismatic porous domains. The solution is derived for the whole porosity range from zero through one, and exhibits acceptable accuracy in the Darcy through Forchheimer flow regions. The validity of the listed assumptions and simplifications

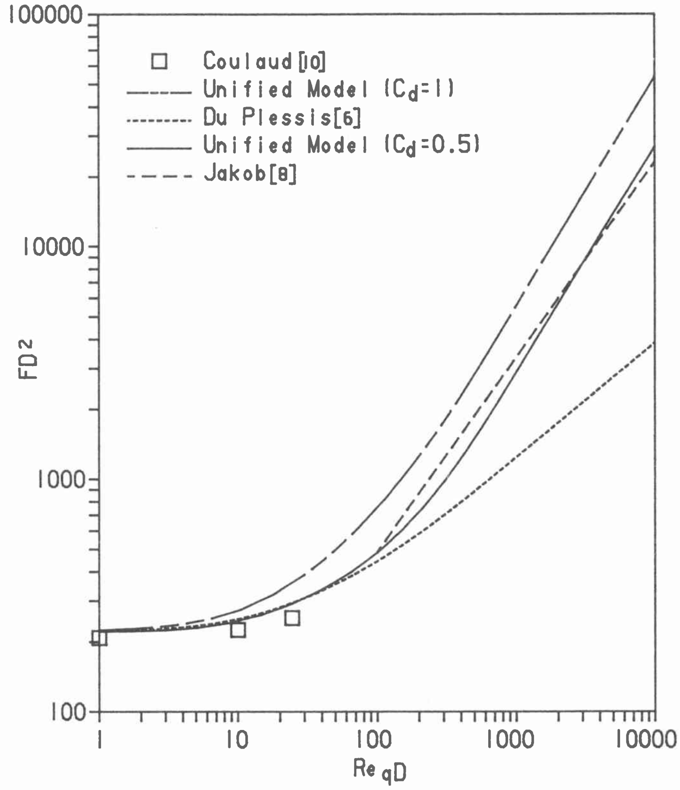

Figure 3: Quantification of prismatic porous media resistance to crossflow for a porosity of 0.43.

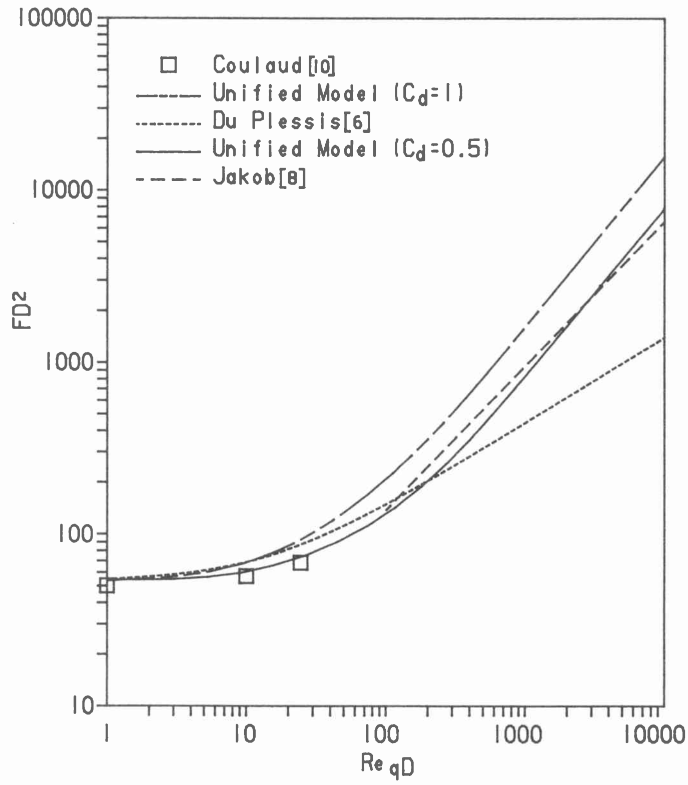

Figure 4: Quantification of prismatic porous media resistance to crossflow for a porosity of 0.61.

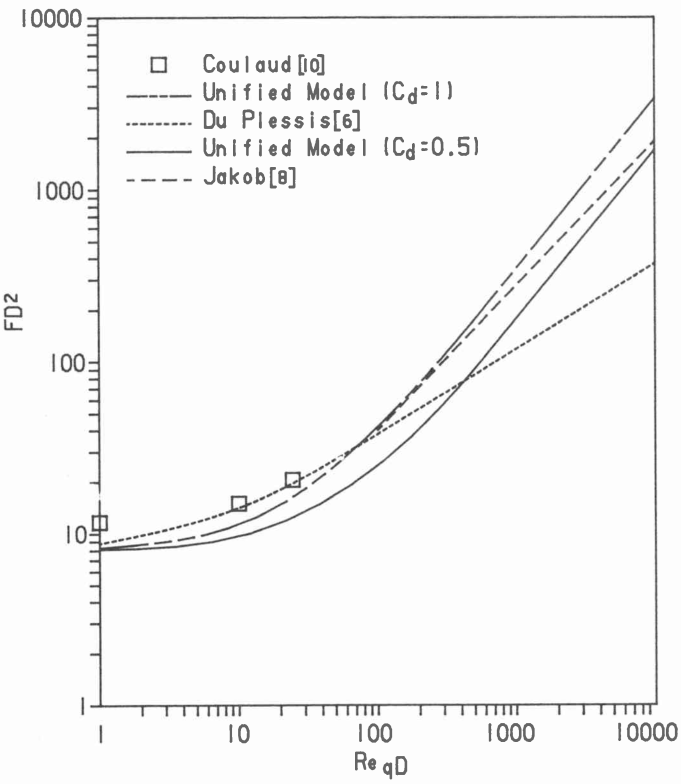

Figure 5: Quantification of prismatic porous media resistance to crossflow for a porosity of 0.81.

at high Reynolds numbers are illustrated through comparison of the presented unified model results with experimental pressure drops over heat transfer tube banks.

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