Heat transfer and friction loss in extended surface heat exchangers for non-Newtonian fluids in laminar flow

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Abstract

Empirical correlations for predicting the heat transfer coefficient and pressure drop in cross-flow annular finned tube heat exchangers with pseudoplastic non-Newtonian fluids in laminar flow are presented. The correlations were derived from measurements conducted on industrial heat exchangers with both staggered and in line configurations, and with different combinations of fin spacings and fin sizes.

Nomenclature

A_o = outside surface area of bare tube
A_t = total surface area of finned tube
a = constant
b = constant
C_N = function of number of tube rows
c_p = specific heat of fluid
D_e = volumetric equivalent diameter
D_f = outside diameter of fins
D_o = outside diameter of tube
dv/dy = shear rate
f = friction factor
h = heat transfer coefficient
K = consistency index
k = thermal conductivity
L = length of flow channel
l = fin height
Nu = Nusselt number
n = flow behaviour index
ΔP = pressure drop
Pr = Prandtl number
Re = Reynolds number
S_t = transversal tube pitch
s = distance between adjacent fins
t = fin thickness
V = actual average velocity
v = superficial velocity
$\in$ = void fraction
η = fin efficiency
μ_a = apparent viscosity
ρ = density
ψ = function of tube layouts
φ = function of fin geometries

Subscripts

f = measured at average film temperature
w = measured at wall conditions

Introduction

During the process of sugar manufacture, the low grade product, consisting of a mixture of fine crystals and molasses, is treated in cooling crystalizers from which it comes out in a supersaturated state. This fluid is pseudoplastic with an apparent viscosity of up to 6000 Pa.s. The apparent viscosity is given by the equatio

$$\mu_a = K\left|\frac{dv}{dy}\right|^{n-1} \tag{1}$$

Where K is the consistency index, and n is the flow behaviour index. The greater the departure of n from unity, the greater the non-Newtonian behaviour of the product. The next step is the separation of the crystals in centrifuges, and to facilitate this operation it is necessary to reduce the viscosity. This is done by warming the product using finned tube heat exchangers while the product is brought back to saturation. As a result of the high viscosity of the fluid and of the low velocities, laminar flow prevails in these heat exchangers.

Empirical correlations, derived experimentally, have been proposed for estimating the heat transfer coefficient in cross-flow extended surface heat exchangers in turbulent flow [1]. These equations are of the type

$$Nu = a.Re^b.Pr^{1/3}.\varphi.\psi.C_N \tag{2}$$

Different equations have been proposed for square and triangular pitch arrangements, and the effect of fin geometry is taken care of by a factor which differs in each equation. Briggs and young [2] used the finning factor s/l, fin spacing/fin height, for high finned tubes, and Schmidt [3] used A_t/A_o, the total area/outside bare tube area.

Similar empirical correlations derived for the Fanning friction factor in turbulent flow include the parameters shown below

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$$f = a.Re^{-b}.\varphi.\psi.C_N \quad (3)$$

Since the correlations that have been proposed do not apply to laminar conditions it was necessary to derive new equations experimentally.

Equipment and procedure

Measurements were conducted on six industrial cross-flow extended surface heat exchangers having the characteristics shown in Table 1, with square annular fins as shown on figure 1. Both square and equilateral triangular pitches were used, and fins were either in line or staggered.

with $$X = \frac{\tanh(ml')}{(ml')} \quad (5)$$

$$m = \sqrt{\frac{2.h}{k.t}} \quad (6)$$

$$Y = X(0{,}7 + 0{,}3X) \quad (7)$$

and the efficiency is given by

$$\eta = Y[0{,}45\ln(D_f/D_0(Y + 1) + 1] \quad (8)$$

Table 1

Characteristics of heat exchangers

	1	2	3	4	5	6
Heating surface – m²	1497	412	542	1212	520	1372
Sectional area – m²	18,1	8,13	9,19	13,0	5,57	15,5
No. of tube passes	60	6	10	12	24	12
Number of tube rows	10	6	10	14	12	8
25,4 mm pitch	6	6	10	10	8	4
38,1 mm pitch	4	–	–	2	4	3
50,8 mm pitch	–	–	–	2	–	1
Fin type	B	B	B	A	B	*
Tube pitch	Triangular	Square	Square	Square	Triangular	Triangular
Fins staggered	Yes	No	Yes	Yes	Yes	No
D_e	0,04833	0,04306	0,05336	0,06027	0,05186	0,05529
ϵ	0,7862	0,7733	0,7908	0,8401	0,8019	0,8161

* First four rows type B, next four rows similar to B but 240 × 240 × 5 mm.

The range of the physical properties and operating conditions that prevailed during the tests are shown in Table 2.

Table 2

Range of physical properties and operating conditions

	Minimum	Maximum
Density – kg/m³	1495	1534
Apparent viscosity – Pa.s	64	2674
Flow behaviour index	0,800	0,935
Superficial velocity – mm/s	0,08	1,11
Hot fluid temperature – °C	46,8	64,5
Cold fluid temperature – °C	32,2	58,6

All the heat exchangers had a single shell pass, and from 6 to 60 tube passes. For this combination of tube and shell passes with both fluids unmixed the corrections to the temperature difference were small. Corrections were also applied for actual fin efficiencies using the method of Weierman [4] which is applicable to both staggered and in-line layouts.

$$l' = l + t/2 \quad (4)$$

Since the heat exchangers had square fins, an average fin diameter based on the area of the fin was used.

The length of the path of the cold fluid on the shell side was assumed to be equal to the height of the tube bundle for the square pitch exchangers and one and a half time the height of the tube bundle for the equilateral triangular pitch arrangement. Fouling resistances on the inside and outside of the tubes were neglected.

Results

Because the fluids were non-Newtonian with pseudoplastic properties, it was necessary to use the generalized Reynolds and Prandtl number

$$Re = \frac{D_e^n.V^{2-n}.\rho}{K}.8.\left(\frac{n}{6n + 2}\right)^n \quad (9)$$

$$Pr = \frac{c_p.K}{8.k}\left(\frac{V}{D_e}\right)^{n-1}\left(\frac{6n + 2}{n}\right)^n \quad (10)$$

Where D_e, the volumetric equivalent diameter, was calculated from

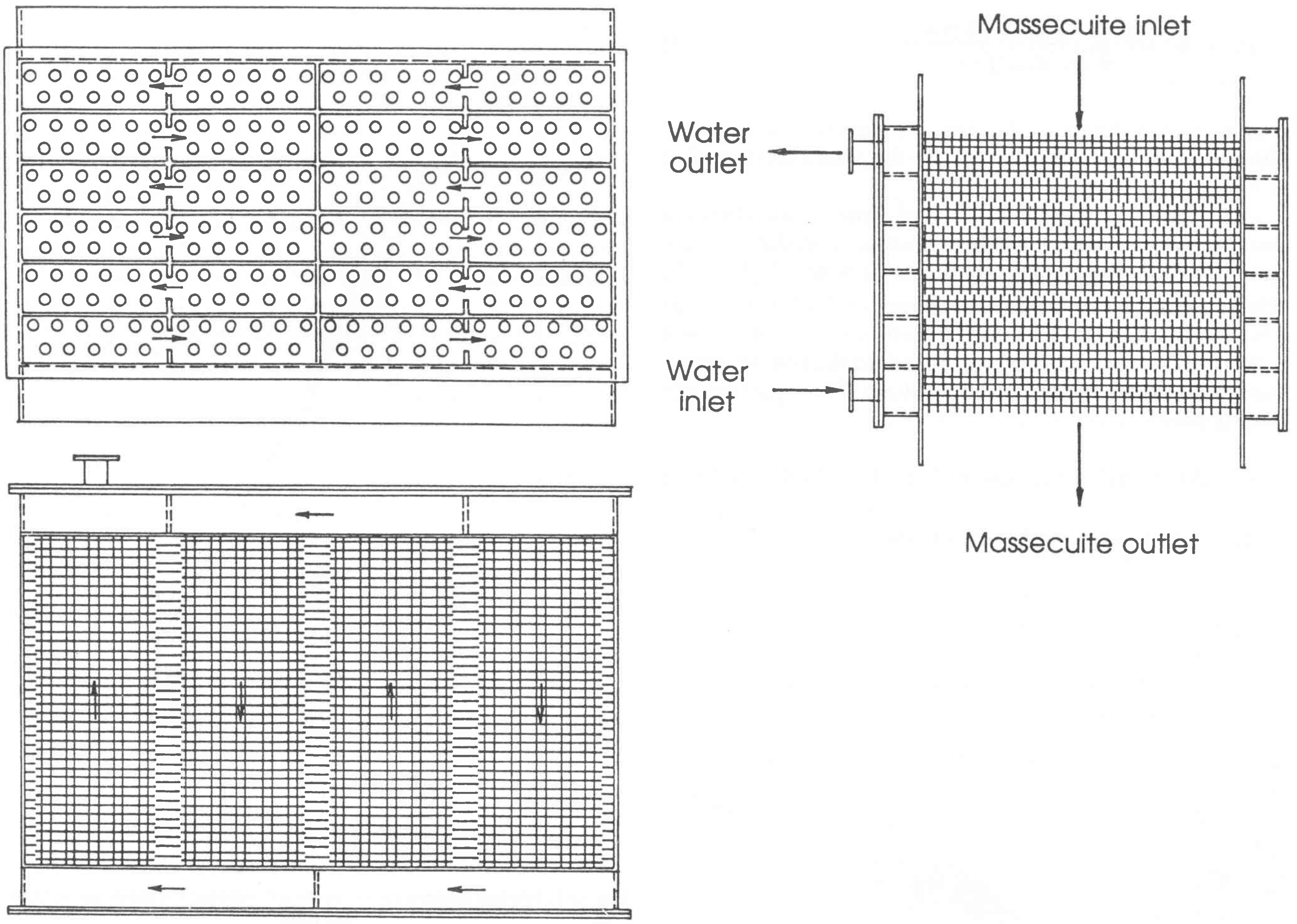

Figure 1. Arrangement of heat exchanger No. 5.

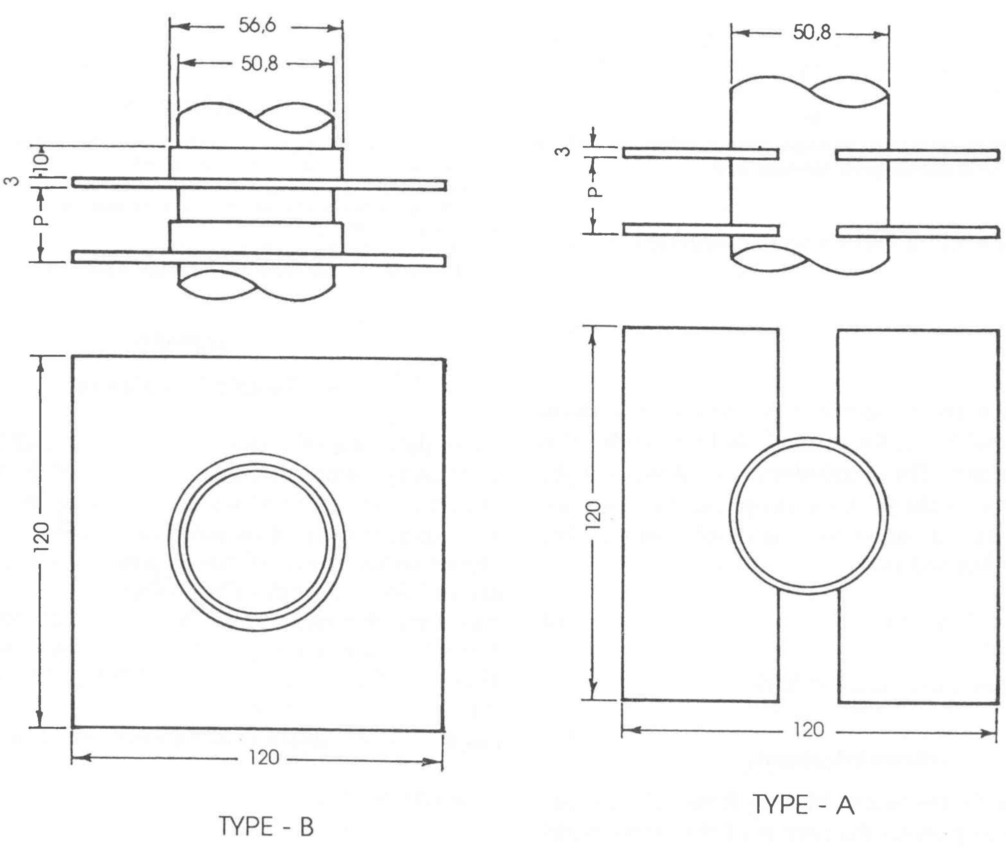

Figure 2. Details of finned tubes.

$$D_e = 4\,\frac{Volume\ of\ flow\ channels}{Wetted\ surface} \qquad (11)$$

The average velocity in the equations above was equal to the superficial velocity divided by the void fraction, that is v/ϵ.

In addition to the Reynolds and Prandtl number, the parameters included in the heat transfer correlation were the ratio D_e/L and the finning factor of Schmidt [3] A_t/A_0, which gave a better correlation than the factor of Briggs and Young [2] s/l. The correlation was also improved when the Prandtl number was based on the film temperature rather than the bulk temperature. The equation obtained was

$$Nu = 1{,}24 \times 10^{-6}(Pr_f)^{1/3}(Re)^{0{,}414}(D_e/L)^{-2{,}43}(A_t/A_0)^{2{,}91} \qquad (12)$$

with a correlation coefficient of 0,95.

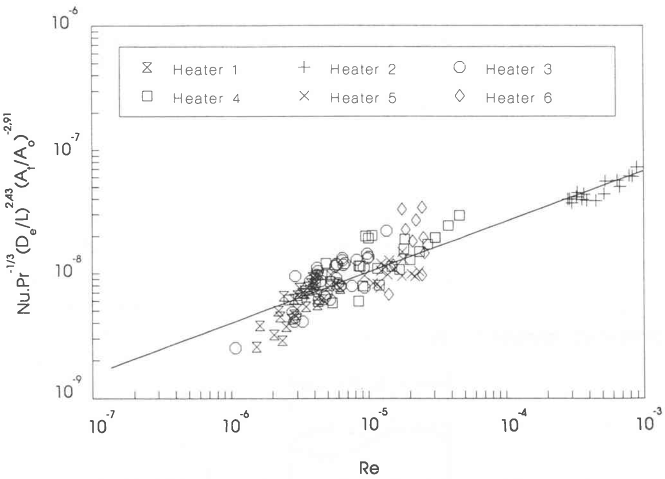

Figure 3. Data correlation for forced convection in extended surface heat exchangers, laminar flow.

The Fanning friction factors were calculated from

$$f = \frac{2.\Delta P.D_e}{V^2.L.\rho} \qquad (13)$$

and correlated with the generalized Reynolds number and the parameter S_t/D_e, the ratio of the tube pitch to the equivalent diameter. The consistency ratio K/K_f or K/K_w, which is included in the pressure drop equation of Gunter-Shaw [4], was not found to be a significant variable. The equation obtained is

$$f = 71{,}7\ Re^{-1{,}17}(S_t/D_e)^{3{,}51} \qquad (14)$$

with a correlation coefficient of 0,97.

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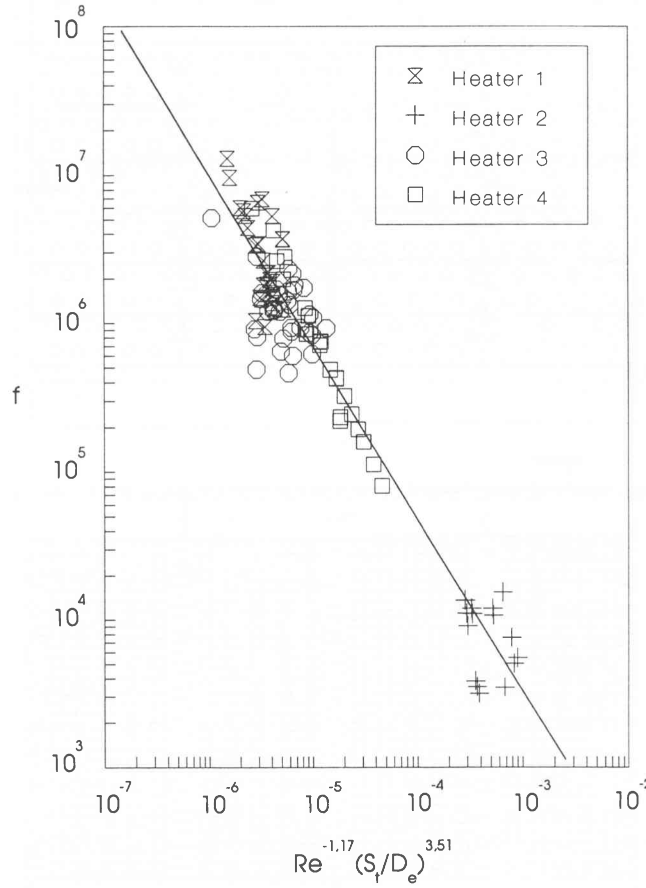

Figure 4. Friction factors for cross flow finned tube exchangers, laminar flow.

References

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2. Briggs, D. E., and Young, E. H., "Convection heat transfer and pressure drop of air flowing across triangular pitch banks of finned tubes", Chem. Eng. Prog. Symp. Ser., 59, 1963, p. 1.
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4. Gunter, A. Y., and Shaw, W. A., Trans. ASME, 1945, p. 643.

Appendix

Sample Calculations

Mass flow rate of water	= 19,30 kg/s
Inlet temperature of water	= 59,20 °C
Outlet temperature of water	= 58,08 °C
Inlet temperature of massecuite	= 46,6 °C
Outlet temperature of massecuite	= 58,0 °C

Overall Heat Transfer Coefficient

Enthalpy of water at 59,2 °C	= 247 786 J/kg
Enthalpy of water at 58,08 °C	= 243 104 J/kg

Heat transferred = Q = 19,30 (247 786-243 104) = 90 363 W

ΔT_{lm} = [(59,2 − 58,0) − (58,8 − 46,6)]/ln[(59,2 − 58,0)/(58,8 − 46,6)]

= 4,55 °C

Correction factor

$$CF = \frac{\sqrt{R^2 + 1}.\ln[(1 - S)/(1 - RS)]}{(R - 1).\ln\left[\dfrac{2 - S(R + 1 - \sqrt{R^2 + 1})}{2 - S(R + 1 + \sqrt{R^2 + 1})}\right]}$$

Where

$R = (58,0 - 46,6)/(59,2 - 46,6) = 0,9048$
$S = (59,2 - 58,08)/(58,0 - 46,6) = 0,09825$

Then

$CF = 0,9982$

Heat transfer area (including tube plates) = 1497 m²

$$U = \frac{Q}{A.\Delta T_{lm}.CF}$$

$U = 90\,363/(1497 \times 4,55 \times 0,9982)$
$= 13,29$ W/m².°C

Water side heat transfer coefficient
Average temperature = (59,20 + 58,08)/2
= 58,64 °C

Then

$k = 0,653$ W/m.°C
$\mu = 4,81 \times 10^{-4}$ Pa.s
$Pr = 3,08$

D_i tubes = 0,0447 m
Number of tubes per pass = 12
Mass flow rate of water =
$19,30/(12 \times \pi/4 \times 0,0447^2) = 1024,9$ kg/m².s

$Re = (0,0447 \times 1024,9)/4,81 \times 10^{-4}$
$= 95\,243$

Using Dittus Boelter equation

$Nu = 0,023\ (95\,243)^{0.8}\ (3,08)^{0.4}$
$= 346,9$

$h_w = (346,9 \times 0,653)/0,0447$
$= 5068$ W/m².°C

Massecuite side heat transfer coefficient corrected for fin efficiency
Assume corrected massecuite side heat transfer coefficient = h_m = 16,11 W/m².°C
From equation (6)
t = fin thickness = 0,003 m
k_t = thermal conductivity of tube = 53 W/m.°C

$m = [(2 \times 16,11)/(53 \times 0,003)]^{1/2}$
$= 14,24$

D_{of} = diameter at base of fins = 0,0566 m
D_f = outside diameter of fins = $(0,12^2 \times \pi/4)^{1/2} = 0,1354$ m
$l = (0,1353 - 0,0566)/2 = 0,0394$ m

From equation (4)

$l' = 0,0394 + 0,003/2$
$= 0,0409$

From equation (5)

$X = 0,9005$

From equation (7)

$Y = 0,9005(0,7 + 0,3 \times 0,9005)$
$= 0,8736$

From equation (8)

$\eta = 0,8736[0,45 \times \ln(0,1354/0,0566)(0,8736 - 1) + 1]$
$= 0,8301$

$$h_m = \frac{1}{(A_f.\eta + A_t)\left(\dfrac{1}{U.A_o} - \dfrac{1}{A_i.h_w} - \dfrac{\ln(D_{oav}/D_i)}{2\pi.k_t.N_t.L_t}\right)}$$

N_t	= number of tubes	=	720
L_t	= length of tubes	=	1,98 m
A_o	= Outside area of finned tubes	=	1476,5 m²
A_i	= inside area of finned tubes	=	200,2 m²
A_f	= Area of fins	=	1272,1 m²
A_t	= Area of bare tubes	=	204,4 m²
D_{oav}	= average outside tube diameter	=	0,0627 m

$h_m = 1/[(1272,1 \times 0,8301 + 204,4)(1/(13,29 \times 1476,5) -$
$1/(200,2 \times 5068) - \ln(0,0627/0,0447)/$
$(2\pi \times 53 \times 720 \times 1,98)]$
$= 16,13$ W/m².°C

This result is close enough to the assumed value.
Tube surface temperature

$$T_w - \frac{T_{wi} + T_{wo}}{2} - \Delta T_{av}.\frac{R_w + R_t}{R_w + R_t + R_m}$$

Where

R_m	= massecuite resistance	=	$4,929 \times 10^{-5}$
R_t	= tube resistance	=	$7,128 \times 10^{-7}$
R_w	= water resistance	=	$9,856 \times 10^{-7}$

$T_w = (59,2 + 58,0)/2 - (6,34)(7,128 + 9,856) \times$
$10^{-7}/(492,9 + 7,128 + 9,856) \times 10^{-7}$
$= 58,39$°C

Film temperature

$T_f = [(58,0 + 46,6)/2 + 58,39]/2$
$= 55,40$°C

K_f = 2414,4 Pa.s
k_f = 0,3201 W/m.°C
C_p = 1443,6 J/kg.°C

ρ = 1505,4 kg/m^3
D_e = 0,04833

Nusselt number

$$Nu = (16{,}13 \times 0{,}04833)/0{,}3201 = 2{,}435$$

Volumetric flow rate of massecuite
= 90 363/(1443,6)(58,0 − 46,6)(1505,4)
= 3,647 × 10^{-3} m^3/s

Sectional area of heat exchanger = 18,1 m^2

Void fraction = 0,7862

Massecuite velocity
= 3,647 × 10^{-3}/(18,1 × 0,7862)
= 2,563 × 10^{-4} m/s

n = flow behaviour index = 0,8003

From equation (10) the Prandtl number is

$$Pr_f = \frac{1443{,}6 \times 2414{,}4}{8 \times 0{,}3201}\left(\frac{2{,}563.10^{-4}}{0{,}04833}\right)^{(0{,}8003-1)}\left(\frac{6 \times 0{,}8003 + 2}{0{,}8003}\right)^{0{,}8003} = 2{,}148 \times 10^7$$

Average bulk temperature of massecuite
= (58,0 + 46,6)/2 = 52,3°C
K = 2625,5 Pa.s

From equation (9) the Reynolds number is

$$Re = \frac{(0{,}04833)^{0{,}8003}\,(2{,}563.10^{-4})^{2-0{,}8003)}\,(1505{,}43)}{2625{,}5}.8.\left(\frac{0{,}8003}{6 \times 0{,}8003 + 2}\right)^{0{,}8003} = 3{,}601 \times 10^{-6}$$

Viscosity ratio

$$K/K_f = 2625{,}5/2414{,}4 = 1{,}087$$

L = Length of flow channell = 1,42 m

$$D_e/L = 0{,}04833/1{,}42 = 0{,}03404$$

Finning factor

$$A_t/A_o = 204{,}4/1476{,}52 = 0{,}1384$$