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Using games to develop number sense in early grade maths clubs

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Background: South African learners face the double disadvantage of living in low socio-economic conditions with access to few resources and attending schools with challenging learning conditions. Mathematics performance reflects such conditions with extreme performance gaps between wealthier and poorer learners. The need for early intervention is increasingly acknowledged.

Aim: In this conceptual article, we draw out the features of mathematics games that can be used to develop learners' foundational number sense that will impact their learning trajectories.

Setting: Based on our experience in working with learners from low socio-economic backgrounds in after-school mathematics clubs, we propose that these are opportune spaces for intervention.

Methods: Drawing both on the research literature and our experience, we exemplify and discuss key features of mathematics games and argue why these are important to support the development of number sense.

Results: We argue that to meaningfully develop young learners' number sense, activities need to be sufficiently engaging to retain their interest and be presented in a manner that club practitioners, who might not be teachers, are able to facilitate. Drawing on our experiences of designing and running mathematics clubs, we discuss why components like built-in checks, readily available resources, connection to calculation strategies and transferability to home settings are important design features.

Conclusion: Particular design features are important to enable maths games to support learners' development of efficient and effective calculation strategies.

Contribution: This article contributes a framework of key features for effective use of games to develop mental mathematics strategies for number sense.

Keywords: mental mathematics; mathematics games; number sense; mathematics clubs; primary mathematics.

Introduction

Many South African learners from low socio-economic backgrounds face a double disadvantage in that not only do they attend schools that are often overcrowded and with limited resources, but they are also less likely to have access to resources at home to support learning (Reddy et al. 2022). The benefit experienced by South African learners with a resourced home background and quality schooling has been equated to about 4 years of learning in numeracy at Grade 3 level (Spaull 2016). There is furthermore increasing awareness that intervening in this ever-growing learning lag in the early grades of schooling is critically important (eds. Spaull & Taylor 2022). Involving learners in quality after-school programmes has been suggested as one way to close the educational gap for disadvantaged learners (Schoeman 2019).

In this conceptual article, we propose that after-school mathematics clubs are opportune spaces for intervening with early grade learners from low socio-economic backgrounds to support the development of number-sense with mental mathematics as a key aspect (and enabler) of number sense. We argue that for after-school programmes to contribute meaningfully to developing learners' number sense in the early grades, the activities need to be sufficiently engaging to retain the interest of young learners after the end of the school day and presented in a user-friendly manner so that after-school practitioners, who might not be teachers, are able to effectively

Note: Special Collection: Mental mathematics and number sense in the early grades.

facilitate their implementation. Furthermore, activities should be able to be replicated in the home space to provide further opportunity to engage with mathematics and develop fluency with it. Dice and card games are particularly useful in developing mental mathematics as they are no or low cost, and readily available.

Drawing on our experiences of developing and running after-school mathematics clubs (Bowie et al. 2022; Graven 2011, 2015; Stott & Graven 2013a), we discuss the contribution these clubs can make to developing number sense through developing mental mathematics strategies, particularly through engagement with mathematical fluency and strategy games that support effective and efficient use of mental strategies. The design of our clubs builds broadly on Lev Vygotsky's sociocultural perspective on learning, where language and more knowledgeable others are key in the mediation of learning – as such clubs maximise opportunity for learner talk and engagement with mathematical ideas. Progression of mathematical ideas and content build on the broad base of early years mathematics learning progressions or trajectories (e.g. Clements & Sarama 2020; Wright, Martland & Stafford 2006).

In particular, we explore, in relation to Foundation Phase (FP) clubs:

- The rationale for the use of mathematical games in engaging young learners and helping them develop fluency in mental calculations.
- The importance of creating links between the various elements of the maths clubs programme so that the fluencies developed in playing the games are coherently linked to the mental calculation and problem-solving work done in further activities (Askew 2009, 2015).
- Design features of the material required to support practitioners to implement game-based and other activities focussed on mental mathematics strategies while simultaneously developing their own number sense.

While not an empirical article, we build on previous empirical studies that point to this design leading to improvements in club learners' mental fluencies. See for example, Stott and Graven (2013b), Stott et al. (2017) and Stott, Baart and Graven (2019) for data showing strong learner improvements across multiple clubs.

Rationale, conceptual framing and literature review

Why use after-school spaces for developing mental mathematics and number sense?

We have been working with after-school primary mathematics clubs since 2011 when the second author piloted her first Grade 3 club with 10 learners in one school that catered for learners from low socioeconomic status (SES) backgrounds. Since then, after-school mathematics clubs have expanded rapidly across the country and a consortium of maths clubs has been formed which the two authors and their respective organisations actively participate in (see www.mathsclubs.co.za for the work of this consortium).

Some of the rationale for setting up after-school clubs particularly for learners from poorer backgrounds included the following:

- Learners are several grades behind (Spaull & Kotze 2015) and thus grade level classroom workbooks are not always at the appropriate level for learners in a grade.
- Large classes in schools tend to have little space and opportunity for individualised learner attention and learner feedback (Hoadley 2012). Club facilitators could work with learners in smaller groups and focus on specific learner needs informed by where learners are at and remediate missing foundational competencies and pervasive negative learning dispositions (Graven & Heyd-Metzuyanim 2014).
- Club learners may become catalysts in their classrooms showing new ways of engaging with mathematics (see Graven 2015).
- Take-home activities and games taught (to be played at home) would extend the time learners engaged with mathematics (see Graven 2015).

Several key design features of clubs (such as the inclusion of games and hands-on activities) have persisted throughout the expansion (Bowie et al. 2022; Graven 2015). These features have been captured in the current mathematics clubs facilitator guides that are available for open source use (Maths Clubs 2023) and are being used both by the authors' own organisations to run mathematics clubs as well as by a variety of partner non-governmental organisations (NGOs) who use them to support their after-school work. Figure 1 shows the description of mathematics clubs given in the club facilitator guides (Maths Clubs 2023).

The bullets presented in Figure 1 resonate with a socio-cultural perspective on learning and link clearly to literature that argues for the importance of mathematics talk (Boaler 2015) using every day and home language as a key learning resource (Planas 2018; Planas & Setati-Phakeng 2014), adopting a growth mindset (Claro, Paunesku & Dweck 2016; Dweck 2006), and having a positive disposition towards mathematics (Boaler 2015; Kilpatrick, Swafford & Findell 2002). The South African Numeracy Handbook for Foundation Phase Teachers: Grades R-3, building on Kilpatrick et al.'s (2001) five-stranded definition of mathematical proficiency, states (Department of Basic Education 2012:11) that being numerate means:

- Understanding what you are doing (conceptual understanding).
- Being able to apply what you have learnt (strategic competence).
- Being able to reason about what you have done (adaptive reasoning).

1 What is an after-school maths club?

The clubs are supportive communities where sense making, active mathematical engagement and participation, and mathematical confidence building are emphasised.

Clubs can provide learners with a space where they can ask their own questions, produce their own mathematics, talk mathematics, explain mathematics and enjoy mathematics!

In the clubs, learners are free to:

- Talk about mathematics
- Argue about mathematics
- Explain how something was worked out
- Ask questions
- Make mistakes as learning happens by making and discussing mistakes
- Speak their own language
- Cross things out, be untidy
- HAVE FUN

FIGURE 1: Introduction to maths clubs in facilitator guides.

- Recognising that you need to engage (productive disposition) with a problem in order to solve it.
- Being able to calculate or compute (procedural fluency) with confidence.

Our work in the clubs and the design of our activities, including the mental maths games that are in focus here, address all five intertwined aspects of mathematical proficiency that are considered together to enable number sense – an 'at-homeness with numbers' (Cockcroft 1982). As the strands are intertwined, our games discussed in this article simultaneously address multiple strands even while some strands might dominate in a particular game.

Why focus on mental mathematics strategies for number sense?

A widely and long recognised challenge of primary school mathematics learning is the persistence of unit counting for calculating far beyond when appropriate and extending to number ranges that do not lend themselves to unit counting. Schollar's (2008) work clearly highlighted this as a key challenge in the development of number sense for South African learners and yet this persists (Graven & Heyd-Metzuyanim 2014; Petersen, Vermeulen & McAuliffe 2017; Weitz & Venkat 2013). The absence of attention to mental mathematics strategies is problematic given they are included in the FP curriculum. The South African Curriculum and Assessment Policy Document for Mathematics for the FP states that a key function of mathematics learning in these years is to develop 'mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute to decision making' (Department of Basic Education 2011:8–9). Yet, it is generally not given the attention it deserves and was not included in the annual national assessments (Graven et al. 2013). More recently, the implementation of the teaching of mental strategies within the 10 min warm-up start of lessons (in the form of the Mental Starters Assessment Programme [MSAP]) with linked diagnostic pre- and post-assessments in Grade 3 classrooms in Department of Basic Education (DBE) schools addresses this absence (see Askew, Graven & Venkat 2022; Department of Basic Education 2021; Graven & Venkat 2021). Both authors are contributing members of MSAP. The emphasis on mental mathematics calculation strategies (and particularly through the use of games) in after-school mathematics clubs predates the MSAP work and to an extent our experiences of focussing on mental mathematics in the clubs has guided our MSAP input.

We use the term mental mathematics synonymously with mental calculation. Mental calculation differs from written calculation procedures such as written algorithms, written representations (such as drawing out ones or drawing the calculation out on a number line) or using manipulatives such as the abacus – in mental calculation, the 'problem' is done mentally (even while imagined manipulatives or representations support the mental calculating process). That said, in our clubs, we do extensive work with key representations or models such as arrays, the empty number line, the part-part-whole bar model, the hundreds chart, and ten frames.

Askew (2015), working with notions introduced by the Freudenthal Institute, distinguishes between 'models of', 'models for' and 'tools for'. He exemplifies these concepts by discussing how a teacher may use counting on a number line as a model of learners' counting of physical objects to do addition. Through joint activity and repeated exposure to the teacher's model, the learners may come to take the teacher's model on for themselves, and at this stage, it becomes a 'model for', that is a model they can use themselves. Finally, through repeated use of this model over time, learners, for example come to be able to use the number line in their heads as a tool for thinking. Thus, where possible, we incorporate the models we feel are powerful and useful into our club activities and draw on learners' emerging 'mental models' in our games.

As in the case of MSAP, we use these to develop understanding and fluency in various calculation strategies such as bridging through 10, jump strategy, rounding and adjusting, re-ordering, doubling, and halving and using the relationship between addition and subtraction. While these strategies can be drawn out on number lines and arrays (with pen on paper), the aim of these strategies is for them to become mental representations that support fluent mental calculation strategies. Thus, in the context of the mental mathematics games, these serve as mental representations that students use for calculations required in the games.

Why use games to develop mental mathematics?

A number of affective factors are cited as a rationale for the inclusion of games in learning mathematics. These include an improved attitude towards mathematics (Bragg 2007; Nisbet & Williams 2009) as well as motivation and engagement (Oldfield 1991). Research evidence indicates that games do indeed have positive effects on these affective factors with, for example, Bragg (2012a) finding more on-task engagement in lessons involving games and Graven and colleagues showing participation in maths clubs with game activities having a positive influence on learners' mathematical identities and learning dispositions (Graven 2015; Hewana & Graven 2015; Hokonya 2021). It has also been suggested that games can stimulate mathematical discussion (Oldfield 1991; Wiersum 2012) with Bragg (2012b) finding greater incidents of learner talk about mathematics in classes where mathematical games were being played.

A number of authors have investigated the efficacy of games in teaching mathematics. Bragg's (2012b) review cites studies that show positive effects of games on learners' mathematics performance in the area of number operations, early numeracy, decimal fractions and mental computation. The research into the effectiveness of games for stimulating mathematical learning extends to pre-schoolers from low income backgrounds. For example, Ramani and Siegler (2008) found that playing board games in which numbers were arranged in order linearly, improved the number identification, comparison, and number line estimation of the learners.

However, the research on the efficacy of games in teaching mathematics also raises some caveats. For example, both Booker (1996) and Onslow (1990) suggest that for games to help learners overcome misconceptions or gain conceptual understanding, they need to be accompanied by discussion. Asplin, Frid and Sparrow (2006:46) allude to the importance of the game being 'scaffolded by teacher designed learning structures'. In this article, the authors argue that the way in which learners had to progress through different levels and were exposed to different strategies when they rotated through different playing partners, supported their engagement with mental computations and their development of fluency in relation to computations. The desire to become an efficient and fluent calculator connects to the reward of becoming good at various computational games.

Lange, Brenneman and Sareh (2021) followed up on Ramani and Siegler's (2008) studies on the efficacy of linear board games by testing them in a natural classroom setting and in a family home setting. While they found that the game playing had a positive effect on learners' numeral identification, it did not have a similar effect on their number line estimation or judgement of numeral magnitude. They hypothesise, based on their observations and the work of a related study (Laski & Collins 2015 in Lange et al. 2021) that the absence of explicit instructions for the teacher or parent to use comparison language (e.g., which is more?) was a likely contributor to the muted effects on number line estimation skills and sense of numeral magnitude. Thus, the nature of mediation linked to games becomes important for the full learning potential of games to be realised.

Pivotal across these discussions is the importance of identifying the educational and mathematical goals of the games. Researchers and practitioners have put forward various potential categorisations of mathematical games. Some, for example the work of Pan et al. (2022), who conducted a systematic review of the literature on learning games for mathematics education from 2008 to 2021 identified categorisations according to multiple game variables: game genre (e.g., puzzles were the most prevalent while strategy games were next), game platform (most were computer games), game function (most functioned as supplemental tools for instruction involving drill and practice), game play mode (individual, collaborative or mixed), and game math learning content. They further noted differences in the learning theory employed (behaviourist, constructivist or experiential) and levels of cognitive demand (higher or lower). Others, like Lim-Teo (1991), focus on the educational purpose of the games: drill and practice, concept reinforcement, as starting point of a mathematical investigation, application of mathematical knowledge or fun. The games that we play in our clubs are multi-purpose strategy and fluency games that we argue address concept reinforcement, practice for fluency, application of knowledge and 'having fun'.

Russo, Russo and Bragg (2018:30), drawing on the research on mathematical games, identify five principles of educationally rich mathematical games. These are:

1. Mathematical games should be engaging, enjoyable and generate mathematical discussion.
2. Mathematical games should appropriately balance skill and luck.
3. Exploring important mathematical concepts and practising important skills should be central to game strategy and gameplay.
4. Mathematical games should be easily differentiated to cater for a variety of learners, and modifiable to cater to a variety of concepts.
5. Mathematical games should provide opportunities for fostering home-school connections.

Although we concur with these principles, and our club games embody each of these, we would add that, given we are working mostly in resource-constrained contexts, the games should not require elaborate equipment and/or time to set up. We have restricted ourselves to readily available resources like dice, playing cards, counters (bottle tops, beans, fingers) and paper and designed games that can be played in multiple settings (e.g. home, on the playground, in the taxi) with little setting-up time and simple rules. In this way, learners can quickly teach and include others in playing the games increasing the likelihood of their being played repeatedly beyond the club context. Thus in our work, we have found that for games to be effective, there are some important considerations. In what follows, we draw on the literature and our experience of working with games to emphasise and illuminate key features that develop mental maths strategies for supporting foundational and robust number sense.

Exemplifying design features of mental maths games for various purposes

In this section, we share a range of FP games that we play in clubs to highlight different design features of:

- games for practice and fluency
- games for concept formation and reinforcement.

Note that while games often support all the above, the primary aim of the games differs. Finally, we expand on how the games we play link to various other activities in the clubs.

Features of games for practice and fluency

Regular and extensive practice is necessary for learners to become fluent in mental calculations (Codding et al. 2016). We have opted for the phrase 'practice towards fluency' rather than 'drill and practice' as the latter often has behaviourist connotations which we do not align with. Indeed, in our mental mathematics games, we practise developing fluency in basic facts while simultaneously building on these to develop fluency in computational strategies. We pay careful attention to sequencing so that we help learners become sufficiently fluent in a set of basic facts, for example bonds of 10, to be able to readily draw on that in a computational strategy like bridging through 10. The strength of mathematical games in engaging learners make them a good vehicle for the kind repeated practice necessary for fluency.

Some game design features we have experienced as supporting learning focussed on practice towards fluency are that they:

- Have simple basic rules so that they can be taught to others and played across a range of settings.
- Need no resources or only use readily available resources so they can be quickly set up and played across a range of settings.
- Have a built-in check on answers.
- Can be played at a pace that keeps students engaged.

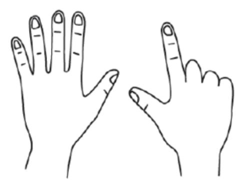
In order for such learning games to be effective, one needs to be certain that learners know and are then practising the correct facts. In certain cases, this can be done by having the teacher lead the game. For example, a popular warm-up game in our clubs is 'Fizz Pop' which involves a call and response routine that can be used to rehearse different facts. For example, playing Fizz Pop bonds of 5 means that when the teacher says a number, the learners will give the number that needs to be added to make 5 or in playing Fizz Pop doubles if the teacher says a number, the learners respond with double that number. In this type of rhythmically paced practice game, the teacher is able to monitor whether learners are answering correctly. This is particularly true if the teacher starts with whole class call and response but then calls on particular groups, pairs or individual learners. Games like bonds to 5 and 10 (and doubling small numbers) can be easily self-checked by learners checking the sensibility of their answer on their fingers.

Games that learners are able to play with their peers or at home with an adult who might themselves not have mastered the facts involved are important for sustained practise. Designing games where there is a built-in check of the correct answer is important. For example, a simple game like 'Spill the beans' allows learners to practise their bonds in a fun way. Their partner starts with 10 beans in a cup and spills a few onto the table. The learner must then state how many beans are left in the cup (a number fact that can be easily checked by seeing, if subitising is appropriate, or counting what remains). This can similarly be played with 5 or 10 objects that are shared between two hands. For example, if there are 10 objects altogether, a learner shows one hand with, for example six of the objects while the other four are hidden in the other hand which is closed. The learner asks another learner – 'so how many are hiding in my closed hand?'. The built-in check is when the objects in the hidden hand are revealed after the learner has answered. While knowing bonds to 5 and 10 is key, this game can also be played with any number of objects to establish part-part-whole number facts. See Jorgensen and Graven (2022) for a detailed description of this game.

For single digit addition, the game 'Flip through 9' has a built-in check. This game involves a learner shuffling 10 cards numbered 1 to 9 and then flipping through them one at a time adding them up while possibly being timed by their partner or caregiver. If their final total is not 45, they know they have made a mistake. For other concepts, it can be harder to build the confirmation of the correct calculation into such mental mathematics games. In these cases, it is sometimes necessary to include a 'referee' in the game. For example, in a game that requires learners to multiply two single-digit numbers together, the referee (a learner or a facilitator who might not be fluent in these facts) can be given a multiplication chart to check the players' answers. This is preferable to a calculator as such charts have various patterns visible in them.

For fluency games to be effective in enabling learners' instant recall of basic facts, they need to be practised and played regularly and consistently. Games that require no equipment and are easy to play are more easily used by teachers in larger classes and by children at home or with friends outside of the club. Games using fingers as the only needed equipment are very useful in this regard. For example, the game 'How many can you see, how many am I hiding' allows learners to practise their bonds of 10 by quickly noting how many fingers of their partner's fingers they can see and how many are hidden by being down (See Figure 2).

As an example of another game that can be played with no equipment, the game '1, 2, 3, show' is a simple game that can be used to practise many different skills with single-digit numbers. In this game, two learners start with their hand(s) behind their backs and saying '1, 2, 3, show' bring their hand(s) out with some fingers raised. The number of fingers each learner has raised will be their number for that round. Depending on the skill being practised, the learners need to compare, add or multiply the numbers represented on their fingers. See Figure 3.



Source: Maths Clubs, 2023, *Maths clubs facilitator guide*, OLICO Mathematics Education

FIGURE 2: A graphic of 'How many fingers can you see, how many am I hiding?' in a maths club facilitator guide.

Features of games for concept formation (and reinforcement)

A key aim in maths clubs is to help learners to move beyond counting in ones towards more efficient strategies for calculation. Both the research base (e.g., Lange et al. 2021; Onslow 1990) and our experience suggest that regular rehearsal alone might not be sufficient to facilitate the adoption of efficient strategies. In the maths clubs, we introduce learners to different calculation strategies; the challenge, however, has been to ensure that these strategies remain in focus when learners are practising calculations while playing the games so that they do not simply revert to older, more familiar, inefficient strategies. Using strategies fluently (such as rounding and adjusting) depends on certain basic facts to be rapidly recalled – that is, fluency of strategies rests on fluency of basic facts (Askew 2009). Thus, in the maths clubs, we play a range of games that practise basic facts (e.g., bonds to 10 or multiplying by 10) and this basic fact fluency then supports developing fluency in games that practise using strategies for calculation. For example, below we explain how we can combine fluencies with multiplying by ten along with fluency with subtracting 9 from a multiple of ten to develop fluency with multiplying by 9. The mechanisms we have introduced to achieve this are discussed and exemplified below.

Practise for fluency of the strategy: Using concrete manipulatives and drawn arrays, we introduce learners to the idea that they are able to calculate $9 \times x$ by saying $10 \times x - x$. First of all, learners must be fluent in $\times 10$ (knowing the pattern enables this). Playing a quick quiz game like fizz pop with $\times 10$ can check whether learners are fluently able to multiply by 10 (Fizz, pop, 9, 90; Fizz, pop, 8, 80 etc.). For learners who may not be fluent in this, the rhythm of the game soon establishes the easy pattern for multiplying by 10.

To reinforce the strategy for calculating 9 times a number, we include a game in the form of a call and response song. The teacher (or leader of the activity who might be club facilitator or a club learner) sets up a clapping rhythm with the class and then accompanies it with the chant:

Teacher: Times by 9

Learners: Times by 9

Teacher: 9 by 6

Learners: 60 minus 6; 54

Teacher: 9 by 8

Learners: 80 minus 8; 72, among others.

The rehearsal of the strategy is thus built into the activity.

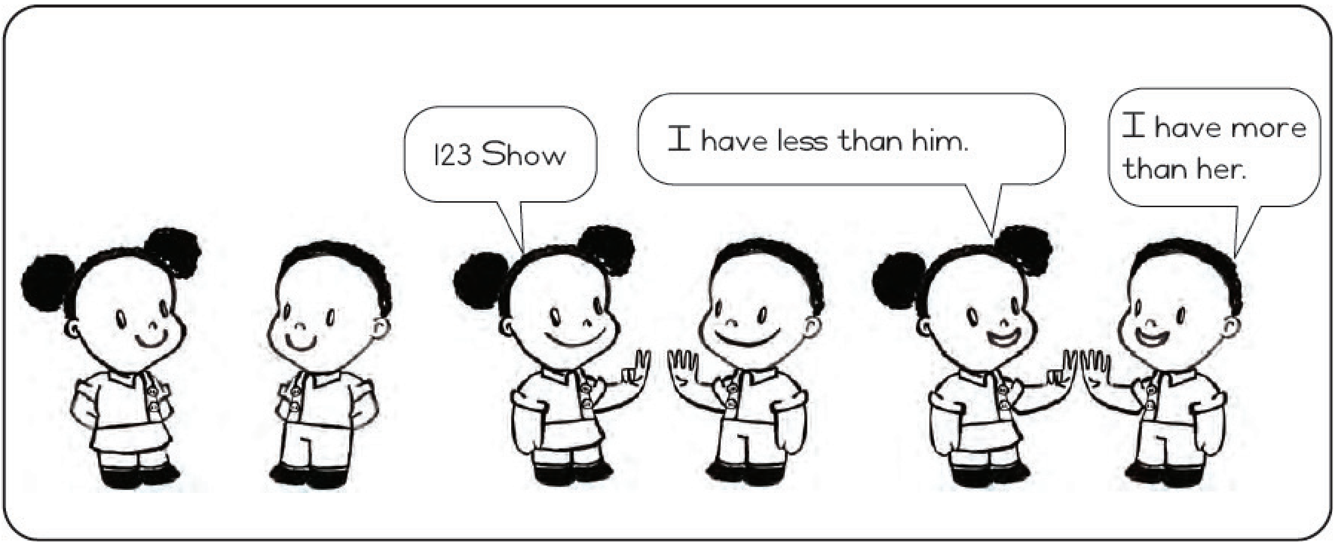
The concept underlying a strategy can be built into the game: It is also possible to build underlying mathematical concepts into games. In learners' work with additive relations, an important concept they need to master is the fact that addition and subtraction are inverse operations. In the game 'Salute', two learners put cards facing out on their foreheads so that they can see each other's number, but not their own number. A third learner then tells the two learners the sum of the numbers and then both of the learners need to figure out what their number is. Although the intent of this game is to focus learners' attention on the relationship between addition and subtraction, it is possible for learners to use guessing as a strategy for working out their number. Following on the suggestions from Lange et al. (2021), we include explicit instruction in the manual for club facilitators to link the game with the linked addition and subtraction number sentences (see Figure 4).

In a game like 'Two clips addition', two paper clips are placed on two parts. Figure 5 shows the explanation of the game in the club facilitator guide. On their turn, a player can move either paper clip, but only one of them, to a new part. They add the two parts together and claim that block on the board. The first player to claim three blocks in a row (horizontally, vertically or diagonally) wins. Although learners might initially play this game without strategy, some of the key learning comes when they start to think about what blocks on the board they want to capture and thus how they should move their paper clips. The club facilitator is often crucial in prompting this. This can be done by asking questions during play, such as 'What number do you want to get next? Is there a way you can do that?'

We have used the game 'How many children in my house?' to help learners move from count all to count on in the learning trajectory for addition (Baroody 1987). In this game, one learner puts some counters, for example 3, under their hand and tells their partner 'I have 3 children in my house'. The learner then takes some more counters, for example 4, and puts them next to their hand and says 'These 4 children also want to come into my house. How many children will be in my house?'. The partner must then determine the answer. The design of the game, with the initial number of counters covered by the learner's hand, encourages learners to use the count on strategy.

Linking game strategies and fluencies to written work

Simple games like 'Fizz Pop' (discussed earlier) are useful for getting learners fluent in basic facts. However in the absence of explicit links, learners might not connect these fluencies to related concepts or to the same idea in written form. For example, we have seen learners who have confidently and fluently worked with bonds of 10 in 'Fizz Pop' revert to counting in ones when asked to do 6 + 4 or be unable to



Source: Maths Clubs, 2023, *Maths clubs facilitator guide*, OLICO Mathematics Education

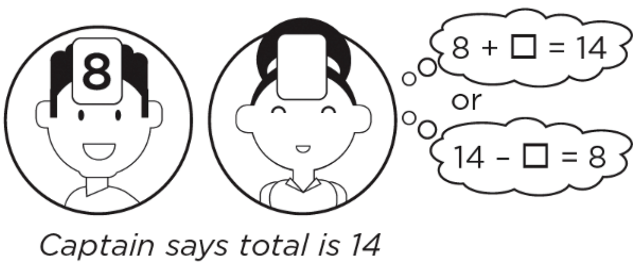
FIGURE 3: Graphic showing 123 show compare in a maths club facilitator guide.

Discourage guessing.

The captain says 14.

How do you get the answer?

You can add or subtract to get the answer.



Source: Maths Clubs, 2023, *Maths clubs facilitator guide*, OLICO Mathematics Education

FIGURE 4: Graphic showing 'Salute' in a maths club facilitator guide.

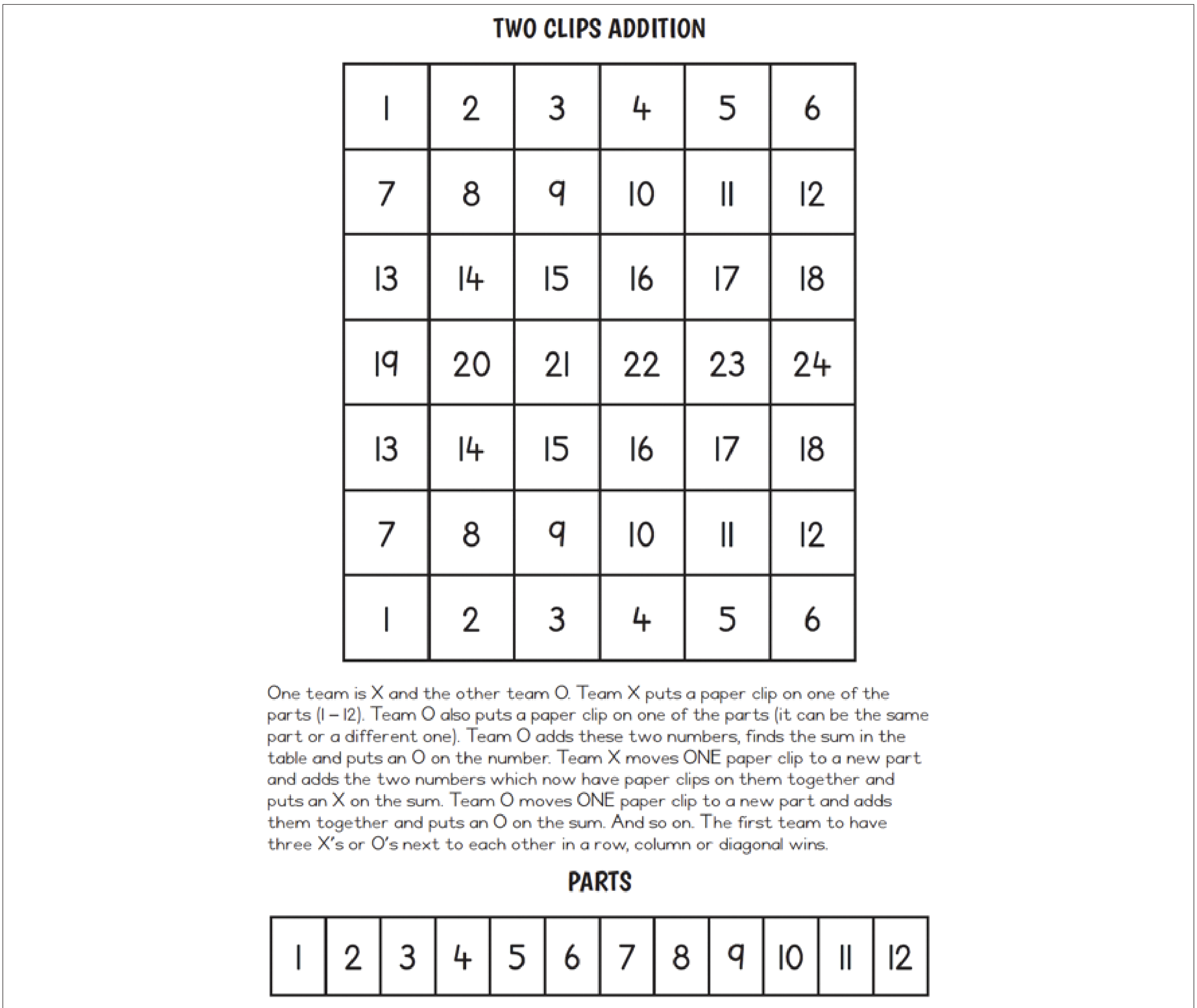
answer 6 + □ = 10 on a worksheet. It is thus important to help learners see the connections. Drawing on the design principles suggested by Porteus (2022) who recommends embedding instructional suggestions to the teacher in the learners' workbook, we include 'talking heads' (see Figure 6) on learner worksheets that emphasise the links between what learners have worked on in the games and the tasks on the worksheet. We thus include such written or worksheet type activities in our clubs and for home use.

It is also possible to design worksheet activities so that they mirror the game and thus reinforce the strategies used in the game. For example, following the 'How many children in my house' game, a set of worksheet activities similar to the one shown in Figure 7 can be used to reinforce the count on strategy used in the game.

It is particularly powerful if the connections between game and strategy can be built into an activity. For example, the game 'Clap up and down in tens' involves learners working with a partner and, along with a pat-a-cake type clapping routine, count up (or down) in tens starting from any two-digit number. This game is followed by an activity where learners add or subtract two two-digit numbers using the number-ten (NT) method (Askew 1998), where the first number is kept intact and the second number is partitioned into tens and ones for example 34 + 57 = 34 + 50 + 7 = 84 + 7 = 91.

Source: Maths Clubs, 2023, *Maths clubs facilitator guide*, OLICO Mathematics Education

FIGURE 6: Graphic showing 'Talking head' in a maths club facilitator guide.



TWO CLIPS ADDITION

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
13	14	15	16	17	18
7	8	9	10	11	12
1	2	3	4	5	6

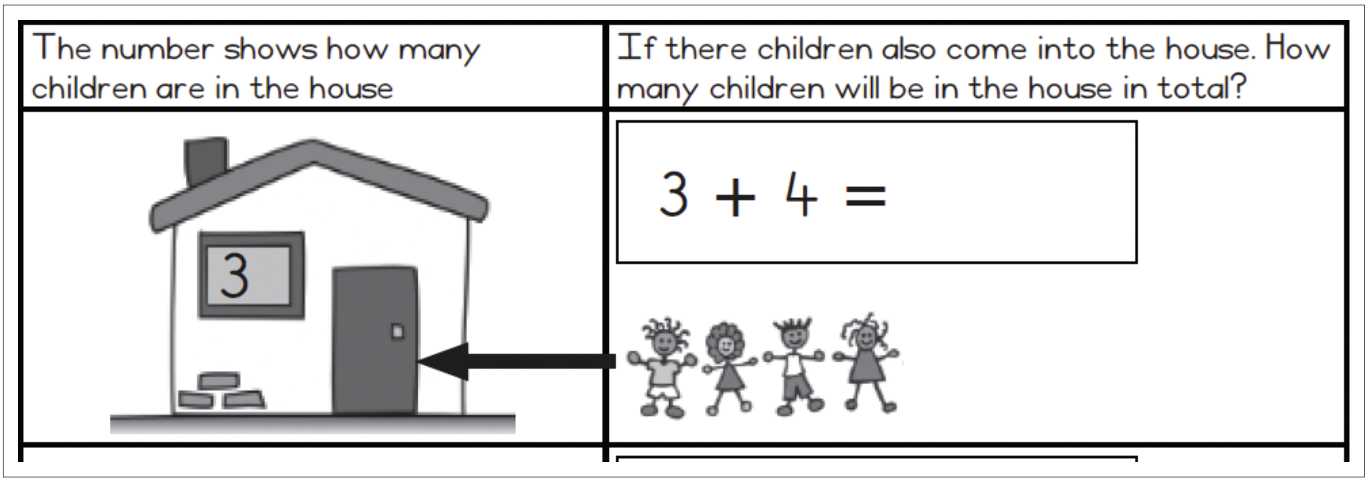
One team is X and the other team O. Team X puts a paper clip on one of the parts (1 – 12). Team O also puts a paper clip on one of the parts (it can be the same part or a different one). Team O adds these two numbers, finds the sum in the table and puts an O on the number. Team X moves ONE paper clip to a new part and adds the two numbers which now have paper clips on them together and puts an X on the sum. Team O moves ONE paper clip to a new part and adds them together and puts an O on the sum. And so on. The first team to have three X's or O's next to each other in a row, column or diagonal wins.

PARTS

1	2	3	4	5	6	7	8	9	10	11	12

Source: Maths Clubs, 2023, *Maths clubs facilitator guide*, OLICO Mathematics Education

FIGURE 5: Board game for 2 clips additions.



Source: Maths Clubs, 2023, *Maths clubs facilitator guide*, OLICO Mathematics Education

FIGURE 7: Graphic showing worksheet activity linked to 'how many children in my house game' in a maths club learner worksheet.

In the initial stages of the activity, learners practise calculations like 34 + 50 by clapping up in tens from 34 thus linking what they have practised in the game to the calculation they are doing.

Although prompts and suggestions can be built into the games and worksheets to help learners make the connections, ultimately teacher or club facilitator is critical in making the links, and thus it is important that they are able to see the links clearly and are encouraged to adopt a connectionist approach (Askew et al. 1997) to their teaching.

Conclusion

In this conceptual article, we have argued that games (with no or only readily available resources), played aurally, are a particularly good way to develop mental mathematics which supports the development of foundational number sense. We have shared multiple games to demonstrate key design features within them that, in our experience of using them in after-school maths clubs with learners in low economic backgrounds with access to few resources, enable the games to meet the key mathematics learning goal of developing number sense. Developing number sense through mental mathematics games requires paying attention to all of the strands of mathematical proficiency. They develop the following: fluency with basic facts and fluency in using key strategies (e.g., multiplying by 10 and using it for the strategy of multiplying by 9); conceptual understanding of concepts (e.g. part-part-whole); strategic thinking and adaptive reasoning (e.g., using rounding and adjusting for × by 9 or for adding 19, 29, 39 etc. to any number), and importantly, a productive disposition (seeing mathematics as something they can do and enjoy doing). We have argued that while some games might foreground practice of important 'rapid recall' facts, others emphasise understanding of key concepts and then using these to develop efficient mental mathematics strategies. We have argued that having built-in checks so that learners playing the games can self-assess or a 'referee' who can assess the validity of their answers is important and that for games to be transported across multiple settings (such as playing them in a taxi, on the playground or at home), they need to require minimal resources and be quick to set up. Thus, for example, aural games with no equipment are ideal, but a pair of dice or a pack of cards is also easily transportable in a learner's pocket to various settings. This transportability enables the games to be played repeatedly and enables the learning embedded in the games to continue beyond school and club spaces.

Pre-coronavirus diease 2019 (COVID-19) local research pointed to learners in low socio-economics backgrounds (which is the majority of learners) being two to three grades behind by the time they start intermediate phase. Given that post COVID-19 this lag has likely increased further, it is critical to provide additional 'time on task' in order to address these backlogs. This requires finding new spaces and times outside of school where learners can explore foundational mathematics ideas and practise these. We argue that mental mathematics games are an ideal means of enabling increased learner mathematical engagement outside of school because of their high motivational factor and because they are easily transported to multiple contexts and can be played with peers and family members across age ranges. Such games also maximise opportunities for learners to talk mathematics in social settings which, from our socio-cultural perspective on learning, is key to enabling meaningful learning and sense-making.

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Competing interests

The authors declare that they have no financial or personal relationship(s) that may have inappropriately influenced them in writing this article.

Authors' contributions

L.H.B., and M.H.G both contributed to the conceptualising and writing of the article.

Ethical considerations

This article followed all ethical standards for research without direct contact with human or animal subjects.

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Data availability

The data that support the findings of this study are not openly available due to human data and are available from the corresponding author, L.H.B., upon reasonable request.

Disclaimer

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