GEOGEBRA, A DYNAMIC SOFTWARE FOR CONCEPTUAL UNDERSTANDING AND VISUALISATION – MULTI-DIRECTIONALITY OF INFLUENCE

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ABSTRACT

This study delves into the dynamic interplay between GeoGebra, a prominent interactive digital tool, and science, technology, engineering, and mathematics (STEM) cognition, focusing on teachers. Anchored by the objective to unravel how GeoGebra influences conceptual understanding and visualisation, the research hypothesises that GeoGebra significantly influences cognitive outcomes in STEM cognition. Thus, aiming to assess the hypothesis that GeoGebra significantly influences cognitive outcomes, a Multi-Group Analysis (MGA) using Partial Least Squares-Structural Equation Modelling (PLS-SEM) was employed. Data was collected through a survey designed to capture the experiences and interactions of 71 teachers with GeoGebra. The significant direct pathways (Analogical Comparison Principle (ACP) and Error and Misconceptions Reflection Principle (EMR) and between Mathematical and Computational Algorithms (MCA) and Analogical Comparison Principle (ACP)) established empirical evidence to the theory that GeoGebra can facilitate conceptual understanding as well as cognitive development. Additionally, the indirect pathways with no significant effects such as Mathematical Cognition (MAS) to ACP to EMR was indicative of the fact that influence of GeoGebra operates more through direct interactions with the software as opposed to through mediated learning processes. The conclusion is that influence of GeoGebra is not uniform across all contexts, indicative of the fact that factors such as geographical location play a critical role in shaping the software's effectiveness in enhancing STEM education. In sum, while GeoGebra is a potent interactive digital tool for enhancing STEM cognition, the results call for tailored strategies in integrating digital tools like GeoGebra, by considering the specific needs of different learning context, particularly paying attention to the contrast between urban and rural educational settings.

Keywords: GeoGebra, STEM Education, Cognitive Outcomes, Digital Learning Tools

INTRODUCTION AND LITERATURE REVIEW

GeoGebra is dynamic mathematics software predominantly used as a learning and teaching tool in science, technology, engineering as well as mathematics (STEM) from both gender and age perspectives (see Figure 1) (GeoGebra 2011; GeoGebra nd.). For instance, Figure 1 illustrates the interface of the GeoGebra software showing the exact cubic solver tool. The solver provides a visual representation of the cubic equation along with its solutions.

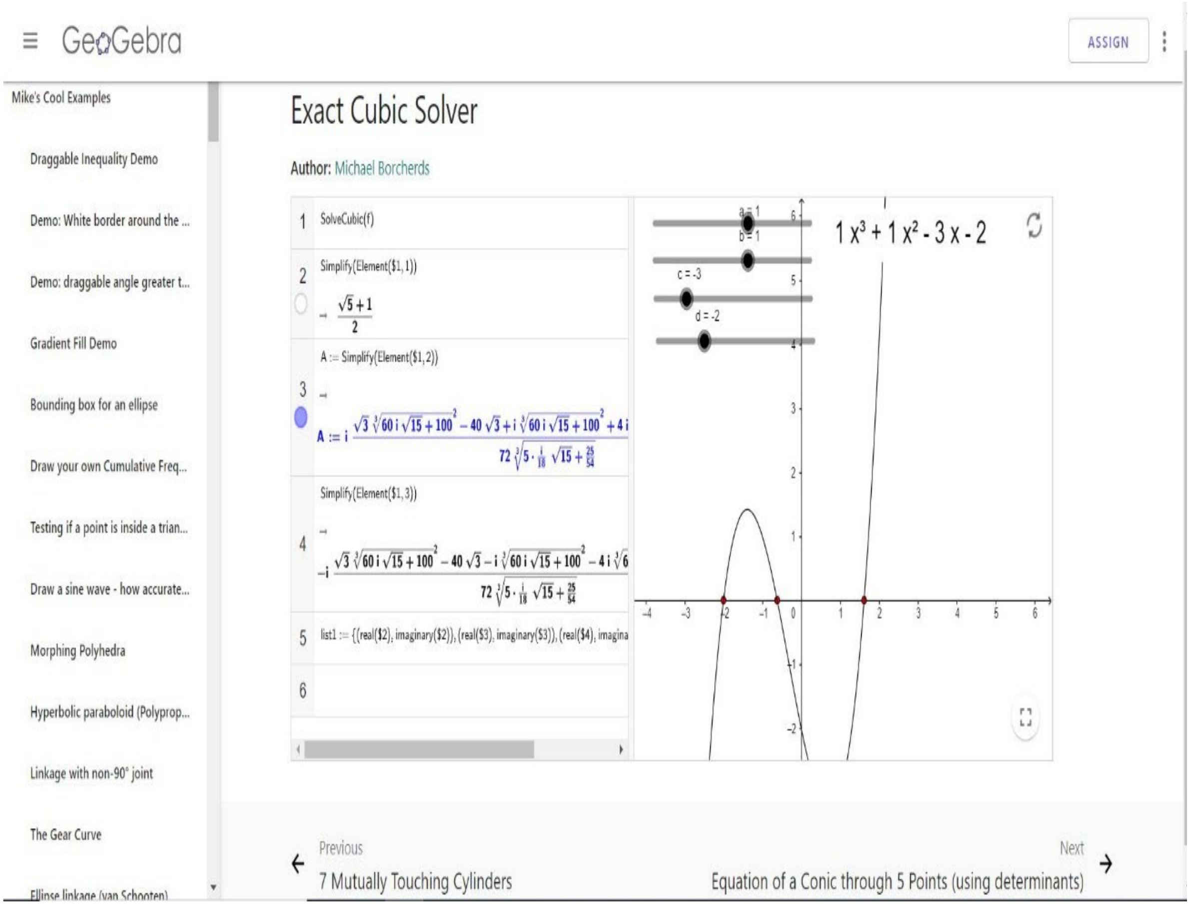

Figure 1: GeoGebra Interface Displaying an Exact Cubic Solver Function (adapted from GeoGebra nd.).

Since Geogebra's development, particularly with urban learners, different studies have already been conducted regarding the benefits from different dimensions such as school level or even higher education and in different disciplines (Engineering, Physics and Mathematics). A recent study focusing on urban leaners by Alkhateeb and Al-Duwairi (2019) examined GeoGebra and Sketchpad influence on the students' performance. In Physics, Machromah, Purnomo, and Sari (2019) demonstrated how understanding of calculus with GeoGebra at college does enhance cognitive abilities such as visualisation. Ishartono et al. (2022) examined the integration of Geogebra into the flipped learning to improve students' self-regulated learning as with Kushwaha, Chaurasia and Singhal (2013), who examined dynamic webpage for GeoGebra quiz.

Aforementioned studies have supported the work of Escuder and Furner (2011), which aimed at examining GeoGebra's influence on professional development of mathematics. As with the work of Escuder and Furner (2011), Faradiba, Abidin and Khasanah (2023) used the teaching of derivative through GeoGebra to demonstrate enhancement of teacher's professional development. Other studies include Suweken (2019), who also examined STEM-oriented mathematics learning with GeoGebra, while, Marciuc, Miron, and Barna (2016), examined application of Geogebra in the pedagogy of oscillatory motions. Mussoi (2011) on the other hand examined GeoGebra learning and teaching in both Physics and Mathematics.

Majority of the studies including Escuder and Furner (2011), Faradiba et al. (2023), Marciuc et al. (2016), Machromah et al. (2019) as with Velikova and Petkova (2019) have demonstrated the efficacy of interactive mathematics software such as GeoGebra as an emerging learning technology. Some evidence also supported Navetta (2016) on visualising functions of complex numbers using GeoGebra as well as modelling the geographical studies with GeoGebra-software. Suryawan and Permana (2020) and Walsh (2017) created interactive physics simulations using GeoGebra for example to provide support on how the GeoGebra software enhance teachers' professional development.

Following the earlier studies, Birgin and Yazıcı (2021) in recent times too confirmed the effectiveness of GeoGebra software and thus evidenced the support for instruction method on 8th graders' conceptual understanding and retention. Additionally, Birgin and Yazıcı (2021) evidenced collaborative learning using GeoGebra software on 11th grader in exponential as well as logarithmic functions. So did Sari, Hadiyan, and Antari (2018), by exploring derivatives by means of GeoGebra. Additional research from Maskur (2020) has been conducted on the effectiveness of problem-based learning and improving computational skills on curriculum as well as analysis of learning of computational representation (Septian and Prabawanto, 2020), through GeoGebra-assisted project-based models and various computational principles such as but not limited to Mathematical and Computational Algorithms (MCA), Mathematical Principles (MP), Mathematical Modelling and Simulation (MMS), Analogical Comparison Principle (ACP), and Error and Misconceptions Reflection Principle (EMR):

In the context of MCA, the development and application of algorithms to solve mathematical problems may differ based on the educational setting. In rural areas with limited access to technology, there may be a hindrance to the widespread application of computational algorithms. Thus, raising debatable issues whether there should be emphasis on mathematical algorithms without heavy reliance on AI-enabled techniques and tools is more suitable for rural learners. The same could be compared to urban learners with reasonable availably of AI-enabled environment, may thus be able to integrate advanced computational algorithms to facilitate efficient problem-solving. On the other hand, foundation of mathematical reasoning together with problem-solving which is termed as MP, tends to prompt different consideration in different settings. As with MP and MCA, the application of MMS, ACP, and EMR do lay fundamental question whether learning and understanding from example mistakes and misconceptions (EMR) are effectively achieved in different educational settings? Such debates tend to add more nuances to the discourse of STEM cognition by acknowledging the potential variations in approach and outcome across diverse settings.

In conclusion, in rural settings, mathematical education emphasises basic principles and practical applications, using analogies rooted in familiar activities. Urban education, with a focus on abstract principles and diverse models, leverages a wide range of analogies but may require structured approaches for addressing errors and misconceptions due to larger class sizes (Septian and Prabawanto 2020).

Yet, many gaps remained rife for further research particularly from different geographical location such as rural verse urban learners. The key being examination of GeoGebra as a dynamic software for conceptual understanding and visualisation from a multi-directionality of influence or interaction between the components' perspective from rural learners' perspective compared with that of urban. For instance, we do not fully comprehend the interaction between the components; MAS, MCA, MP, MMS, ACP, and EMR from both settings (rural and urban). While there can be multiple directions of influence, as summarised in Table 1, we do not comprehend how MAS affects ACP, which intends leads to MM, nor do we understand the relation between MAS and MP and potential consequence on MMS.

Table 1: Overview of Research on GeoGebra in STEM Education

Theme	Research Objective	Hypothesis	Main Research Gaps	Associated Sources
Exploration of the dynamic relationship between GeoGebra, a digital tool, and STEM cognition	To investigate how GeoGebra influences conceptual understanding and visualisation in STEM education, and to assess its impact on cognitive outcomes.	GeoGebra significantly impacts cognitive outcomes in STEM education, with possible differing effects in urban versus rural educational contexts.	– Insufficient understanding of the interaction between components like MAS, MCA, MP, MMS, ACP, and EMR. – Lack of comprehension of the multi-directionality of influence among these components. – Limited research on the differential impact of GeoGebra in urban and rural settings. – Scarcity of substantial evidence on the complex interplay between digital technologies like GeoGebra in STEM cognition and student outcomes. – Need for integrated approaches in GeoGebra usage for effective STEM education.	GeoGebra (2011; nd), Alkhateeb and Al-Duwairi (2019), Escuder and Furner (2011), Faradiba, Abidin, and Khasanah (2023), Suweken (2019) – Marciuc, Miron, and Barna (2016), Mussoi (2011), Velikova and Petkova (2019), Navetta (2016), Birgin and Yazıcı (2021), Sari, Hadiyan, and Antari (2018), Maskur (2020), Septian and Prabawanto (2020), Arbain and Shukor (2015), Dwiranata, Pramita, and Syaharuddin (2019), Noor and Timeless (2022), Samura (2023), Suryawan and Permana (2020)

Other hypothetical standpoint worthy of research include how MMS influences MAS, which consequently affects ACP. In the proposed model therefore, the directional arrows signify the hypothetical influence of each construct on the others. MAS is shown to potentially impact ACP, MCA, and MP. MCA has a central role, with hypothesised direct effects on all other constructs, including MP, which is posited to influence EMR, as well as having a direct effect on EMR itself. MCA also is theorized to affect ACP, which in turn may impact EMR. MMS is represented as influencing MP and EMR both directly and indirectly. The multi-group analysis (MGA) approach enables the examination of these relationships separately within urban and rural groups, allowing for the assessment of whether and how these hypothesised paths may differ based on the geographical context of the respondents. This analytical approach provides insights into the potential variability of the structural model's dynamics across different residential settings.

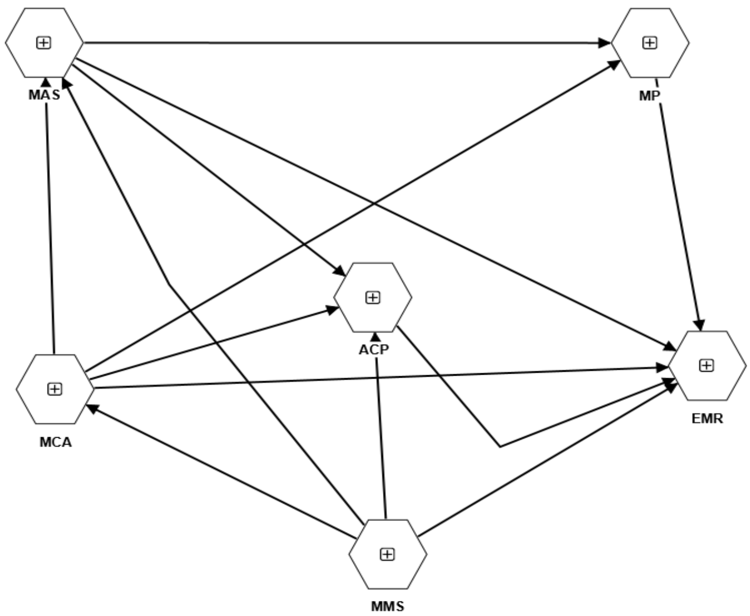

Figure 2: Multi-directionality of influence model for conceptualisation and visualisation of dynamic Software (GeoGebra) – Own model

Additionally, we are yet to comprehend the context of multi-group analysis using encompassing the potential differences in path relationships between urban and rural groups within the structural model, hence the following research objective. A potential hypothesis can thus be tested using MGA to evaluate whether the path coefficients for these relationships are significantly different between the two groups, thereby providing insights into the moderating effect of the residence on the structural model's dynamics.

Collectively, the literature indicates numerous unexplored areas, especially concerning geographical disparities such as those between urban and rural learners. The interactions between components such as MAS, MCA, and others remain insufficiently understood.

These gaps in literature underscore the need to evaluate the influence of GeoGebra on pre-service teachers' pedagogical content knowledge and their capacity to design engaging STEM lessons for a diverse student body.

In fact, Arbain and Shukor (2015) noted the scarcity of substantial evidence on the complex interplay between digital technologies like GeoGebra in STEM cognition and student outcomes. While Alkhateeb and Al-Duwairi (2019) explored calculus concept construction through GeoGebra-assisted learning, and Dwiranata, Pramita, and Syaharuddin (2019) developed interactive 3D mathematics materials for Android, the overarching consensus is that the multifaceted influence of GeoGebra remains under-researched. Though, we know through the study of Septian et al. (2020) that GeoGebra-assisted problem based learning does improve mathematical problem-solving ability, just as student's mathematical connection ability through GeoGebra assisted project-based learning model (Septian, 2022). The aforementioned studies highlight need for learners' conceptual knowledge development and attitudinal change towards using GeoGebra (Yimer and Feza 2020).

Amidst the aforementioned positive revelations, Noor and Timeless (2022), however expressed concerns about the cultivation of higher-order thinking skills in mathematics education at the high school level, while Samura (2023) and Suryawan and Permana (2020) recognised additional hurdles in GeoGebra's application. The research body collectively underscores the potential of GeoGebra in augmenting teachers' pedagogical content knowledge, yet it also calls for an integrated approach that synthesises modelling, visualisation, and programming within GeoGebra for effective STEM education.

Therefore, this study proposes to employ MGA to investigate the varied influences of GeoGebra as a dynamic educational tool. The literature suggests that these components can be quantified through direct and indirect observations, with potential moderation and mediation effects. Figure 2 depicts a summarised structural model employing MGA, segmenting the sample into urban and rural subgroups.

Research objective

The study's objective is to investigate the complex interplay between digital technologies (GeoGebra) in STEM cognition and students' cognitive outcomes. Understanding these relationships will inform the development of targeted interventions to enhance STEM education. Through survey design, the research aims to investigate the effectiveness of GeoGebra as a digital technological as a tool for conceptual understanding and visualisation in teaching to preservice teachers.

Hypotheses

The relationship strengths among MAS, MCA, MP, MMS, ACP, and Error and EMR will differ between urban and rural residents within the structural model.

Specific pathways, such as MAS to ACP, MCA to MP, and MMS to EMR, including indirect influences through MP on EMR, will demonstrate significant variations in impact when comparing urban to rural contexts.

These principles are hypothesised to exert differential effects on learners from varied residential backgrounds (urban vs. rural), leading to distinct educational outcomes when implementing GeoGebra as a dynamic software for conceptual understanding and visualisation.

RESEARCH METHODOLOGY

This research sought to evaluate the influence of AI-enhanced STEM cognition by conducting a questionnaire survey with 71 students. The questionnaire, divided into two sections, delved into participants' socio-demographic backgrounds, drawing from existing literature. A questionnaire was developed through a four-point Likert scale and directed by the core hypothesis which posits that these factors exert differing influences on learners in urban and rural settings, leading to varied outcomes during the utilisation of GeoGebra-a dynamic software designed for conceptual understanding and visualisation. Guided by Hair et al.'s (2022) minimum size of 50, and to ensure accurate data collection process, the 71 survey design questionnaires were tested for both convergent and discriminant validity accuracy (see results section for details). In terms of ethical consideration, respondents' privacy and confidentiality and anonymisation of relevant information were protected at all times, thus ensuring that the ethical procedures followed from XXX University's Review Board in South Africa, referenced as H21-EDU-PGE-026.

Due to the hypothesis, data analysis applied PLS-SEM, with a focus on MGA-PLS. The procedure examined both direct and indirect relationships found in the hypothesised mode and guided by the conceptual framework (Figure 2). MGA-PLS was a preferred choice because of the small sample and non-normal data distribution using SmartPLS™ software 4.0.9 for the analysis. Additionally, Measurement invariance across groups was verified in stages, assessing configural, compositional invariance, and equality of composite means and variances. Subsequently, path coefficients were compared across groups via the Henseler PLS-MGA method to detect significant differences (Fornell and Larcker 1981; Hair et al. 2022).

DATA ANALYSIS RESULTS

Measurement model assessment

In the assessment of the measurement model (Table 2), the constructs demonstrated satisfactory levels of reliability and convergent validity. Cronbach's alpha (α) and composite reliability (CR) were employed for reliability assessment, while the average variance extracted (AVE) was used to evaluate convergent validity. For the complete sample, the ACP construct indicated high reliability (α = .88, CR = .88) and good convergent validity (AVE = .68). The EMR also showed strong reliability (α = .86, CR = .87) and convergent validity (AVE = .64). The MCA construct had slightly lower reliability (α = .83, CR = .85), with an AVE of .60, which is acceptable for convergent validity. The MP construct recorded the lowest reliability (α = .75, CR = .75), but an adequate AVE (.67) suggested sufficient convergent validity. Subgroup comparisons revealed the following constructs: urban -ACP showed strong reliability (α = .86, CR = .89) associated with a convergent validity (AVE = .64). In contrast, rural-demonstrated higher reliability (α = .90, CR = .90) with associated convergent validity (AVE = .71). The EMR construct maintained consistent reliability and convergent validity across urban (α = .85, CR = .87, AVE = .63) and rural (α = .87, CR = .88, AVE = .66) subgroups. The MCA construct presented steady reliability and convergent validity (AVE = .60) in both subgroups (urban: α = .83, CR = .88; rural: α = .84, CR = .87).

Table 2: Reliability and convergent validity

Constructs	Complete			Urban			Rural		
	Alpha	CR	AVE	Alpha	CR	AVE	Alpha	CR	AVE
ACP	0.88	0.88	0.68	0.86	0.89	0.64	0.9	0.9	0.71
EMR	0.86	0.87	0.64	0.85	0.87	0.63	0.87	0.88	0.66
MCA	0.83	0.85	0.6	0.83	0.88	0.6	0.84	0.87	0.6
MP	0.75	0.75	0.67	0.75	0.83	0.67	0.76	1.17	0.63

The measurement model's discriminant validity was assessed using the Heterotrait-Monotrait ratio (HTMT) criterion. For the complete sample, the HTMT value between the EMR and ACP constructs was close to the threshold (0.89). In the urban sample, this threshold was exceeded (HTMT = 0.96), suggesting potential issues with discriminant validity, while the rural sample showed better validity (HTMT = 0.86). When comparing the MCA construct with ACP, acceptable discriminant validity was indicated in the complete (HTMT = 0.77) and rural (HTMT = 0.73) samples. However, the urban sample showed a higher value (HTMT = 0.91), hinting at possible overlap. The MCA construct demonstrated strong discriminant validity with the MAS construct across all samples (Complete: HTMT = 0.06; Urban: HTMT = 0.27; Rural:

HTMT = 0.2). For MAS ↔ ACP, low HTMT values (Complete: 0.12; Urban: 0.2; Rural: 0.26) suggested strong discriminant validity. This was similarly observed for MAS ↔ EMR, MMS ↔ ACP, MMS ↔ EMR, and MMS ↔ MAS.

However, the MP construct, particularly in the urban sample, showed high HTMT values with EMR (HTMT = 0.94) and ACP (HTMT = 0.78), indicating possible issues with discriminant validity. In the rural sample, MP maintained strong discriminant validity (e.g., MP ↔ ACP: HTMT = 0.27). Using the Fornell-Larcker criterion, discriminant validity was confirmed in the complete dataset, with square roots of AVEs (ACP = .83, EMR = .80, MAS = 1.00, MCA = .77, MMS = 1.00, MP = .82) exceeding inter-construct correlations. Challenges were noted in the urban sample, especially between ACP (AVE = .80) and EMR (correlation = .86), and between MCA (AVE = .78) and EMR (correlation = .78). In the rural sample, discriminant validity was maintained across all constructs. These results indicate generally distinct constructs (Hair et al. 2022).

In the study (Table 3), the R-square (R^2) and adjusted R-square (R^2 adjusted) values revealed the model's varied explanatory power across different samples. For the Analogical Comparison Principle (ACP), the complete dataset showed an R^2 of 0.56 (adjusted R^2 = 0.54), while the urban sample had a higher R^2 of 0.75 (adjusted R^2 = 0.70), and the rural sample was like the complete dataset. The EMR demonstrated strong R^2 (complete (0.68) and urban (0.86)), associated with slightly lower adjusted R^2s. However, MAS and MCA showed minimal explanatory power across all samples while MP had moderate explanatory power in the urban sample (R^2 = 0.32, adjusted R^2 = 0.25), however lower in complete and rural samples. The results reveal significant variation in explanatory power, signifying the importance of context in analysing these results.

Table 3: R-Square

Complete			Urban		Rural	
	R-square	**R-square adjusted**	**R-square**	**R-square adjusted**	**R-square**	**R-square adjusted**
ACP	0.56	0.54	0.75	0.7	0.54	0.5
EMR	0.68	0.66	0.86	0.81	0.68	0.64
MAS	0.02	-0.01	0.1	0	0.05	0
MCA	0.02	0.01	0	-0.05	0.06	0.04
MP	0.11	0.09	0.32	0.25	0.02	-0.02

Assessment of structural model

The Variance Inflation Factor (VIF) examined potential collinearity within the structural model across complete, urban, and rural samples. Results suggest that collinearity is not a significant concern across samples (Hair et al. 2022). Across complete, urban, and rural samples, as detailed in Table 4, the structural model was evaluated. In the complete sample, the path from ACP to EMR demonstrated a strong positive relationship (β = 0.95, SD = 0.10, t = 9.68, p < .001) indicating substantial positive influence of ACP on EMR. However, the urban exhibited a weaker, albeit significant, relationship for the same path (β = 0.58, SD = 0.29, t = 2.02, p = .04), and the rural sample demonstrating a similarly strong influence as the complete sample (β = 1.02, SD = 0.14, t = 7.02, p < .001). For the path from MAS) to ACP, the relationship was not statistically significant in the complete (β = 0.08, SD = 0.09, t = 0.88, p = .38) and urban samples (β = 0.06, SD = 0.12, t = 0.52, p = .60), with a marginally stronger but still non-significant relationship in the rural sample (β = 0.15, SD = 0.13, t = 1.13, p = .26).

Interestingly, the path from MCA to ACP was significant and strong in all samples, with the highest beta coefficient observed in the urban sample (β = 0.86, SD = 0.09, t = 9.32, p < .001), suggesting that MCA significantly predicts ACP, especially in urban contexts. The relationship between MMS and ACP was also significant in the complete sample (β = 0.29, SD = 0.09, t = 3.27, p < .001), indicating a moderate positive influence. However, this relationship was not as strong in the urban and rural samples. Other pathways, such as MAS -> EMR, MAS -> MP, and MMS -> MAS, showed non-significant relationships across all samples, indicating that these predictors do not significantly influence the respective outcome variables in this model.

Meditated relationships – Total indirect and specific indirect effects

Mediated relationship through total indirect and specific indirect effects across complete, urban, and rural samples were evaluated using the beta coefficients (β), standard deviations (SD), t-values, and p-values (Hair et al. 2022).

Total indirect effects as of complete sample path from MAS to EMR through other constructs indicated a non-significant effect (β = 0.07, SD = 0.09, t = 0.76, p = 0.45). Similarly, for MCA influencing ACP, the indirect effect was also non-significant (β = 0.01, SD = 0.02, t = 0.37, p = 0.71).

Notably, a significant indirect effect was observed for MCA -> EMR in the complete (β = 0.66, SD = 0.08, t = 8.84, p < .001) and urban samples (β = 0.68, SD = 0.24, t = 2.79, p = 0.01), but with a slightly weaker effect in the rural sample (β = 0.58, SD = 0.12, t = 4.91, p < .001).

Specific indirect effects path for MAS -> ACP -> EMR demonstrated non-significant effects across all samples (e.g., complete sample: β = 0.07, SD = 0.09, t = 0.86, p = 0.39). So was MMS -> MCA -> ACP -> EMR and MMS -> MCA -> MAS -> MP showing non-significant effects, an indication that these specific mediated pathways do not significantly contribute to the endogenous constructs.

Table 4: Direct relation

	Complete				Urban				Rural			
	Beta	**SD**	**T value**	**P values**	**Beta**	**SD**	**T values**	**P values**	**Beta**	**SD**	**T values**	**P values**
ACP -> EMR	0.95	0.1	9.68	0	0.58	0.29	2.02	0.04	1.02	0.14	7.02	0
MAS -> ACP	0.08	0.09	0.88	0.38	0.06	0.12	0.52	0.6	0.15	0.13	1.13	0.26
MAS -> EMR	-0.02	0.08	0.27	0.79	0.09	0.14	0.69	0.49	-0.09	0.11	0.78	0.43
MAS -> MP	-0.02	0.15	0.14	0.89	0.01	0.2	0.03	0.97	0.03	0.2	0.16	0.87
MCA -> ACP	0.65	0.07	9.18	0	0.86	0.09	9.32	0	0.54	0.09	5.72	0
MCA -> EMR	-0.18	0.13	1.45	0.15	0.1	0.28	0.35	0.72	-0.21	0.18	1.18	0.24
MCA -> MAS	0.08	0.13	0.59	0.56	-0.2	0.22	0.93	0.35	0.22	0.18	1.23	0.22
MCA -> MP	0.34	0.16	2.19	0.03	0.57	0.16	3.45	0	0.14	0.25	0.58	0.56
MMS -> ACP	0.29	0.09	3.27	0	0.24	0.16	1.45	0.15	0.33	0.12	2.83	0
MMS -> EMR	-0.24	0.09	2.73	0.01	-0.12	0.19	0.63	0.53	-0.26	0.1	2.48	0.01
MMS -> MAS	-0.13	0.12	1.1	0.27	-0.26	0.2	1.32	0.19	-0.07	0.16	0.45	0.65
MMS -> MCA	0.14	0.13	1.08	0.28	-0.06	0.26	0.23	0.82	0.25	0.15	1.71	0.09
MP -> EMR	0.13	0.09	1.44	0.15	0.37	0.19	1.97	0.05	0.11	0.12	0.91	0.36

In contrast, the MCA -> ACP -> EMR path showed a significant effect in the complete sample ($\beta = 0.62$, SD = 0.08, t = 7.97, $p < .001$), indicating a strong mediation effect through ACP on EMR with less pronounced effect in the urban and rural samples.

Both the direct and indirect pathways demonstrate the complexity of the relationships across different population samples. The variability reflects the significance of contextual factors in comprehending the mediated relationships.

Assessment of Measurement Invariance of Composite Models (MICOM)

In this MGA using PLS, the assessment of measurement invariance of composites (MICOM) revealed several key findings (Table 5). The original correlations for ACP, EMR, MAS, MCA, MMS, and MP were very high (ranging from 0.93 to 1.00), indicating strong positive relationships. These were consistent with the correlation permutation means, which were equally high. In MICOM Configural Invariance (Step 2), permutation p-values varied, with ACP (p = .03), EMR (p = .25), MAS (p = .02), MCA (p = .41), MMS (p = .61), and MP (p = .30), suggesting that for some variables, the original correlations were not significantly different from the permutation correlations. For MICOM Composite Mean Invariance (Step 3a), the original and permutation mean differences in means were found to be small and within narrow confidence intervals for all variables. In MICOM Composite Variance Invariance (Step 3b), examining the variance, a similar pattern was observed with minimal differences between the original and permutation mean differences, as indicated by the permutation p-values (ACP: p = .34, EMR: p = .95, MAS: p = .44, MCA: p = .98, MMS: p = .19, MP: p = .82). These results suggest a strong and consistent relationship across the variables, with the observed correlations and mean differences being stable and not significantly different from those obtained through permutation, indicating the robustness of these relationships (Hair et al. 2022)

Bootstrap MGA, Parametric Test, Welch Satterthwaite

The Bootstrap MGA (see Table 6) compared path coefficients between urban and rural groups (Hair et al., 2022). The assessment also used parametric and Welch Satterthwaite tests. The analysis focused on differences in path coefficients (Urban – Rural), with associated one-tailed and two-tailed p-values for the parametric test and t-values and p-values for the Welch Satterthwaite test. For the path from Analogical Comparison Principle (ACP) to Error and Misconceptions Reflection Principle (EMR), a negative difference was observed (-0.44), but it was not statistically significant as indicated by the p-values (parametric

Table 5: Bootstrapping MGA – MICOM FOR MGA PLS

	MICOM STEP2				MICOM STEP3a (mean)					MICOM STEP 3b (variance)				
	Original correlation	**Correlation permutation mean**	**5.00%**	**Permutation p value**	**Original difference**	**Permutation mean difference**	**2.50%**	**97.50%**	**Permutation p value**	**Original difference**	**Permutation mean difference**	**2.50%**	**97.50%**	**Permutation p value**
ACP	1	1	1	0.03	0.07	0.01	-0.49	0.53	0.79	0.32	-0.04	-0.86	0.6	0.34
EMR	1	1	0.99	0.25	0	0.01	-0.47	0.55	0.99	-0.02	-0.04	-0.92	0.71	0.95
MAS	1	1	1	0.02	0.19	0.01	-0.44	0.55	0.5	0.37	-0.07	-0.93	0.77	0.44
MCA	0.99	0.99	0.97	0.41	0.34	0	-0.49	0.54	0.21	0.02	-0.07	-0.99	0.88	0.98
MMS	1	1	1	0.61	-0.05	-0.01	-0.52	0.54	0.87	0.41	-0.06	-0.71	0.58	0.19
MP	0.93	0.89	0.46	0.3	0.49	0	-0.48	0.54	0.06	0.11	-0.06	-0.94	0.74	0.82

Table 6: Bootstrap MGA, Parametric and Welch Satterthwaite Tests

	Difference (Urban - Rural)	1-tailed (Urban vs Rural) p value	2-tailed (Urban vs Rural) p value	Difference (Urban - Rural)	t value (|Urban vs Rural|)	p value (Urban vs Rural)	Difference (Urban - Rural)	t value (|Urban vs Rural|)	p value (Urban vs Rural)
ACP -> EMR	-0.44	0.93	0.14	-0.44	1.54	0.13	-0.44	1.4	0.18
MAS -> ACP	-0.08	0.68	0.63	-0.08	0.4	0.69	-0.08	0.48	0.63
MAS -> EMR	0.18	0.14	0.29	0.18	0.96	0.34	0.18	1.05	0.31
MAS -> MP	-0.03	0.53	0.93	-0.03	0.08	0.94	-0.03	0.09	0.93
MCA -> ACP	0.32	0.01	0.02	0.32	2.1	0.04	0.32	2.5	0.02
MCA -> EMR	0.31	0.13	0.27	0.31	0.96	0.34	0.31	0.96	0.35
MCA -> MAS	-0.42	0.93	0.14	-0.42	1.38	0.17	-0.42	1.53	0.14
MCA -> MP	0.42	0.07	0.13	0.42	1.08	0.28	0.42	1.44	0.16
MMS -> ACP	-0.09	0.69	0.62	-0.09	0.45	0.65	-0.09	0.47	0.64
MMS -> EMR	0.14	0.24	0.48	0.14	0.69	0.49	0.14	0.64	0.53
MMS -> MAS	-0.19	0.77	0.46	-0.19	0.69	0.49	-0.19	0.76	0.46
MMS -> MCA	-0.31	0.85	0.31	-0.31	1.12	0.27	-0.31	1.06	0.3
MP -> EMR	0.25	0.12	0.24	0.25	1.15	0.26	0.25	1.16	0.26

one-tailed p = 0.93, two-tailed p = 0.14; Welch Satterthwaite p = 0.18). This suggests no significant difference between urban and rural groups for this path. In the comparison of MAS to ACP, a slight negative difference was noted (-0.08), but again, it was not significant (parametric p = 0.63; Welch Satterthwaite p = 0.63), indicating similar relationships in both urban and rural settings. A notable result was found for the path from MCA to ACP, where a positive difference (0.32) was observed, with the Welch Satterthwaite test showing statistical significance (p = 0.02). This implies a stronger relationship in urban compared to rural settings.

Some pathways such as MAS -> EMR, MCA -> EMR, and MMS -> ACP, demonstrated differences but non-statistically significant across the tests. Such pathways included MAS -> EMR, between urban and rural groups (parametric p = 0.29; Welch Satterthwaite p = 0.31). Generally, Bootstrap MGA results suggested certain paths exhibit different strengths in urban versus rural groups, others do not significantly differ between these groups. Path MCA -> ACP highlights the potential influence of geographical in the structural model (Hair et al. 2022).

Importance-Performance Map Analysis (IPMA) within the structural model

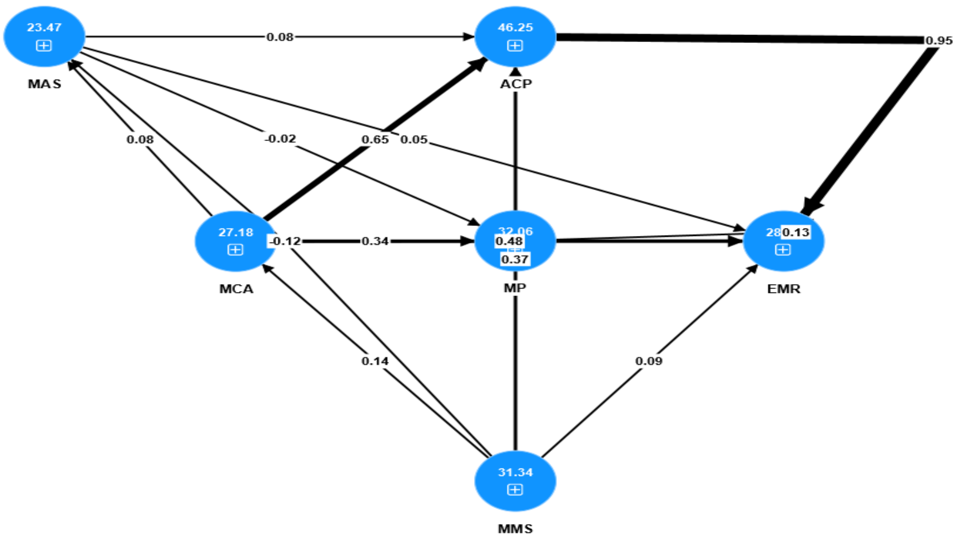

Figure 3: IPMA conceptualisation and visualisation of dynamic Software (GeoGebra)

Figure 3 illustrates an IPMA focused on the conceptualisation and visualization of dynamic software called GeoGebra. The bold lines within the figure represent relationships of greater relative importance between the constructs, as inferred from the thickness of the lines. Example, the bold line between ACP and MP suggests a strong and potentially significant relationship that warrants attention in the context of software performance. On the other hand, the unbolded lines indicate relationships of lesser importance, implying that while they may contribute to the overall model, they are not as critical as those denoted by bold lines. The numeric values near each construct, such as 46.25 for ACP and 27.18 for MCA, represent the importance scores, which reflect the relative importance of each construct in the model.

The inference is to prioritise areas for improvement by leveraging on pathways that could enhance the effectiveness of the dynamic software (Hair et al. 2022).

DISCUSSIONS

Recall that the study examined GeoGebra, a pivotal dynamic software tool in enhancing conceptual understanding and visualisation within STEM cognition.

Prior research acknowledged GeoGebra as a dynamic software across different STEM levels, demonstrating considerable benefits in cognitive abilities such as visualisation (GeoGebra 2011: n.d.). Studies, such as those by Alkhateeb and Al-Duwairi (2019), Machromah et al. (2019), and Faradiba et al. (2023) corroboratively demonstrated the positive influence of GeoGebra on student performance and teacher development. Yet, there is the suggestion that the complex interactions among mathematical cognition components, such as urban and rural learners, are not adequately researched (Arbain and Shukor 2015).

Consequently, the current hypothesis dissected the intricate interactions between various components as reflected in Figures 2 and 3, within different geographical contexts.

The results, through IPMA, indicated by the bold lines associated with ACP and MP suggested areas of high importance and performance within the software, thus requiring prioritisation for improvement (see Figure 3). Noting that the importance scores, visually represented where GeoGebra's implementation can be most effective, advocating for considerable attention to digital tool integration.

The pathways related to both direct and indirect relations which are central to understanding the structural dynamics of GeoGebra's impact on STEM cognition highlighted the multidirectional influence GeoGebra has on different components of mathematical cognition.

Direct Relationships: The analysis found strong direct relationships, such as between the ACP) and EMR, indicating that ACP has a substantial positive influence on EMR. This was particularly evident in the complete sample ($\beta = 0.95$, $p < .001$) and the rural sample ($\beta = 1.02$, $p < .001$), but was weaker in the urban sample ($\beta = 0.58$, $p = .04$). The path from MCA to ACP was also significant across all samples, with the urban sample showing a particularly strong relationship ($\beta = 0.86$, $p < .001$), suggesting that MCA is a significant predictor of ACP, especially in urban contexts.

These findings as summarised in Table 7 underscore the nuanced ways in which GeoGebra impacts cognitive outcomes in STEM education.

Table 7: Overview of the Study on GeoGebra in STEM Education

Key Issues Under Discussion	Main Gaps and Findings	Theoretical Implications	Practical Implications	Highlights	Sources
– Exploration of GeoGebra as a dynamic software tool in STEM education. – Use of MGA PLS to investigate its impact on cognitive outcomes. – Examination of complex interactions among mathematical cognition components, particularly in urban and rural contexts. – Assessment of direct and indirect relationships within the software's impact on STEM cognition.	– Gaps: Limited understanding of interactions among MAS, MCA, MP, MMS, ACP, and EMR components in different geographical contexts; lack of clarity on the multi-directional influence of GeoGebra. – Findings: Strong direct relationships like ACP to EMR; varied significance of indirect relationships like MCA to EMR across samples; differential impact based on geographical location; recognition of GeoGebra as a potent tool in STEM education.	– Digital tools like GeoGebra facilitate conceptual understanding and cognitive development. – Highlights complexity of direct vs. indirect influences on learning. – Enriches debate on digital divide and educational equity. – Suggests need for theories to accommodate variability in digital tool integration.	– Effective GeoGebra implementation enhances teaching strategies in STEM. – Focus on features aligning with ACP and MCA for cognitive outcomes. – Necessity for tailored approaches for urban and rural learners. – Context-specific teacher training and support structures. – Advocacy for ongoing research and development of digital tools.	– New insights into differential impact in urban vs. rural settings. – Original findings on direct and indirect influences of GeoGebra on STEM cognition. – Novel perspective on integration of digital tools in education.	– GeoGebra (2011; nd) – Alkhateeb and Al-Duwairi (2019) – Machromah, Purnomo, and Sari (2019) – Faradiba, Abidin, and Khasanah (2023) – Arbain and Shukor (2015)

Despite the ongoing studies, Arbain, and Shukor (2015) in the examination of the effects of GeoGebra on students' achievement bemoans lack of credible evidence in investigating the complex interplay between digital technologies in STEM cognition and students' cognitive outcomes. Even though others have explored the construction of calculus concepts through worksheet-based problem-based learning assisted by GeoGebra software (Alkhateeb and Al-Duwairi 2019) as well as development of android-based interactive mathematics learning three-dimensional material (Dwiranata et al. 2019), nevetheless, the inference is that GeoGebra as a dynamic software for conceptual understanding and visualisation through a multi-directionality of influence of interconnected variables is under researched.

Indirect Relationships: The mediated relationships were assessed through total indirect and specific indirect effects. Notably, the path from MCA to EMR showed a significant indirect effect in the complete ($\beta = 0.66$, $p < .001$) and urban samples ($\beta = 0.68$, $p = 0.01$), although it was slightly weaker in the rural sample ($\beta = 0.58$, $p < .001$). However, other hypothesised mediated paths, such as MAS to ACP to EMR, did not demonstrate significant indirect effects across the samples, indicating that these specific mediated pathways do not significantly influence the endogenous constructs.

Differential Influence Based on Geographical Location: The Bootstrap MGA and Welch Satterthwaite tests provided insights into how these paths differ between urban and rural groups. For instance, the path from MCA to ACP showed a statistically significant stronger relationship in urban compared to rural settings ($p = 0.02$), highlighting the potential influence of geographical context on the dynamics within the structural model.

The direct relationships (and significant levels) demonstrate strong influence, whereas the indirect pathways (or lack thereof) offer a more complex relationship of various components of mathematical cognition. The conclusion is that influence of GeoGebra is not uniform across all contexts, indicative of the fact that factors such as geographical location play a critical role in shaping the software's effectiveness in enhancing STEM education.

In sum, while GeoGebra is a potent digital tool for enhancing STEM cognition, the results call for tailored strategies in integrating digital tools like GeoGebra, by considering the specific needs of different learning context, particularly paying attention to the contrast between urban and rural educational settings.

CONCLUSION

The significant direct pathways (ACP and EMR, and between MCA and ACP) established empirical evidence to the theory that GeoGebra can facilitate conceptual understanding as well as cognitive development. Additionally, the indirect pathways with no significant effects such as MAS to ACP to EMR was indicative of the fact that influence of GeoGebra operates more through direct interactions with the software as opposed to through mediated learning processes.

The differential influence noted between two education settings (urban and rural) enriched the theoretical stance by highlighting the integration and effectiveness of technological tools in education with varying significance. Thus, underscoring the need to accommodate the variability and complexity of digital tool integration across different learning environments.

In sum, effective GeoGebra implementation enhances teaching strategies in STEM. There is need to focus on features aligning with ACP and MCA for cognitive outcomes. It is also necessary for tailored approaches for urban and rural learners and finally, context-specific teacher training and support structures.

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